PRAYAS

JEE 2025

Lecture - 04

Physics

Circular Motion

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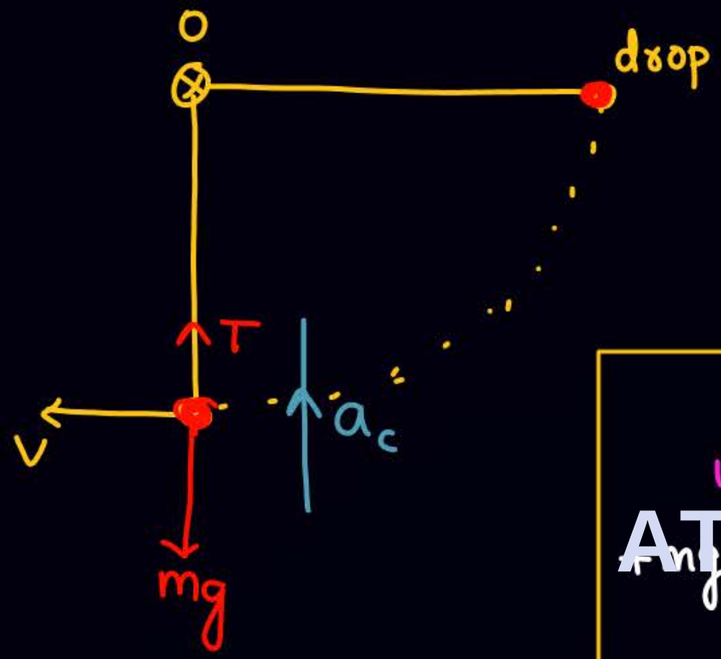
Topics to be covered

1. Circular motion dynamics
2.
3.
4.

(1)



$$T - mg = ma_c = \frac{mv^2}{R}$$

$$T = mg + \frac{mv^2}{R}$$

$$a_c = v^2/R$$

$$a_t = 0$$

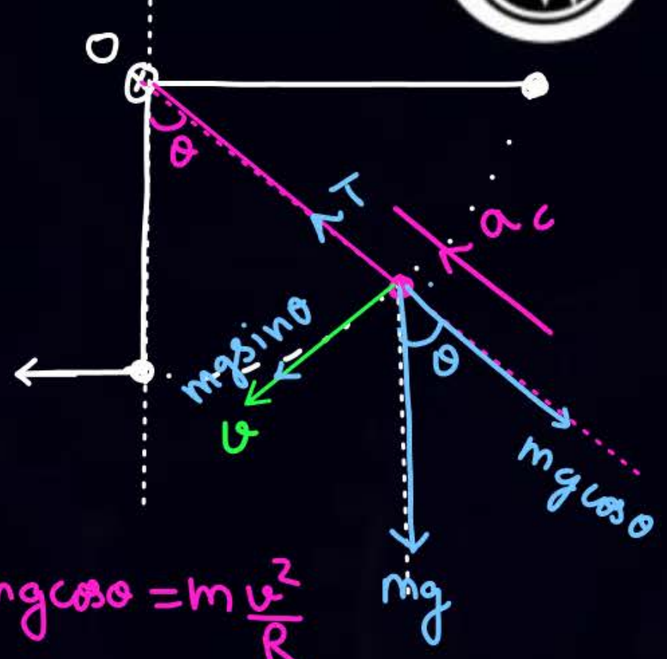
next chapter

$$W_g + W_T = \Delta K.E.$$

$$mgR + 0 = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{2gR}$$

(2) R



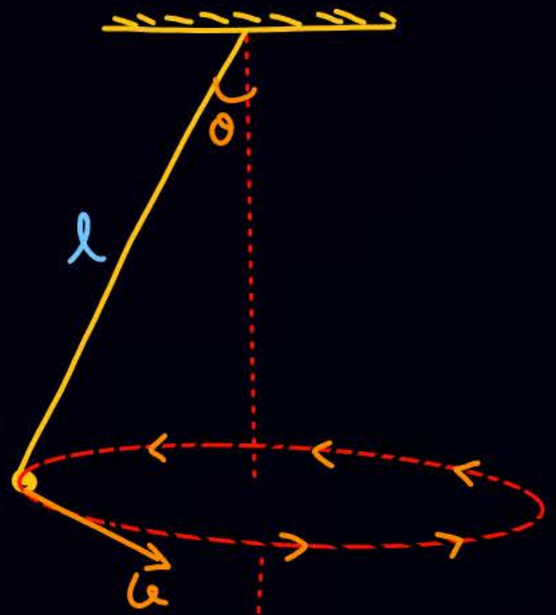
$$T - mg\cos\theta = m\frac{v^2}{R}$$

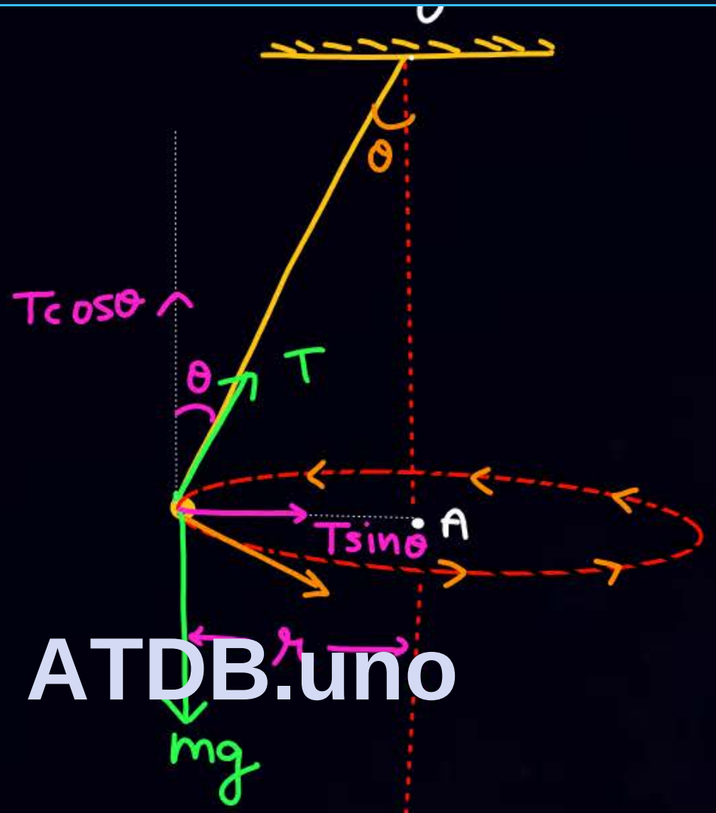
$$mg\sin\theta = ma_t$$

(3)

Conical pendulum

(moving with const speed)





$$T\cos\theta = mg$$

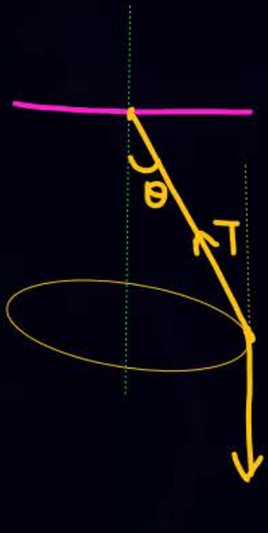
$$T\sin\theta = \frac{mv^2}{r}$$

$$T\sin\theta = mr\omega^2$$

$$\tan\theta = \frac{v^2}{rg}$$

$$T\sin\theta = m \cdot l\sin\theta \cdot \omega^2$$

$$T = ml\omega^2$$



④ A particle is moving with const speed in a circular path as shown in करोरा

$N\cos\theta = mg$

$N\sin\theta = m r\omega^2 = \frac{mv^2}{r}$

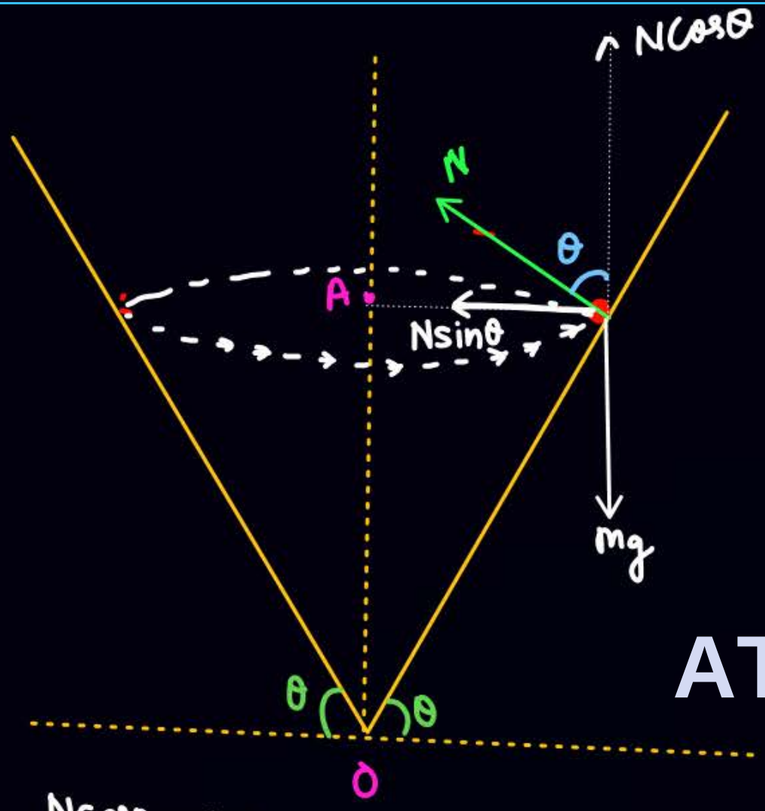
↓ radius of circle

$R\sin\theta = r$

$N\sin\theta = mR\sin\theta\,\omega^2$

$\boxed{N = mR\omega^2}$

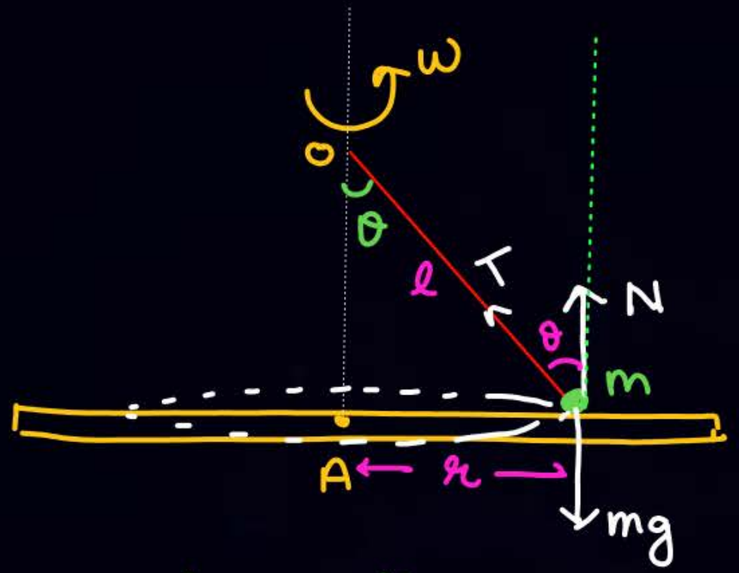
(5)



$N\cos\theta = mg$

$N\sin\theta = \frac{mv^2}{r}$

(6)



$T\cos\theta + N = mg$

$T\sin\theta = m r \omega^2$

$T\sin\theta = m l \sin\theta\, \omega^2$

$T = m l \omega^2$

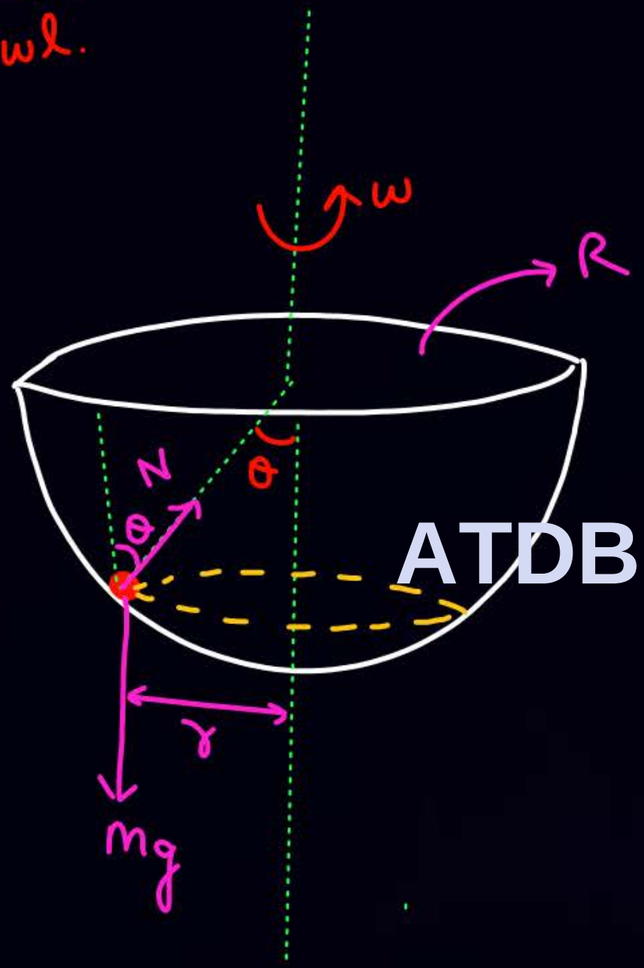
Platform is rotating with ω & mass m is at rest wrt platform. find ω at which contact loose.

put $N = 0$,

$T\cos\theta = mg$

$T\sin\theta = m.l\sin\theta.\omega^2$

(7) कटोरा Bowl is rotated with ω abt Vertical axis s.t mass m is at rest wrt Bowl.



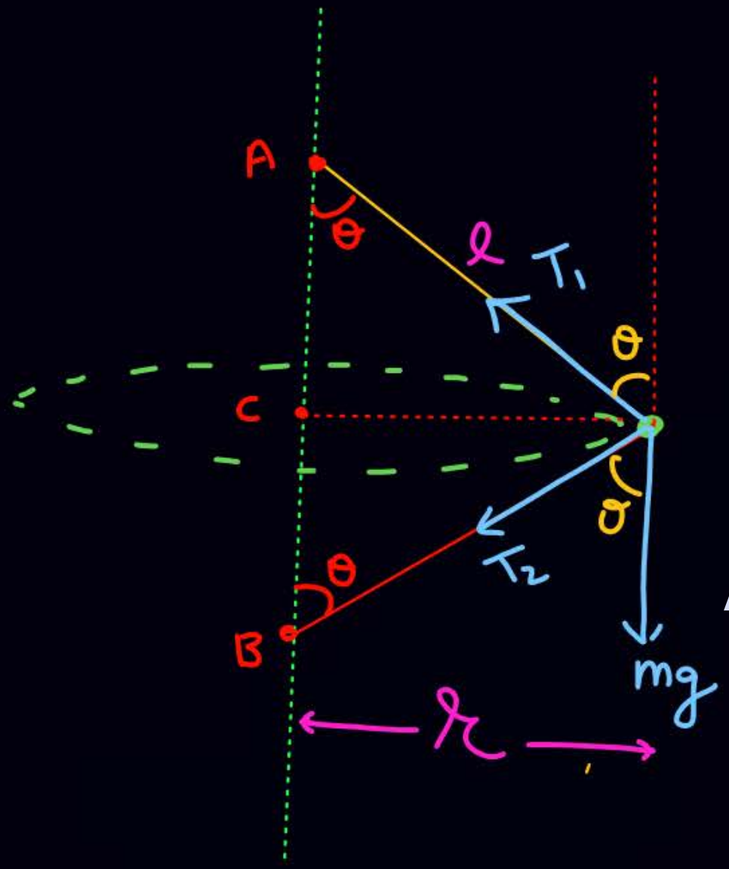
$N\cos\theta = mg$

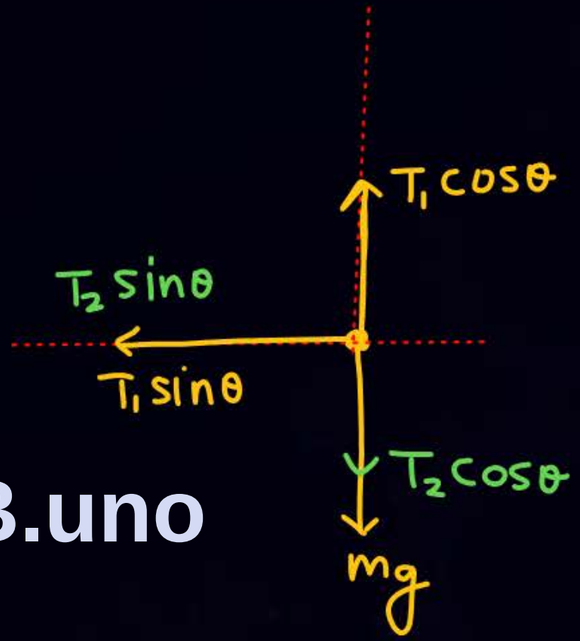
$N\sin\theta = mr\omega^2$

$R\sin\theta = r$

$N\sin\theta = m.R\sin\theta.\omega^2$

(8)





$T_1\cos\theta = mg + T_2\cos\theta$

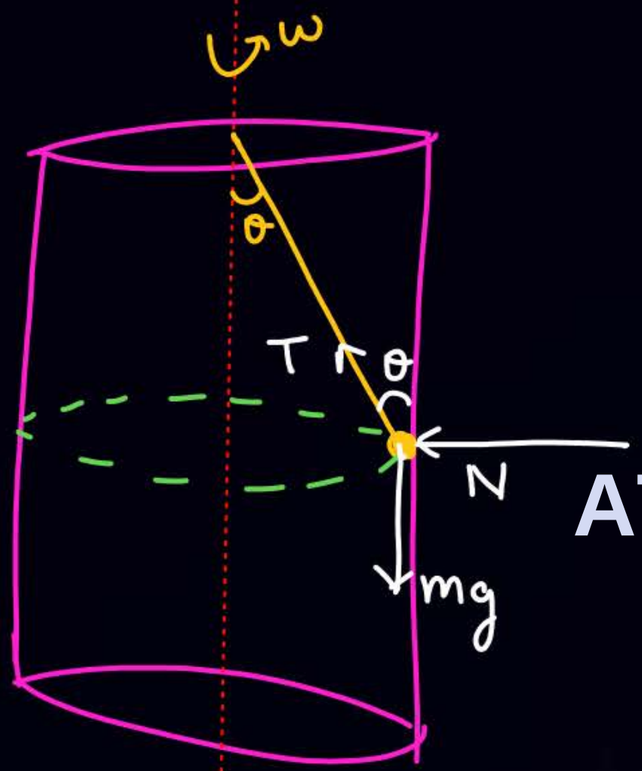
$T_2\sin\theta + T_1\sin\theta$

$= m r \omega^2$

$= m \ell \sin\theta\, \omega^2$

(9)

mass is at rest wrt Cylinder.

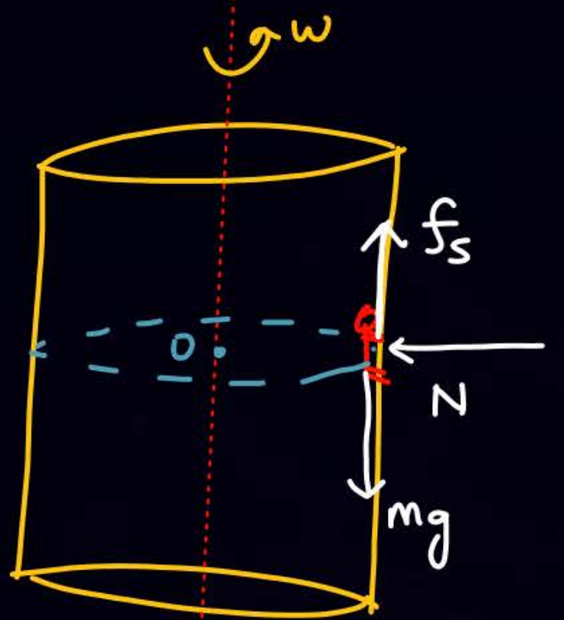


$T\cos\theta = mg$

$N + T\sin\theta = mR\omega^2$

$\ell \sin\theta = R$

(10)



$N = mR\omega^2$

$f_s = mg$

find μ_{min} so that man remains at rest wrt Cylin

$mg \leq (f_s)_{max}$

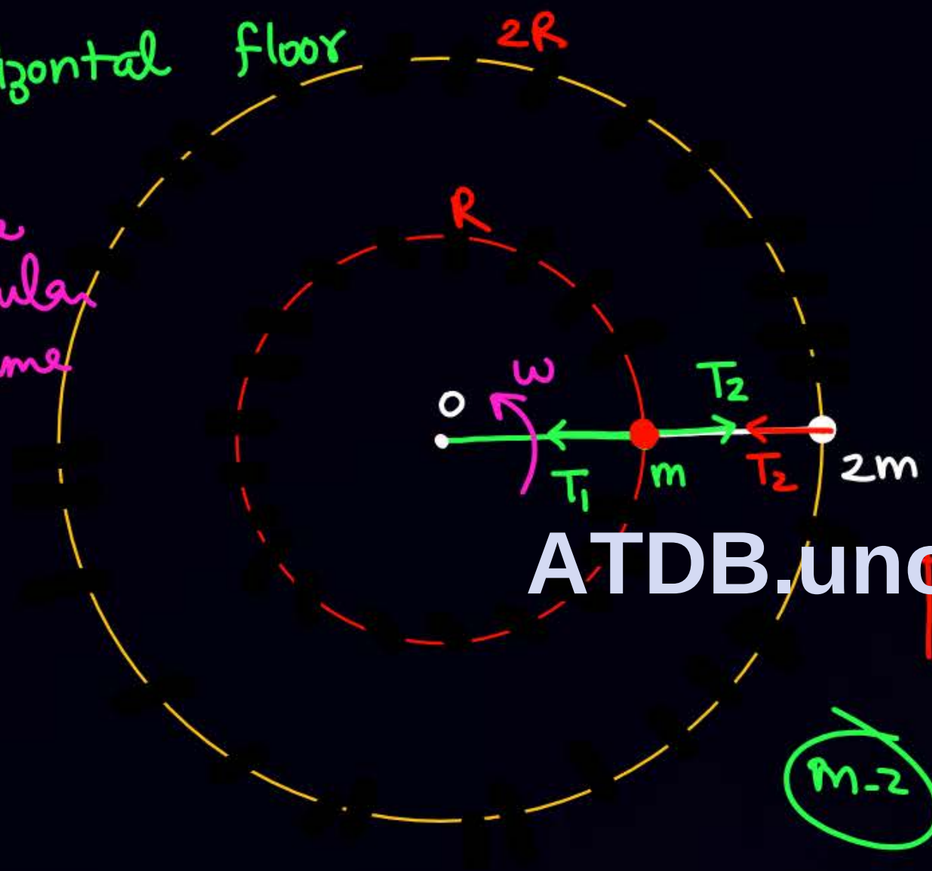
$mg \leq \mu_s N$

$mg \leq \mu_s \cdot mR\omega^2$

(11) Q

Horizontal floor

Both masses are moving in circular path with same ω.



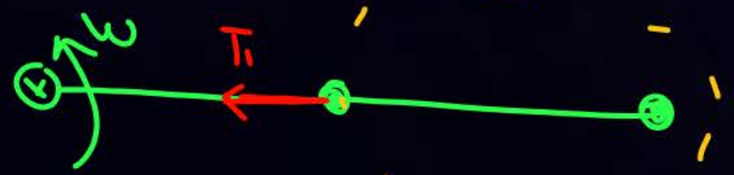
mg ⟶ अंदर
N ⟶ बाहर

$T_1 - T_2 = mR\omega^2$

$T_2 = (2m)(2R)\omega^2$

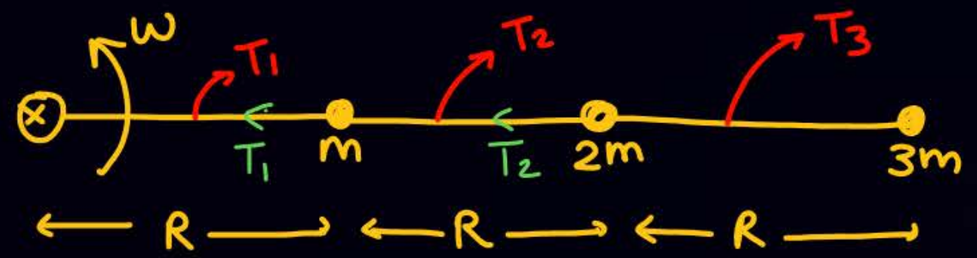
$T_2 = 4mR\omega^2$, $T_1 = 5mR\omega^2$

m-2



$T_1 = mR\omega^2 + 2m.2R\omega^2$,

Q



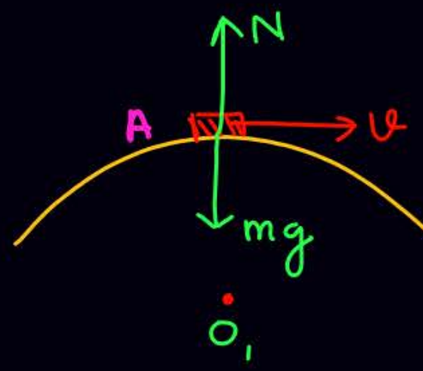
find $T_1 : T_2 : T_3$

$T_1 = mR\omega^2 + 2m\,2R\,\omega^2 + 3m\,3R\omega^2 = 14mR\omega^2$

$T_2 = 2m \cdot 2R \cdot \omega^2 + 3m \cdot 3R \cdot \omega^2 = 13mR\omega^2$

$T_3 = 3m3R\omega^2 = 9mR\omega^2$

13



$$mg - N_A = \frac{mv^2}{R}$$

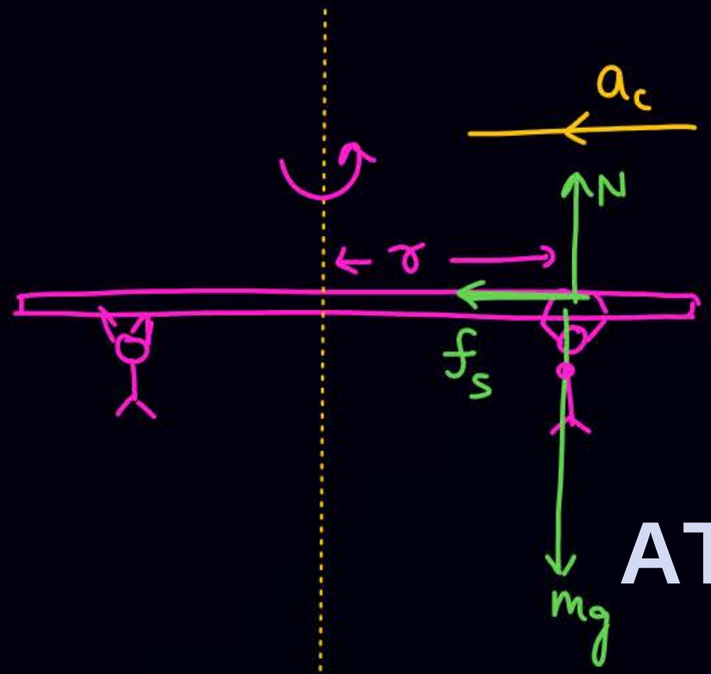
$$N_A = mg - \frac{mv^2}{R}$$

$$N_B > N_A$$

$$N_B - mg = \frac{mv_B^2}{R_2}$$

$$N_B = mg + \frac{mv_B^2}{R}$$

(14)



$$f_s = m r \omega^2$$

μ_{min}

$$\mu_s N = m r \omega^2$$

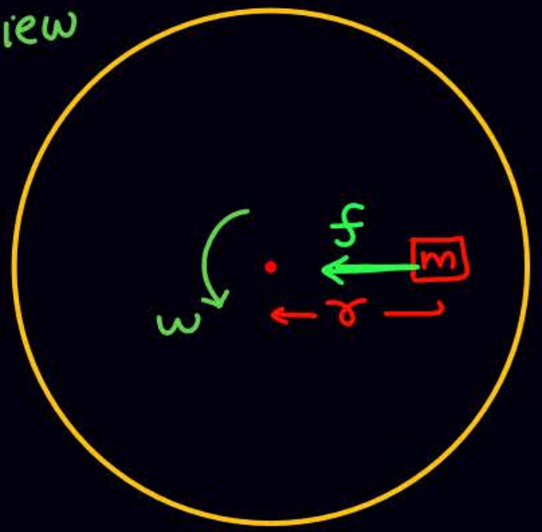
$$\mu_s \rho h g = \rho h r \omega^2$$

(Horizontal turn table)

μ_s

15

Top View



find ω_{max} so that mass remains at rest wrt turn table.

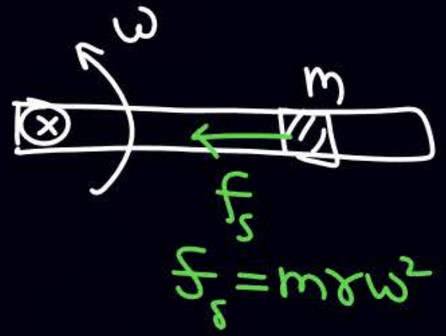
Sol^n

mg अंदर
N बाहर

$f_s = m r \omega^2$

max max

Q



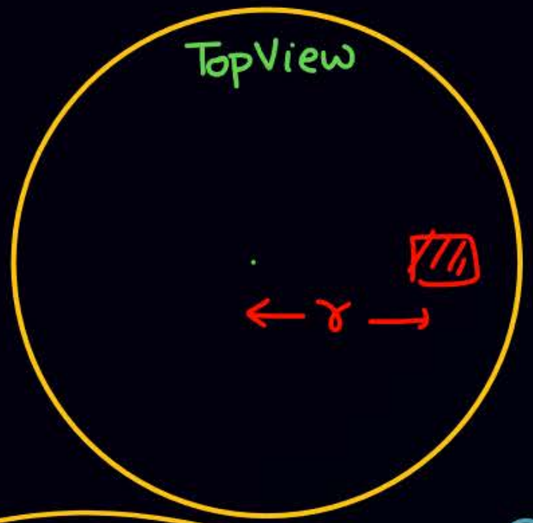
$f_s = m r \omega^2$

$\mu_s mg = m r \omega^2$

Q A turn table start rotating from rest, with const angular acc α at $t=0$. Find when block will slip. (μ_s, m)

$a_t = r\alpha$

Sol^n when block is just abt to slip



$(f_s)_{max}\cos\theta = mr\omega^2_{max}$

$(f_s)\sin\theta = mr\alpha$

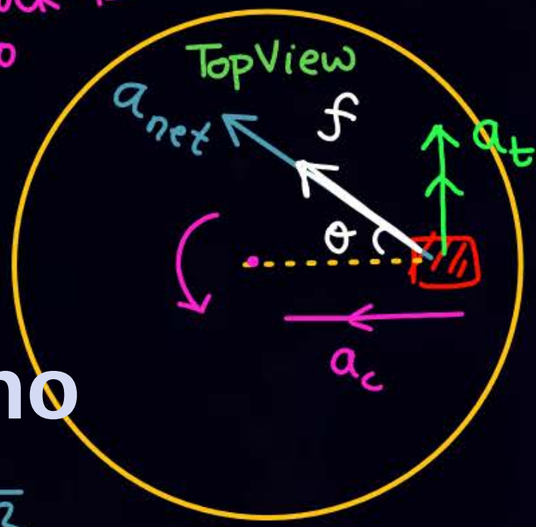
Square & add

$(f_s)_{max} = m\sqrt{(r\alpha)^2+(r\omega^2)^2}$

$\omega = \omega_0 + \alpha t$

$\omega_{max} = 0 + \alpha t$

$\mu_s mg$



$a_{net} = \sqrt{a_t^2 + a_c^2}$

$a = \sqrt{(r\alpha)^2+(r\omega^2)^2}$

max, max

$f_s = ma$

$(f_s)_{max} = ma_{max} = m\sqrt{(R\alpha)^2+(R\omega^2_{max})^2}$

$\mu mg = m\sqrt{(r\alpha)^2+(r\omega^2)^2}$

$\downarrow ma_t$

table (A table whose surface can rotate about central axis). Table starts from rest and rotates with constant angular acceleration, $\alpha = 3$ rad/sec^2. The friction coefficient between block and table is $\mu = 0.5$. At time $t = \frac{x}{3}$ sec from starting of motion (i.e. $t = 0$ sec) the block is just about to slip. Find the value of x.

$(f_s)_{max} = .5 \times 200 = 100 = m\sqrt{(R\alpha)^2 + (R\omega^2)^2}$

एक $m = 20$ kg द्रव्यमान का ब्लॉक घूर्णी टेबल (ऐसी टेबल जिसकी सतह केन्द्रीय अक्ष के सापेक्ष घूर्णन कर सकती है) की केन्द्रीय घूर्णन अक्ष से $R = 1$m की दूरी पर रखा है। टेबल विरामावस्था से प्रारम्भ होती है तथा नियत कोणीय त्वरण $\alpha = 3$ rad/sec^2 से घूर्णन करती है। ब्लॉक तथा टेबल के मध्य घर्षण गुणांक $\mu = 0.5$ है। गति के प्रारम्भ से (अर्थात् $t = 0$ sec से) $t = \frac{x}{3}$ sec समय पर ब्लॉक [illegible] x का मान ज्ञात कीजिये।

$100 = 20\sqrt{9 + \omega^4}$

$5 = \sqrt{9 + \omega^4}$, $\omega = 2$

$\omega = \omega_0 + \alpha t$

$2 = 0 + 3t$

$t = \frac{2}{3}$

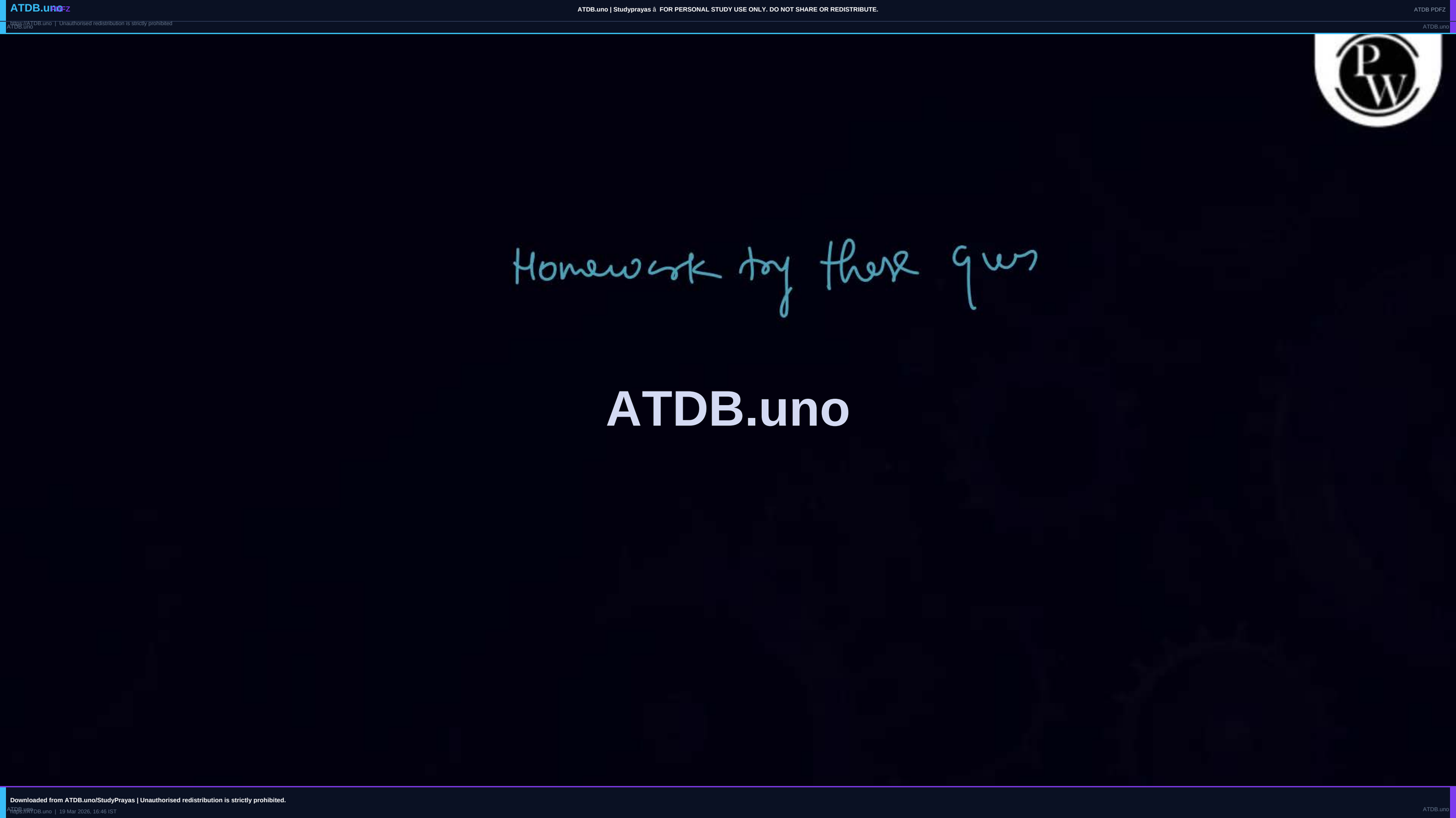
Ans. 2

Banking of road ≡ next class

$$v_{max} = \sqrt{Rg\tan(\theta+\phi)}$$

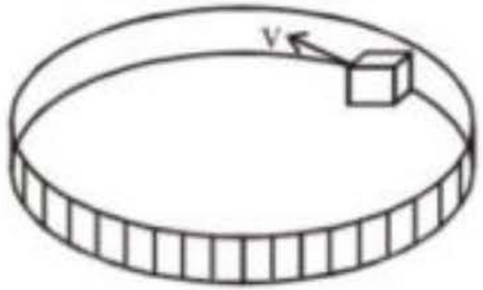
$$v_{min} = \sqrt{Rg\tan(\theta-\phi)}$$

$$\tan\phi = \mu_s$$



A block of mass m moves with speed v against a smooth, fixed vertical circular groove of radius r kept on smooth horizontal surface.

एक ब्लॉक का द्रव्यमान m है। यह चिकनी क्षैतिज सतह पर रखे त्रिज्या r वाले घर्षणरहित, स्थिर ऊर्ध्वाधर वृत्ताकार खाँचें में v चाल से गति करता है



Find :

(i) normal reaction of the floor on the block.

(ii) normal reaction of the vertical wall on the block.

ज्ञात कीजिए :

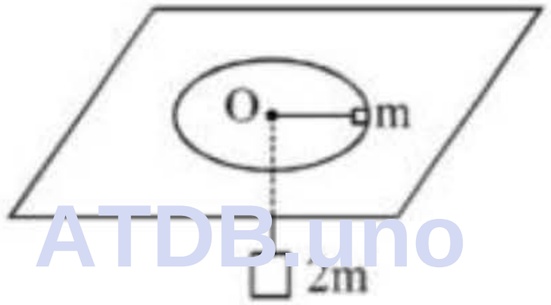
(i) ब्लॉक पर फर्श की अभिलम्ब प्रतिक्रिया।

(ii) ब्लॉक पर ऊर्ध्वाधर दीवार की अभिलम्ब प्रतिक्रिया।

Ans. (i) mg, (ii) $\frac{mv^2}{r}$

stationary mass 2m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.

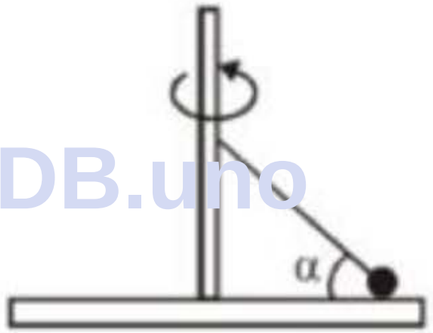
एक घर्षणरहित चिकनी मेज पर एक द्रव्यमान m, 1 m त्रिज्या के क्षैतिज वृत्त में मुक्त रूप से घूर्णन कर रहा है। इस मेज पर एक घर्षणरहित छिद्र O है (चित्र देखें) इसमें से एक रस्सी गुजर रही है। इस रस्सी के एक सिरे पर द्रव्यमान m (जो कि घूर्णन कर रहा है) बंधा हुआ है तथा दूसरे सिरे पर द्रव्यमान 2m (जो कि ऊर्ध्वाधर रूप से लटका हुआ है) जुड़े हैं। घूर्णन का कोणीय वेग ज्ञात कीजिए।



Ans. $\sqrt{2g}$ rad/s

is a ball of mass 1 kg, attached to the long axis of the platform by a thin rod of length 10 cm ($\alpha = 30°$). Find normal force exerted by the ball on the platform (in newton). Friction is absent.

एक वृत्ताकार प्लेटफॉर्म, ऊर्ध्वाधर अक्ष के चारों ओर कोणीय वेग $\omega = 10$ rad/s से घूमता है। प्लेटफॉर्म पर एक 1 kg द्रव्यमान की गेंद रखी है जो 10 cm लम्बी पतली छड़ द्वारा प्लेटफॉर्म की लम्बी अक्ष से जुड़ी हुई है, तथा ($\alpha = 30°$) है। प्लेटफॉर्म पर गेंद द्वारा आरोपित अभिलम्ब बल का मान (न्यूटन में) ज्ञात कीजिए। घर्षण अनुपस्थित है।



Ans. 5

radial acceleration & tangential acceleration are equal. If at $t = 0$ velocity of particle is V_0. Find the speed of the particle after time $t = \frac{R}{2V_0}$

एक कण R त्रिज्या के वृत्ताकार पथ में इस प्रकार गति करता है कि प्रत्येक क्षण इसके त्रिज्यीय त्वरण तथा स्पर्शरेखीय त्वरण का परिमाण बराबर है। यदि $t = 0$ पर कण का वेग V_0 है तो समय $t = \frac{R}{2V_0}$ के पश्चात् कण की चाल ज्ञात कीजिए।

Ans. $2V_0$

HW

of length 0.3m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.

(i) Calculate the frictional force on m_1

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position. $(f_s)_{max} = \mu m g$

(iii) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1

$f = 0$ put

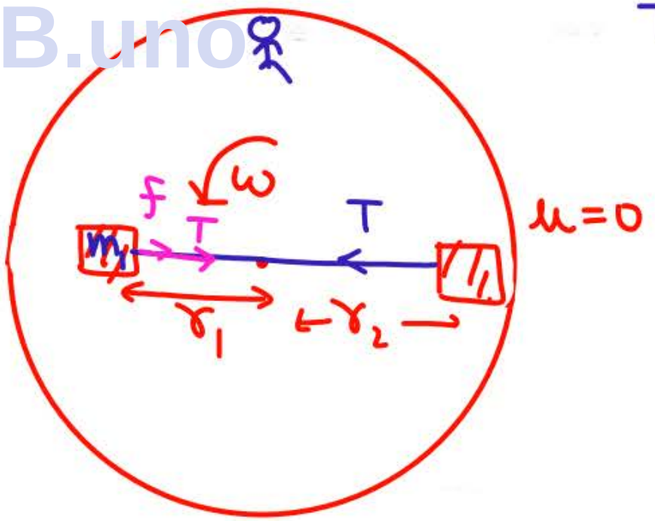
$r_1 = .124$

$r_2 = .3 - .124$

$= .176$

$\mu_1 = .5$

300
124
176



$T + f = m_1 r_1 \omega^2$

$T = m_2 r_2 \omega^2$

Ans. (i) 36N, (ii) 11.66rad/sec, (iii) 0.1m, 0.2m

conical path with its axis same as that of the cylinder with uniform angular velocity. Radius of cylinder is 30 cm, length of string is 50 cm and mass of bob is 400 gm. The bob makes contact with the inner frictionless wall of the cylinder while moving :-

(A) The minimum value of angular velocity of the bob so that it does not leave contact is 5 rad/s

(B) Tension in the string is 5N for all values of angular velocity

(C) For angular velocity of 10 rad/s the bob pushes the cylinder with a force of 9N.

(D) For angular velocity of 10 rad/s, tension in the string is 20N

$T\sin\theta + N = m r \omega^2$

$5 \times \frac{3}{5} + N = .4 \times .3 \times 100$

$N = 9$

$\omega = 5$

$R = 30\,cm$

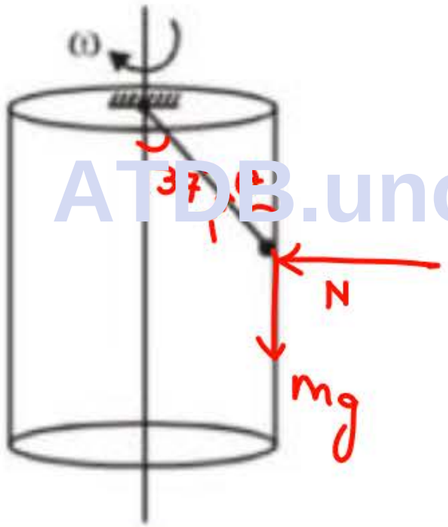
$\ell = 50\,cm$

$m = .4\,kg$

$T\sin\theta + N = mR\omega^2$

$\sin\theta \rightarrow 0$

$5 \times \frac{3}{5} = .4 \times .3 \times \omega^2$



$T\cos 37^\circ = mg$

$T \times \frac{4}{5} = .4 \times 10$

$T = 5$

tension in the ring is :

त्रिज्या r एवं प्रति एकांक लम्बाई द्रव्यमान m वाली एक वलय मुक्ताकाश में ω कोणीय वेग से घूर्णन कर रही है, वलय में तनाव होगा

(A) zero (B) $\frac{1}{2}m\omega^2r^2$ (C) $m\omega^2r^2$ (D) $mr\omega^2$

a vertical axis passing through one end. The tension in the rod at a distance x from the axis is :

m द्रव्यमान एवं ℓ लम्बाई वाली एक समरूप छड़ इसके एक सिरे से गुजरने वाली एक उर्ध्वाधर अक्ष के परित: ω कोणीय वेग से क्षेतिज तल में घूर्णन कर रही है। अक्ष से x दूरी पर छड़ में तनाव है

(A) $\frac{1}{2}m\omega^2 x$ (B) $\frac{1}{2}m\omega^2\frac{x^2}{\ell}$ (C) $\frac{1}{2}m\omega^2\ell\left(1-\frac{x}{\ell}\right)$ (D*) $\frac{1}{2}\frac{m\omega^2}{\ell}[\ell^2-x^2]$

Ans. (D)

with angular velocity ω and angular acceleration α about its end. If μ is coefficient of friction. Mark the correct option. (Rod rotates in the plane of paper.)

(A*) If $\mu = \frac{\omega^2}{\alpha}$ friction on bead is static in nature

(B) If $\mu > \frac{\omega^2}{\alpha}$ friction on bead is kinetic in nature

(C) If $\mu < \frac{\omega^2}{\alpha}$ friction is static

(D) If bead does not slide relative to rod. Friction will not exist between bead and rod.

एक कण (मोती) गुरूत्वविहीन क्षेत्र में स्थित छड़ पर ही चित्रानुसार गति कर सकता है। इस छड़ को इसके सिरे के परित: कोणीय वेग ω तथा कोणीय त्वरण α से घूर्णन कराया जाता है। यदि घर्षण गुणांक μ हो तो सही कथन चुनिये।
(छड़ कागज के तल में घूर्णन करती है)

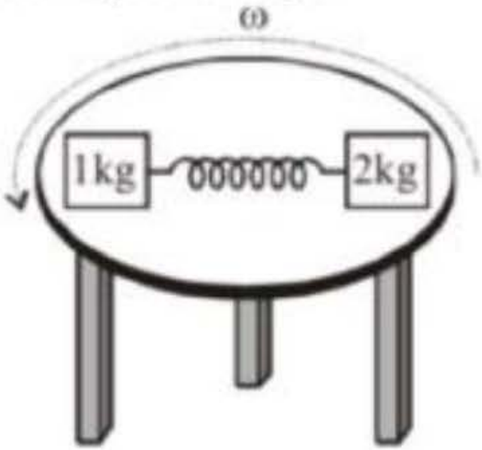
(A) यदि $\mu = \frac{\omega^2}{\alpha}$ हो तो कण पर कार्यरत् घर्षण स्थैतिक प्रकृति का है।

(B) यदि $\mu > \frac{\omega^2}{\alpha}$ हो तो कण पर कार्यरत् घर्षण गतिक प्रकृति का है।

(C) यदि $\mu < \frac{\omega^2}{\alpha}$ हो तो घर्षण स्थैतिक है।

(D) यदि कण, छड़ के सापेक्ष गति नहीं करता है तो कण व छड़ के मध्य घर्षण विद्यमान नहीं होगा।

4. On a circular turn table rotating about its center horizontally with uniform angular velocity ω rad/s placed two blocks of mass 1 kg and 2 kg, on a diameter symmetrically about center. Their separation is 1m and friction is sufficient to avoid slipping. The spring between them as shown is stretched and applied force of 5N. If f_1 and f_2 are values of friction on 1 kg & 2kg block respectively:-



(A) For $\omega = 2$ rad/s, $f_1 = 3N$ & $f_2 = 1N$

(B) For $\omega = 3$ rad/s, $f_1 = 0.5$ N & $f_2 = 4N$

(C) For $\omega = \sqrt{10}$ rad/s, $f_1 = 0$ & $f_2 = 5N$

(D) For $\omega = \sqrt{10}$ rad/s, $f_1 = 0$ & $f_2 = 0N$

H.W

- DPP
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