

Work is said to be done when a force applied on the body displaces the body through a certain distance

WORK DONE BY CONSTANT FORCE

$W = F \cos \theta \times S$
 $= \vec{F} \cdot \vec{S}$

NATURE OF WORK DONE

1) Positive work ($0^\circ \leq \theta < 90^\circ$)

2) Negative work ($90^\circ < \theta \leq 180^\circ$)

2) Zero work
Work done becomes 0 for three conditions

1. Force is perpendicular to displacement
2. if there is no displacement
3. if there is no force acting on the body

WORK DONE BY VARIABLE FORCE

$dW = \vec{F} \cdot d\vec{s}$
 $W = \int \vec{F} \cdot d\vec{s} = \int F ds \cos \theta$

in terms of rectangular components

$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$
 $d\vec{s} = dx \hat{i} + dy \hat{j} + dz \hat{k}$
 $W = \int F_x dx + \int F_y dy + \int F_z dz$

Graphical representation of work done

$dW = F \cdot dx$
 $W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F \cdot dx$

CONSERVATIVE & NON CONSERVATIVE FORCE

Conservative: work done doesnot depend on path followed
Non-conservative: work depends on the path followed

• $W_{A \rightarrow B}$ (Path 1) = $W_{A \rightarrow B}$ (Path 2) = $W_{A \rightarrow B}$ (Path 3)
(for conservative force)

• $W_{A \rightarrow B}$ (Path 1) $\neq$ $W_{A \rightarrow B}$ (Path 2) $\neq$ $W_{A \rightarrow B}$ (Path 3)
(for non conservative force)

Note :
Work done for a complete cycle for a conservative force is zero

WORK DONE BY DIFFERENT FORCES

$W_1 = mgh = mgh$
 $W_2 = mg \times l \sin \theta = mg \times l \times \frac{h}{l} = mgh$
 $W_3 = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4$

Work done by spring force
magnitude of spring force, $F_s = -kx$

$W_s = \int \vec{F}_s \cdot d\vec{x} = - \int_{x_i}^{x_f} kx dx$
 $= -\frac{1}{2} k (x_f^2 - x_i^2)$

THE CHAIN

$L \rightarrow$ Total length
(1/n)th part of length hanging
 $M \rightarrow$ Mass of chain
Work done in pulling the hanging portion back on the table
 $W = \frac{MgL}{2n^2}$

WORK ENERGY & POWER

ENERGY

- Capacity of doing work
- Scalar quantity
- Dimension ML^2T^{-2}

Relation between different units
1eV = 1.6×10^{-19} Joules
1kWh = 3.6×10^6 Joules
1calorie = 4.18 Joules
1 Joule = 10^7 erg

Kinetic Energy

- Energy possessed by virtue of motion
- $K.E = \frac{1}{2} mv^2$
- Always positive
- Depends on frame of reference

Work Energy Theorm

Change in kinetic energy of a body is equal to network done on the body

$K_2 - K_1 = \int \vec{F} \cdot d\vec{r}$

ENERGY WITH OTHER QUANTITIES

Linear momentum:- $P = \sqrt{2mK}$
Variation of graph of kinetic Energy

m constant, $E \propto v^2$
 m constant, $E \propto p^2$
 P constant, $E \propto \frac{1}{m}$
 P constant, $E \propto P$

POTENTIAL ENERGY

- Defined only for conservative force
- Energy possessed by a body by virtue of its position/configuration
- Can either be positive, negative or zero according to point of reference
- Force always acts from higher potential to lower potential

Identifying forces with the help of potential energy

1) Force opposing the motion:-
On increasing x , if U increases
 $\frac{dU}{dx} = \text{positive}$ (BC portion of graph)

2) Force supporting the motion:-
On increasing x , if U decreases
 $\frac{dU}{dx} = \text{negative}$ (AB portion of graph)

3) Zero force:-
On increasing x , if U doesnot change
 $\frac{dU}{dx} = 0$

B, C points on the graph

Types of Potential Energy

- Elastic Potential Energy
- Electric Potential Energy
- Gravitational Potential Energy

• Types of equilibrium

If net force acting on a particle is zero it is said to be in equilibrium

STABLE	UNSTABLE	NOETRAL
• If particle displaced from equilibrium position, force acting will try to bring the particle back to the equilibrium position • Potential energy is minimum at stable equilibrium • $F = -\frac{dU}{dx} = 0$ • $\frac{d^2U}{dx^2} = \text{positive}$	• If particle displaced from equilibrium position, force acting on it tries to displace it further away from equilibrium position • Potential energy is maximum at unstable equilibrium • $F = -\frac{dU}{dx} = 0$ • $\frac{d^2U}{dx^2} = \text{negative}$	• If particle is slightly displaced from equilibrium, then it does not experience a force or continues to be in equilibrium • Potential energy is constant • $F = -\frac{dU}{dx} = 0$ • $\frac{d^2U}{dx^2} = 0$

CONSERVATION OF ENERGY

For an isolated system or body in the presence of only conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion
 $K.E + P.E = \text{constant}$

POWER

- Rate at which body does work
- Average power (P_{av}) = $\frac{W}{t}$
- Instantaneous power
 $(P_{inst}) = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$

Relation between units:

- 1 watt = 1 joule/sec = 10^7 erg/sec
- 1 HP = 746 watt, 1MW = 10^6 watt
- 1 KW = 10^3 watt

- If work done by two bodies is same then power $\propto \frac{1}{\text{time}}$
- Unit of power multiplied by time always gives work
1 KWh = 3.6×10^6 Joules
- Slope of work-time curve gives instantaneous power

$\tan \theta = \frac{dW}{dt} = P_{inst}$

• Area under power-time graph gives work done

$P = \frac{dW}{dt}$
 $\Rightarrow dW = P dt$
 $W = \int_{t_1}^{t_2} P dt$

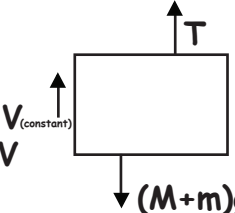
$W = \text{Area under P-t graph}$

Position and velocity in terms of power (P=constant)

- 1) Velocity, $V = \left[\frac{2Pt}{m} \right]^{1/2}$
- 2) Position, $S = \left[\frac{8Pt^3}{9m} \right]^{1/2} + 3/2$

Power delivered by an elevator

$a=0, T=(M+m)g$
 $\vec{P}=\vec{T} \cdot \vec{V}$
 $=TV$
Power, $P=(M+m)gV$



Power of a water drawing pump

- Power, $P = \frac{dW}{dt} = \frac{dm}{dt} \left[gh + \frac{V^2}{2} \right]$
- h = height of water level
- $\frac{dm}{dt} \Rightarrow$ mass flow rate of pump
- $V \rightarrow$ velocity of the water outlet
- Power required to just lift water, $V=0$
 $P = gh \left(\frac{dm}{dt} \right)$
- Efficiency of pump
 $\mu = \frac{\text{Output Power}}{\text{Input Power}}$

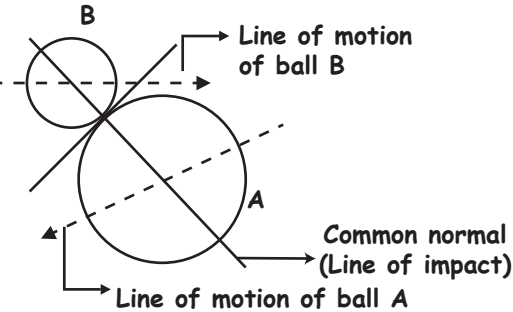


WORK
ENERGY &
POWER

Collision is the event in which impulsive force acts between two or more bodies which results in change of their velocities.

Line of impact

Line passing through common normal to surfaces in contact during impact

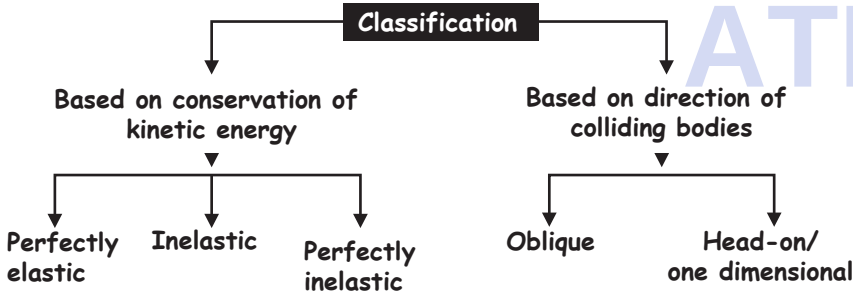


Coefficient of restitution (e)

$$e = \frac{\text{Velocity of separation along the line of impact}}{\text{Velocity of approach along the line of impact}}$$
$$= \frac{\text{Relative velocity along the line of impact after collision}}{\text{Relative velocity along the line of impact before collision}}$$

Conditions

1. For elastic collision: $e=1$
2. For inelastic collision: $e<1$
3. For perfectly inelastic collision: $e=0$



Perfectly elastic collision

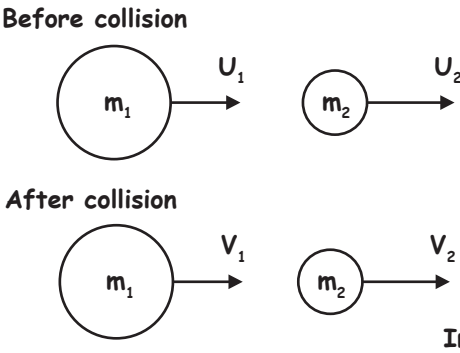
K.E before and after collision is same

Inelastic collision

K.E before and after collision is not same

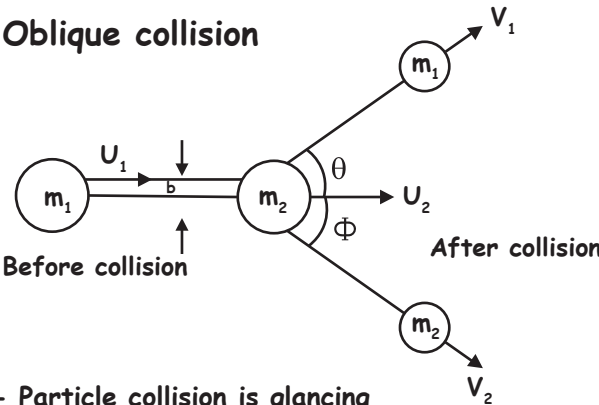
Head-on collision / One dimensional collision

Initial velocities of the bodies are along the line of impact.



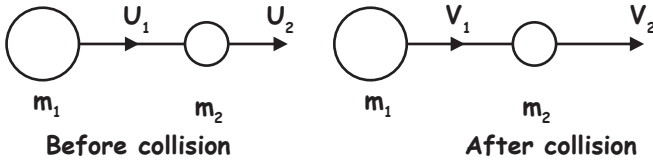
If initial velocities of the bodies are not along the line of impact.

Oblique collision



- Particle collision is glancing
- Directions of motion after collision are not along initial line of motion
- Impact parameter $0 < b < (r_1 + r_2)$ where r_1, r_2 are radii of colliding bodies

Perfectly elastic Head-on collision



Vel. after collision

$$V_1 = U_1 \left[\frac{m_1 - m_2}{m_1 + m_2} \right] + \frac{2m_2 U_2}{m_1 + m_2}$$

$$V_2 = U_2 \left[\frac{m_2 - m_1}{m_1 + m_2} \right] + \frac{2m_1 U_1}{m_1 + m_2}$$

Special cases:

1) Projectile and target having same mass $m_1 = m_2$, then $v_1 = u_2, v_2 = u_1$, the velocities get interchanged.

2) If massive projectile collides with a light target i.e. $m_1 \gg m_2$, then $v_1 = u_1, v_2 = -u_2 + 2u_1$

3) If a light projectile collides with a very heavy target, $m_1 \ll m_2$, then $v_1 = -u_1 + 2u_2, v_2 = u_2$

Energy transfer from projectile to target

1) Fractional decrease in kinetic energy of projectile (If target is at rest)

$$\frac{\Delta K}{K} = \frac{4m_1 m_2}{(m_1 - m_2)^2 + 4m_1 m_2}$$

Greater the difference in masses, less will be transfer of K.E and vice versa

Transfer of K.E will be maximum when difference in masses is maximum

If $m_2 = nm_1$
$$\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$$

Inelastic collision

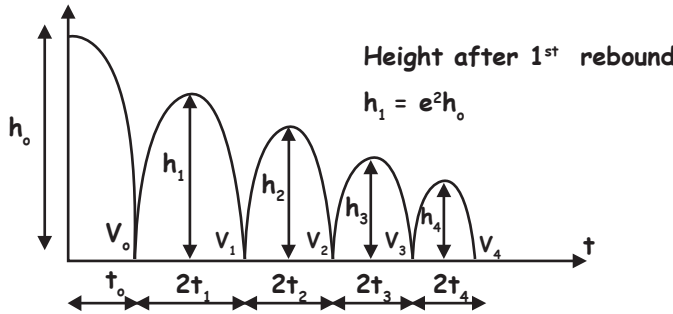
$$e = \frac{V_2 - V_1}{U_1 - U_2}$$
 Relative velocity of separation
Relative velocity of approach

Velocity after collision $V_1 = \frac{(1+e)m_2 U_2}{m_1 + m_2} + \frac{(m_1 - em_2)U_1}{m_1 + m_2}$

Ratio of velocities $V_2 = \frac{(1+e)m_1 U_1}{m_1 + m_2} + \frac{(m_2 - em_1)U_2}{m_1 + m_2}$

$$\frac{V_1}{V_2} = \frac{1-e}{1+e}$$
 Loss in kinetic energy $\Delta K = \frac{1}{2} \left[\frac{m_1 m_2}{m_1 + m_2} \right] (1-e^2) (U_1 - U_2)^2$

Rebounding of ball



Total height covered by the ball before it stops bouncing

$$H = h_0 \left[\frac{1+e^2}{1-e^2} \right]$$

Total time taken by the ball until it stops bouncing $T = \left(\frac{1+e}{1-e} \right) \frac{2h_0}{g}$

Perfectly inelastic collision

Colliding bodies stick together

After collision are moving in the same



Loss in kinetic energy $\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (U_1 - U_2)^2$

Colliding bodies are moving in the opposite direction

$$V = \frac{m_1 U_1 - m_2 U_2}{m_1 + m_2}, \text{ Change in kinetic energy } \Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (U_1 - U_2)^2$$