PRAYAS

JEE 2025

Lecture - 08 | Physics

Oscillations

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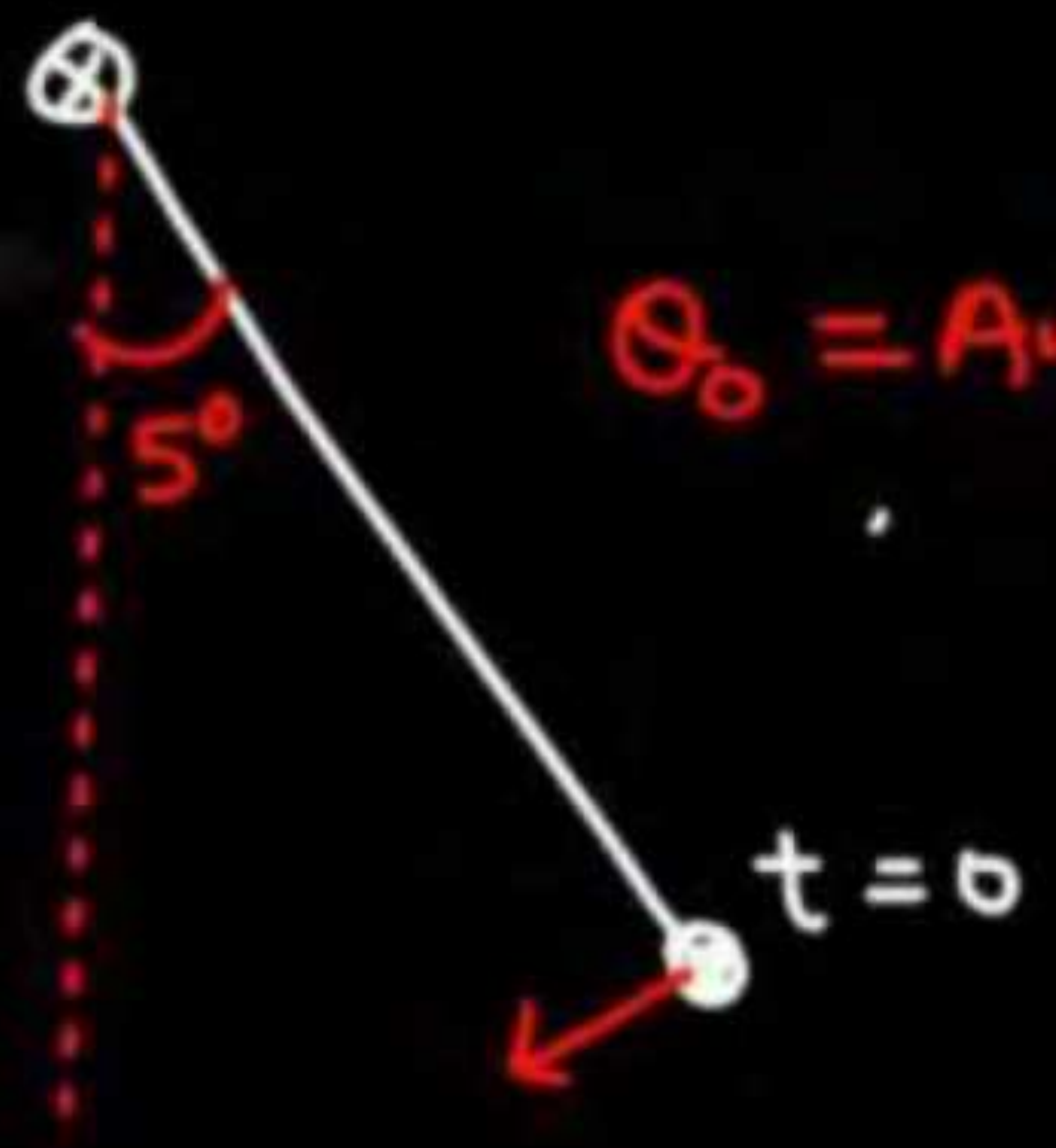
Topics *to be covered*

1. Angular SHM

2. Compound Pendulum & Torsional Pendulum
3. Phasor
4. Superposition of SHM

Linear SHM $\vec{a} = -\omega^2 \vec{x}$	Angular SHM $\vec{\alpha} = -\omega^2 \vec{\theta}$
$\vec{F}_{net} = -K\vec{x}$ $\quad T = 2\pi\sqrt{\frac{m}{K}}$	$\vec{\tau}_{net} = -K'\vec{\theta}$ $\quad \vec{\tau} = I\vec{\alpha} = -K\vec{\theta}$ (Axis)
$\vec{F}_{net} = m\vec{a} = -K\vec{x}$	$T = 2\pi\sqrt{\frac{I}{K'}}$, $\omega = \sqrt{\frac{K'}{I}} = \frac{2\pi}{T}$
$T = 2\pi\sqrt{\frac{m}{K}}$, $\omega = \sqrt{\frac{K}{m}}$	$\theta = \theta_0 \sin(\omega t + \phi)$
$x = A\sin(\omega t + \phi)$; ω → Angular freq, $\omega = \frac{2\pi}{T}$	$\left(\frac{d\theta}{dt} = \Omega\right)$, $\Omega = \omega\sqrt{A^2 - x^2}$; Ω → Angular velocity, ω → Angular freq.
$v = \omega\sqrt{A^2 - x^2}$	
$KE = \frac{1}{2}K(A^2 - x^2)$	$KE = \frac{1}{2}K(\theta_0^2 - \theta^2)$
$PE = \frac{1}{2}KA^2 + U_0$	$PE = \frac{1}{2}K\theta^2 + U_0$

Q Angular SHM



$\theta_0 = \text{Amplitude} = 10^\circ$

$$\boxed{\theta = 10^\circ \sin(\omega t + 150^\circ)}$$

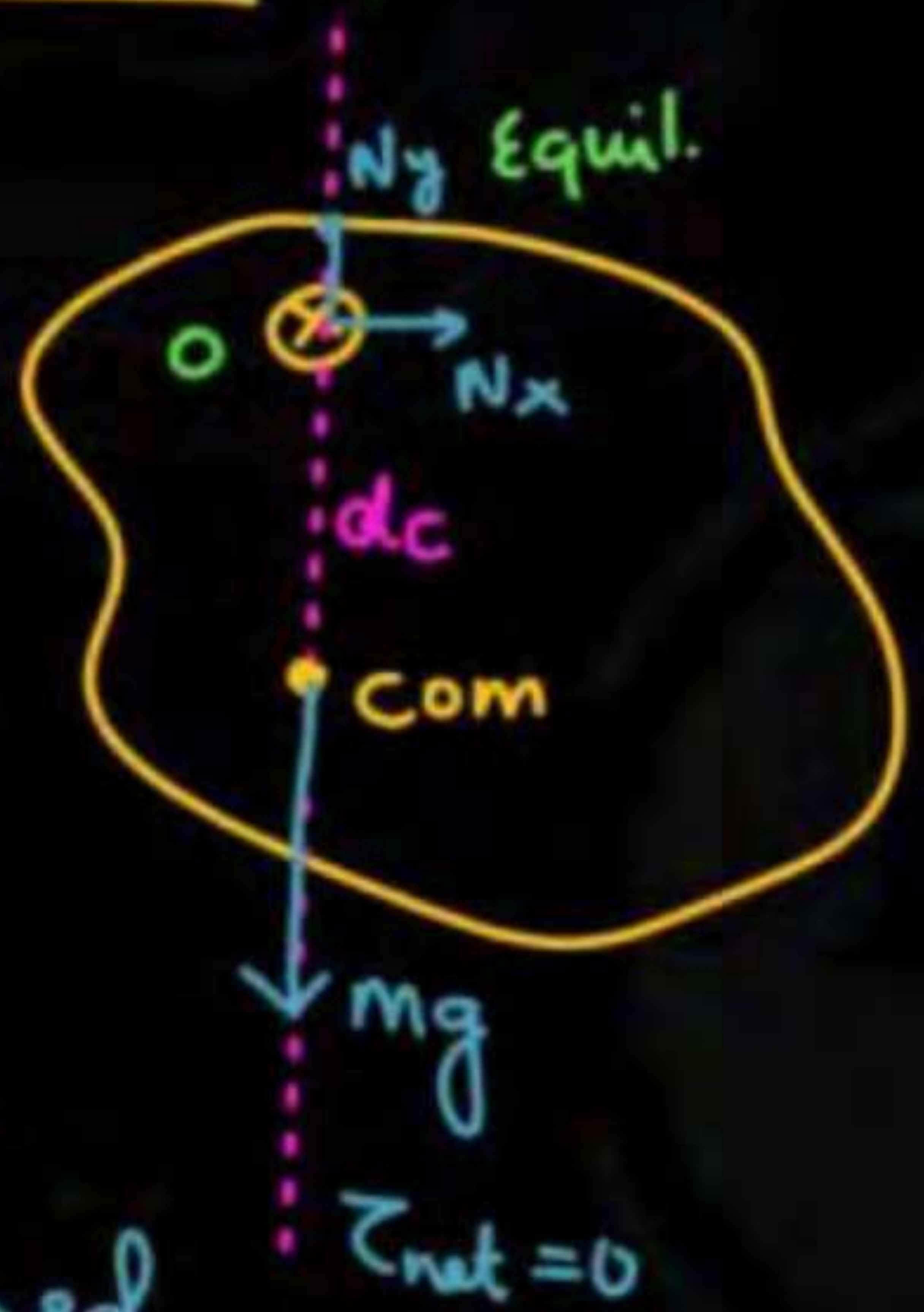
$$\omega = \frac{2\pi}{T}$$

Compound Pendulum

$$T = 2\pi\sqrt{\frac{I}{mgd_c}}$$

$I \longrightarrow$ MoI abt hinged pt

$d_c \longrightarrow$ com की hinged point से distance



$$\tau_0 = mg\sin\theta \times d_c$$

$$\tau = mgd_c\sin\theta$$

If θ is very small

$$\sin\theta \simeq \theta$$

$$\tau = mgd_c\,\theta$$

$$\vec{\tau} = -mgd_c\vec{\theta}$$

$$\vec{\tau}_{net} = -K\vec{\theta}$$

$$T = 2\pi\sqrt{\frac{I}{K}} = 2\pi\sqrt{\frac{I}{mgd_c}}$$

Q

$$T = 2\pi\sqrt{\frac{I}{mgd_c}}$$

$$T = 2\pi\sqrt{\frac{mL^2/3}{mg\cdot\frac{L}{2}}}$$

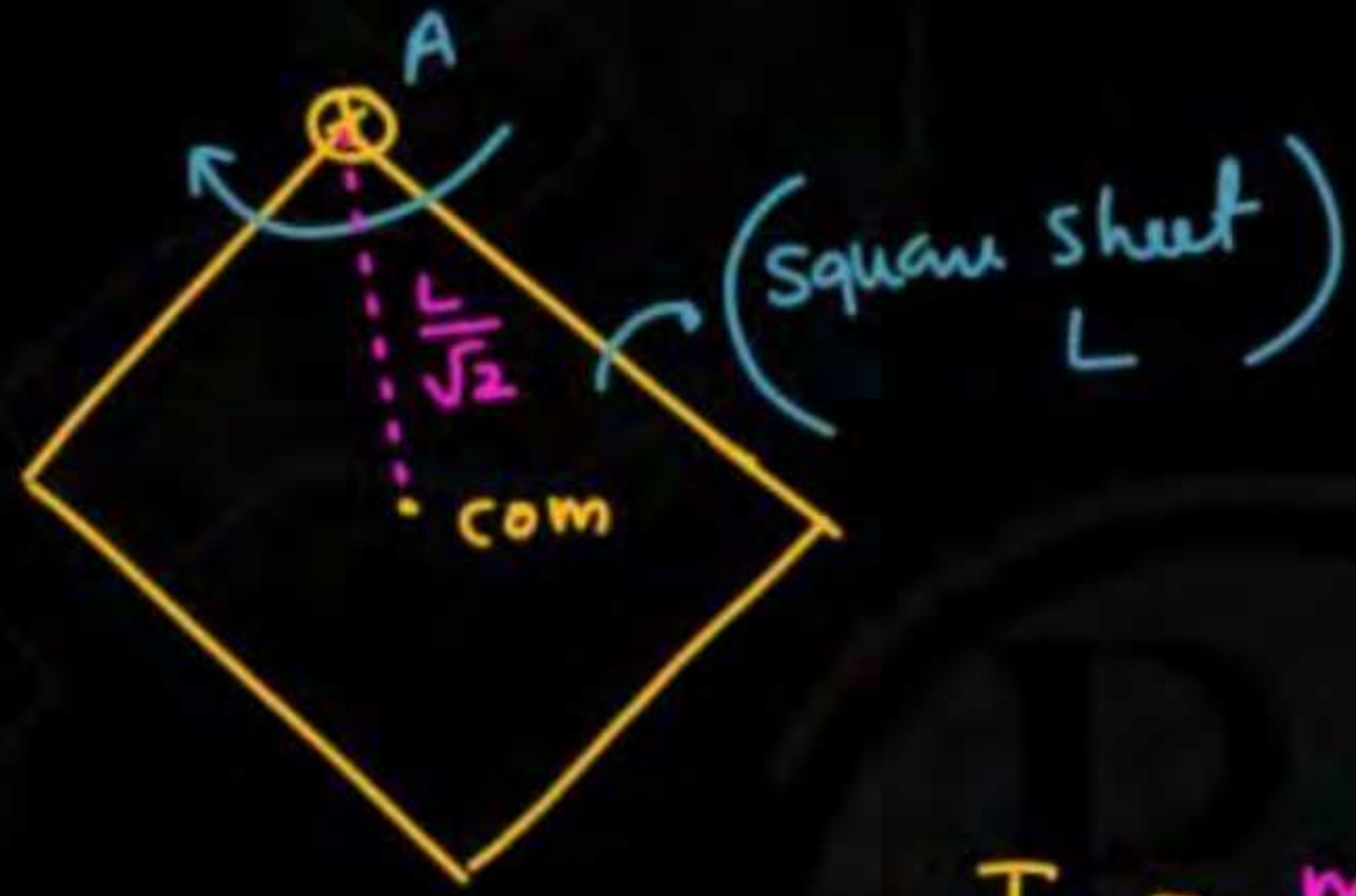
$$T = 2\pi\sqrt{\frac{2L}{3g}}$$

Q Disc(m, R)

$$T = 2\pi\sqrt{\frac{I}{mgd_c}}$$

$$T = 2\pi\sqrt{\frac{mR^2/2 + mR^2}{mgR}}$$

Q

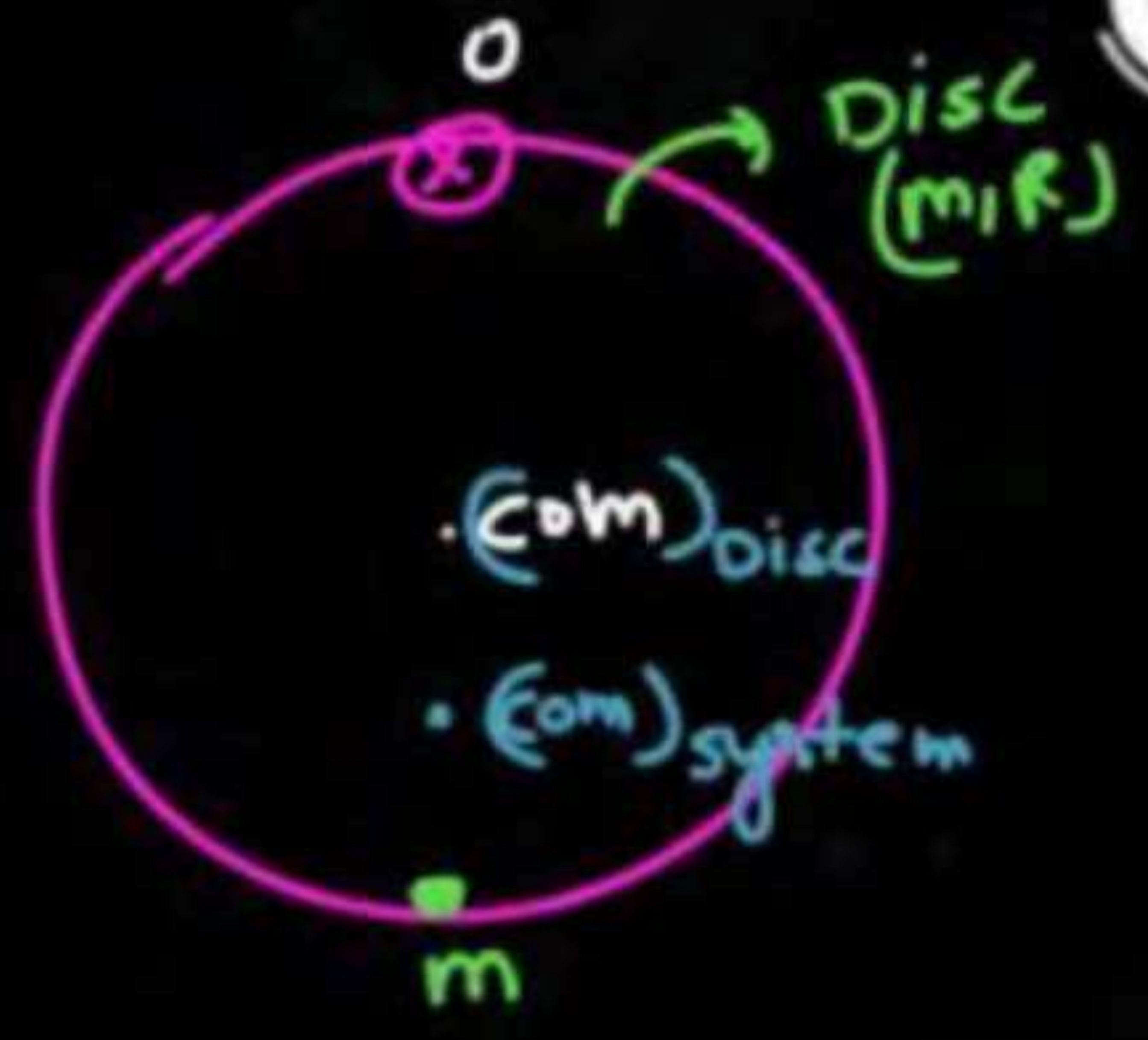


$$T = 2\pi\sqrt{\frac{I_A}{mgd_c}}$$

$$I_A = \frac{mL^2}{6} + m\left(\frac{L}{\sqrt{2}}\right)^2$$

$$d_c = \frac{L}{\sqrt{2}}$$

Q



$$T = 2\pi\sqrt{\frac{I_0}{(2m)g\left(R+\frac{R}{2}\right)}}$$

$$I_0 = \frac{mR^2}{2} + mR^2 + m(2R)^2$$

Q2

$\theta_0 = 10^\circ$

$\phi = 90^\circ$

$$T = 2\pi\sqrt{\frac{I}{mgd_c}} = 2\pi\sqrt{\frac{mL^2}{3mg\frac{L}{2}}} = \checkmark$$

$$\omega = \frac{2\pi}{T}$$

$$\boxed{\theta = 10^\circ \sin(\omega t + 90^\circ)}$$

Simple Pendulum

$$T = 2\pi\sqrt{\frac{I}{mgd_c}}$$

$$T = 2\pi\sqrt{\frac{ml^2}{mgl}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

lift ✓ — $g \equiv g_{eff}$

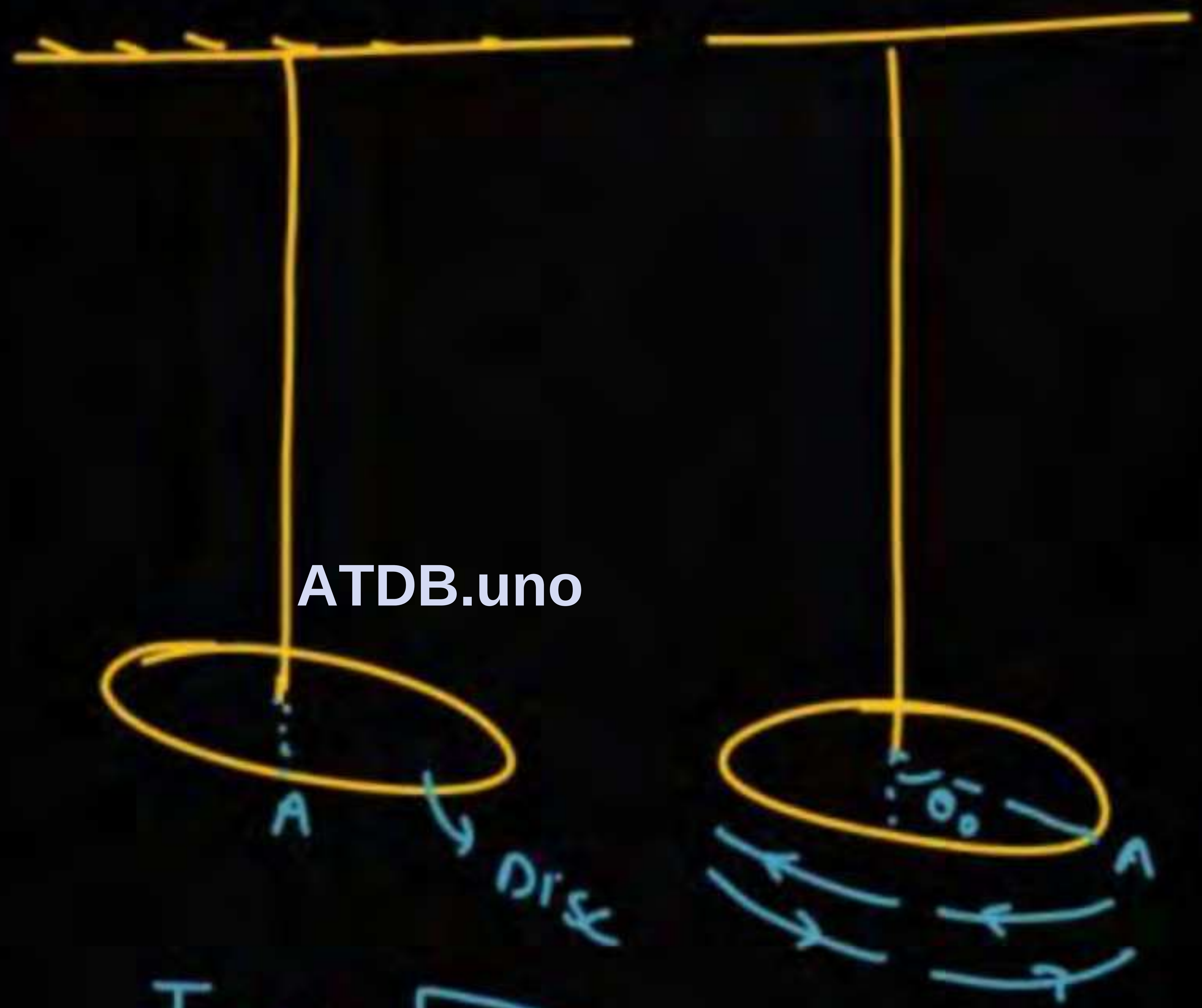
Tortional Pendulum

$$\vec{\tau}_{net} \propto -\vec{\theta}$$

$$\vec{\tau}_{net} = -C\vec{\theta}$$

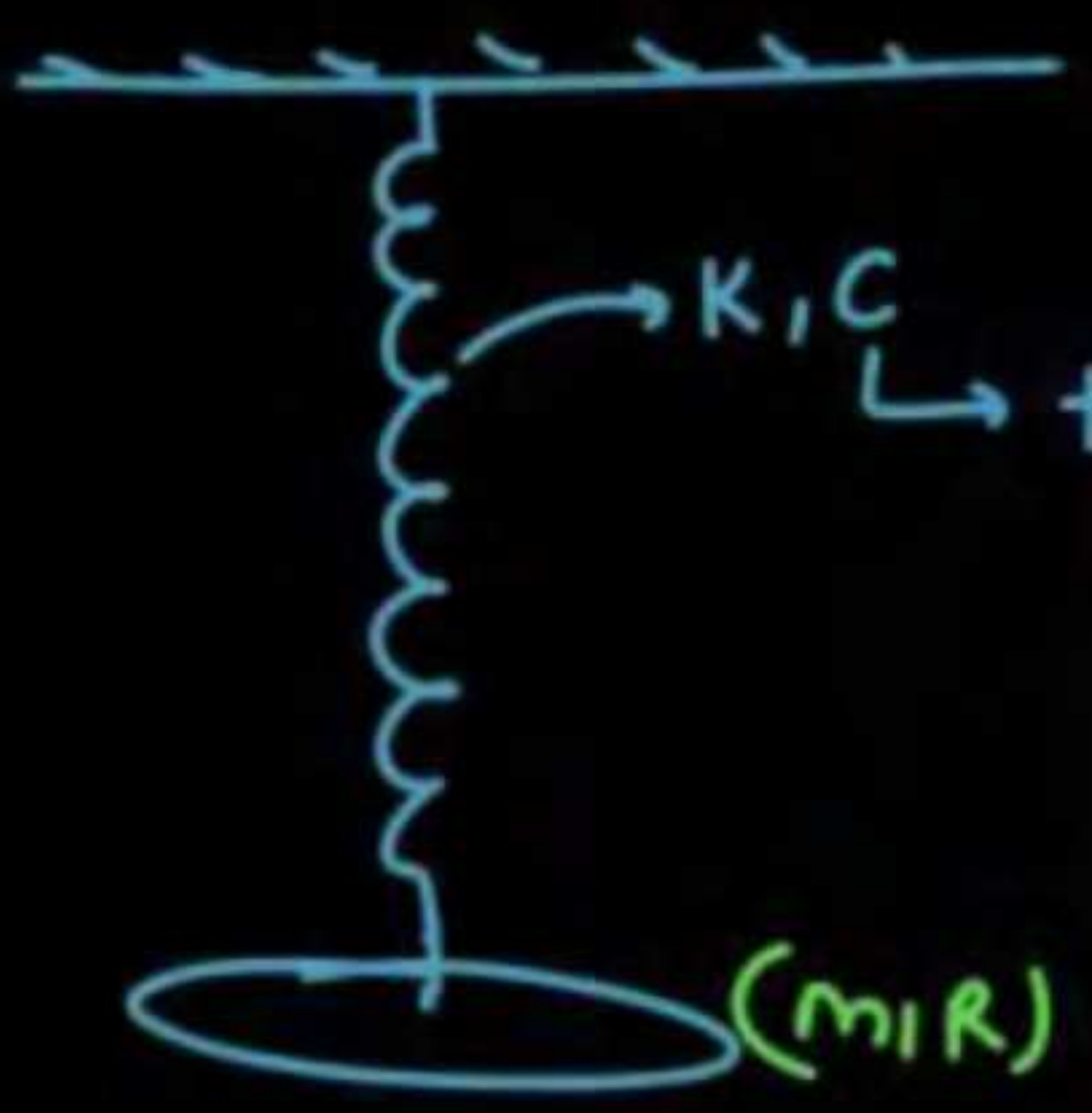
tortional const

$$\boxed{T = 2\pi\sqrt{\frac{I}{C}}}$$



$$T = 2\pi\sqrt{\frac{MR^2/2}{C}}$$

Q2



K, C
↳ torsional const

(m, R)

$$\frac{T_1}{T_2} = ?$$

$T_1 \longrightarrow$ spring block

$T_2 \longrightarrow$ torsional pendulum

$$T_1 = 2\pi\sqrt{\frac{m}{K}}$$

$$T_2 = 2\pi\sqrt{\frac{I}{C}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{m \cdot C}{K \cdot \frac{mR^2}{2}}}$$

phasor

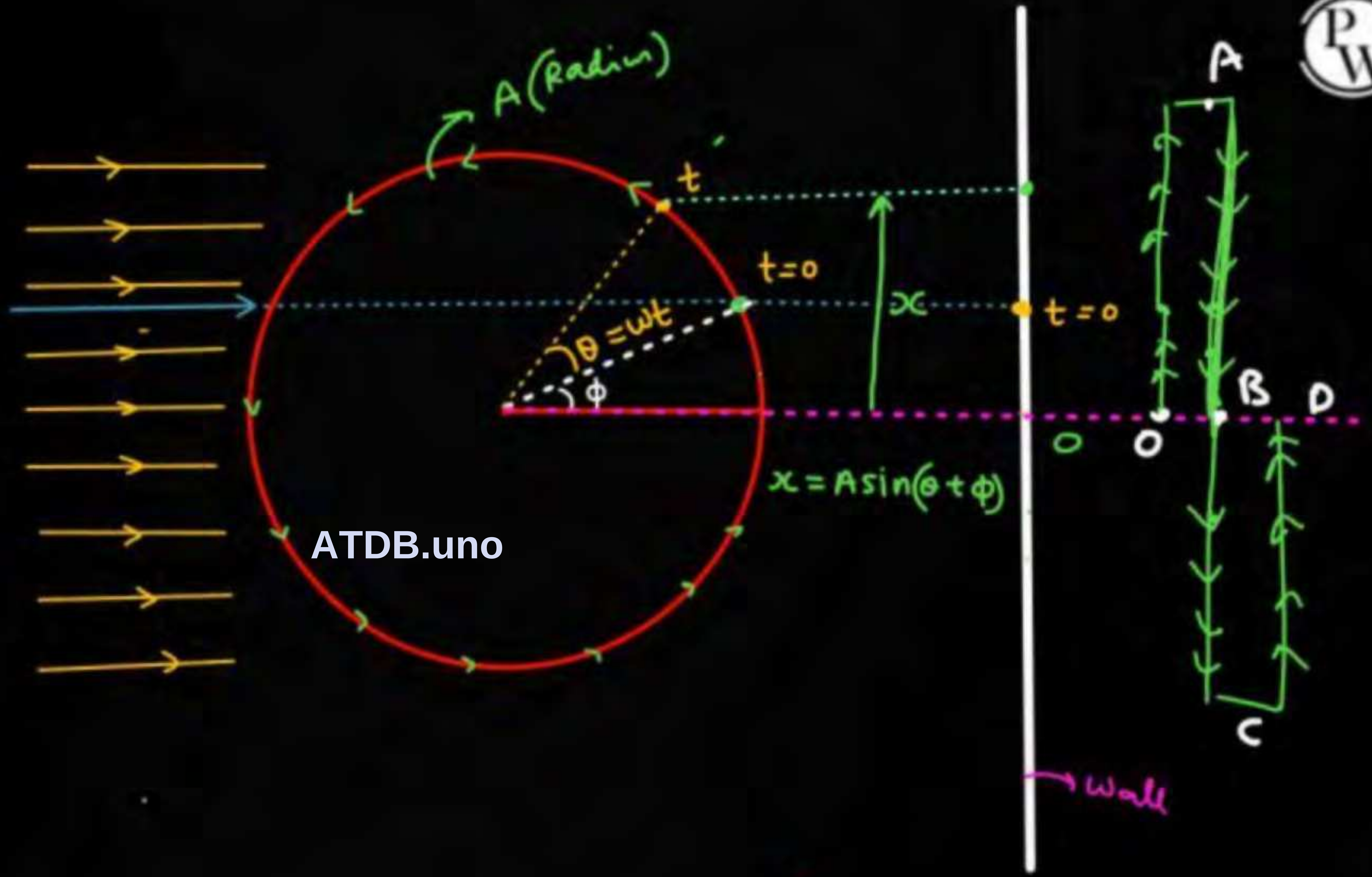
Ⓐ ≡ Uniform circular motion, T, ω

Shadow ⟶ SHM.

$x = A\sin(\theta + \phi)$

$$\boxed{x = A\sin(\omega t + \phi)}$$

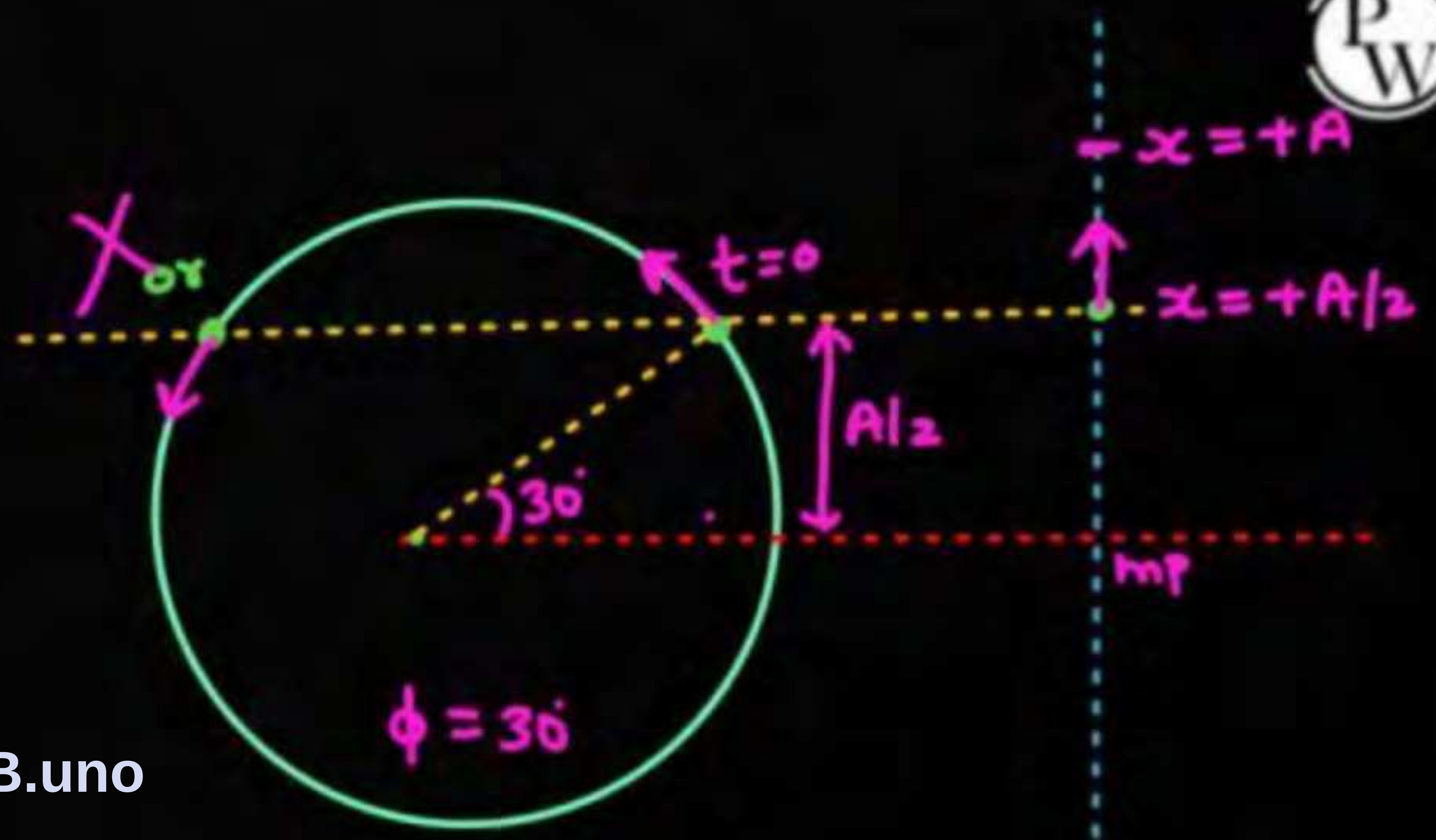
SHM



$t=0,\ x=+\frac{A}{2}$ moving Away from mp

Q

$$x = A\sin(\omega t + 30^\circ)$$



Q Two particle are performing SHM simultaneously in a single line. having same amplitude.

$x_1 = A\sin\omega t$ = A

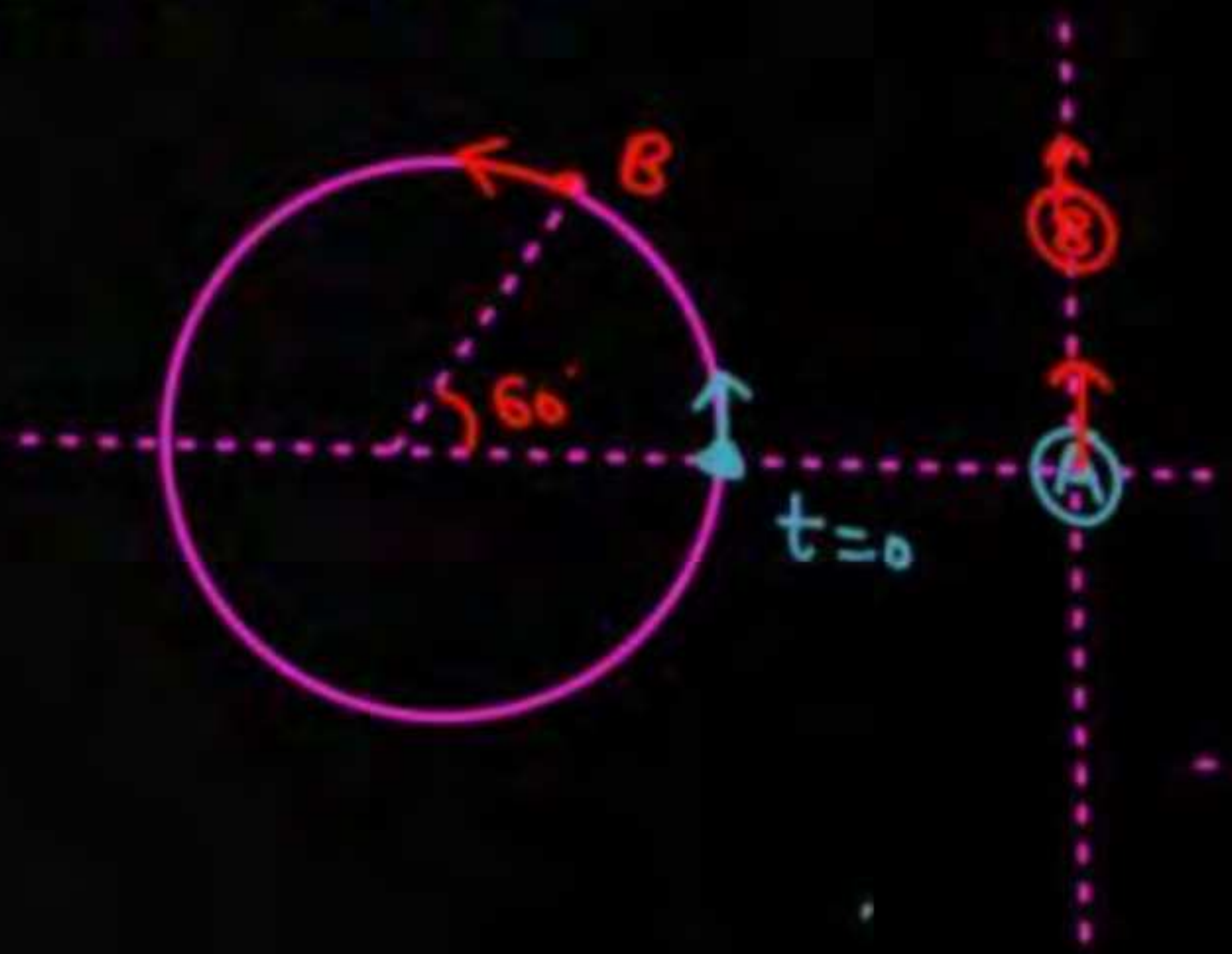
$x_2 = A\sin(\omega t + 60)$ = B

(ω, A $\longrightarrow$ same)

① find when they will meet

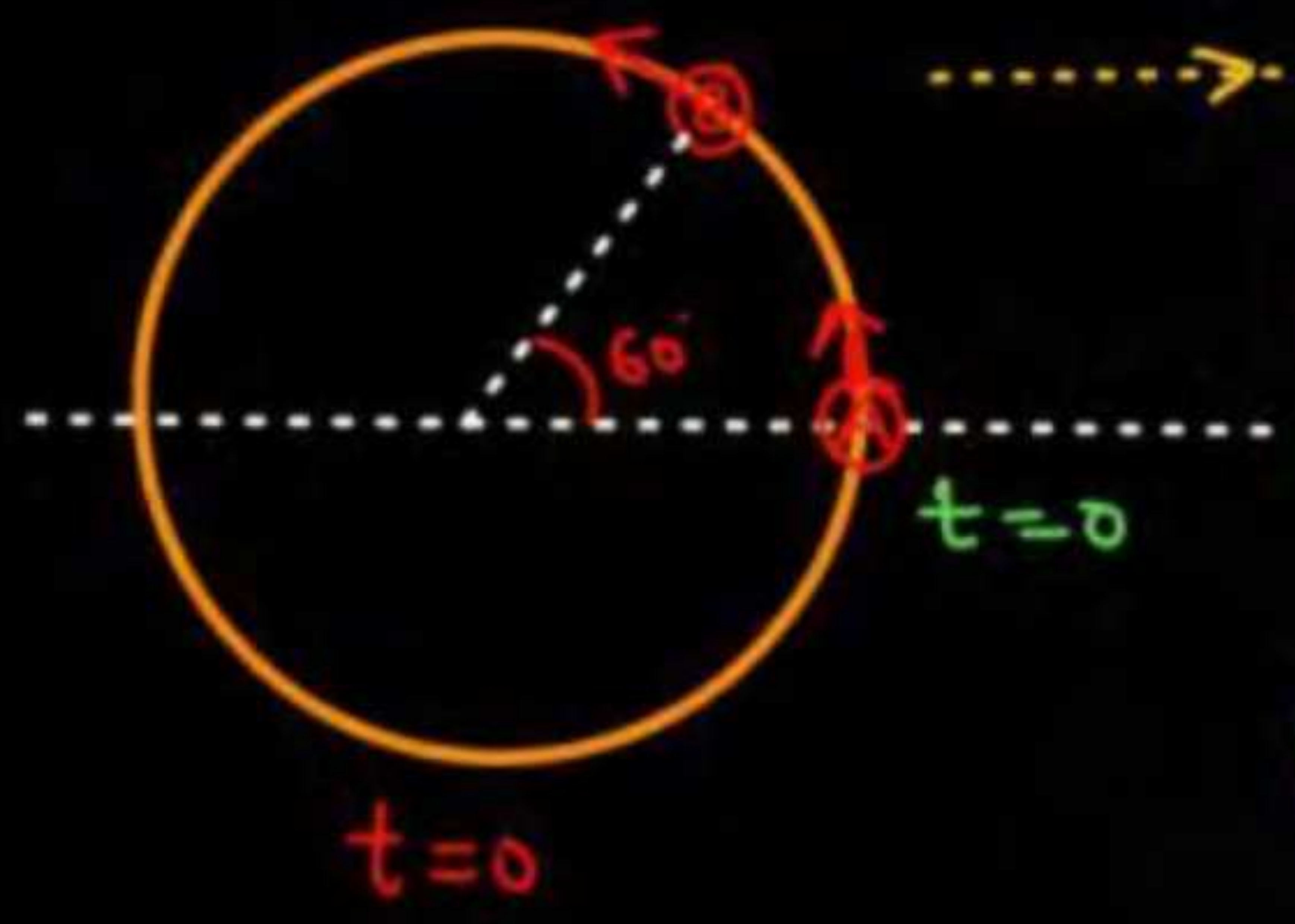
② find max sepr b/w them & when it will be.

$t = 0$



$x_1 = A\sin\omega t \Rightarrow A$

$x_2 = A\sin(\omega t + 60) \Rightarrow B$



(1) when they meet

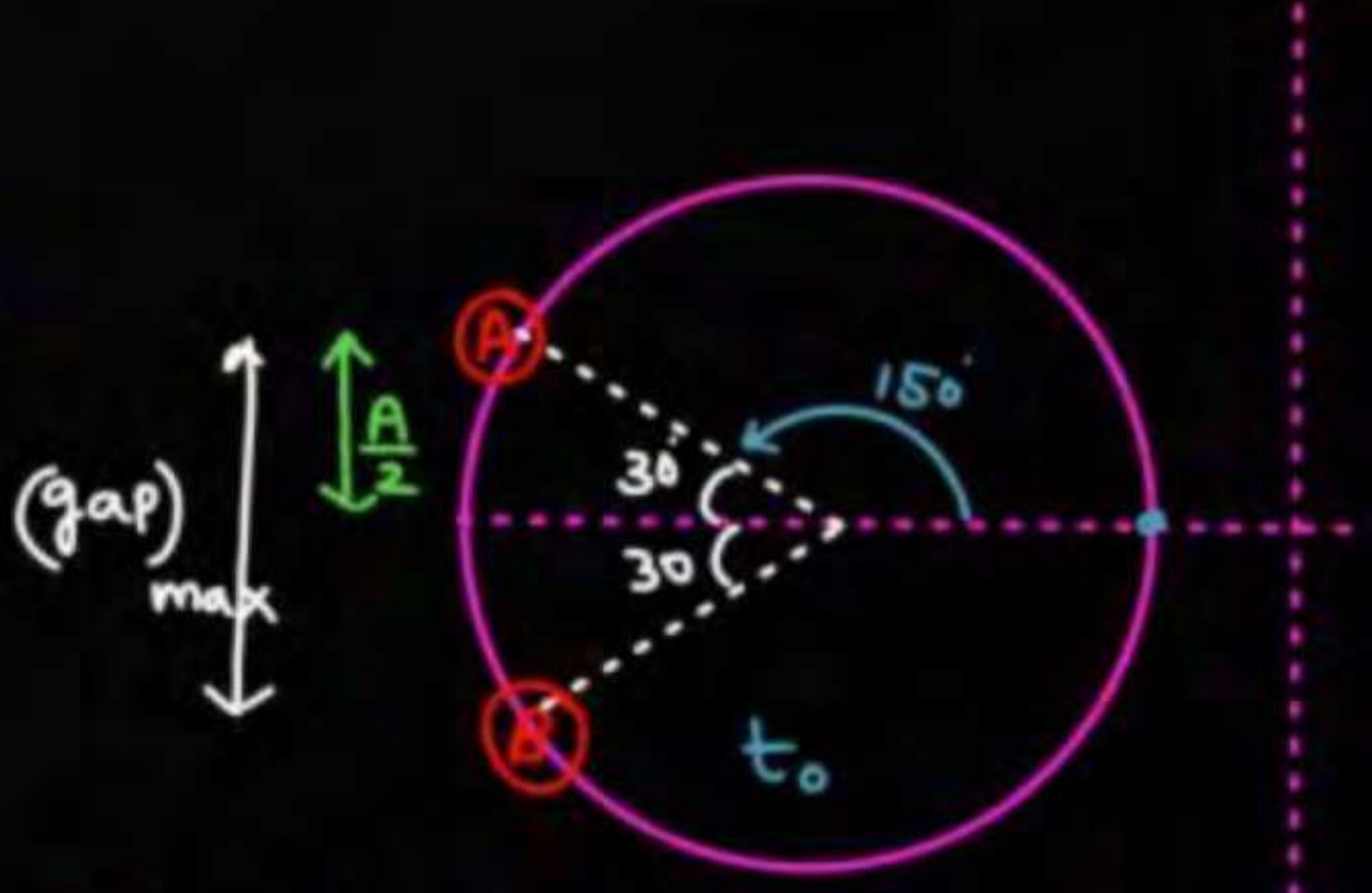
$(A) \equiv 60°$ घूमा

$360° \longrightarrow T$

$60 \longrightarrow T/6$

Ans $T/6$

(2) find max separation b/w them



$gap = 2A\sin 30 = A$

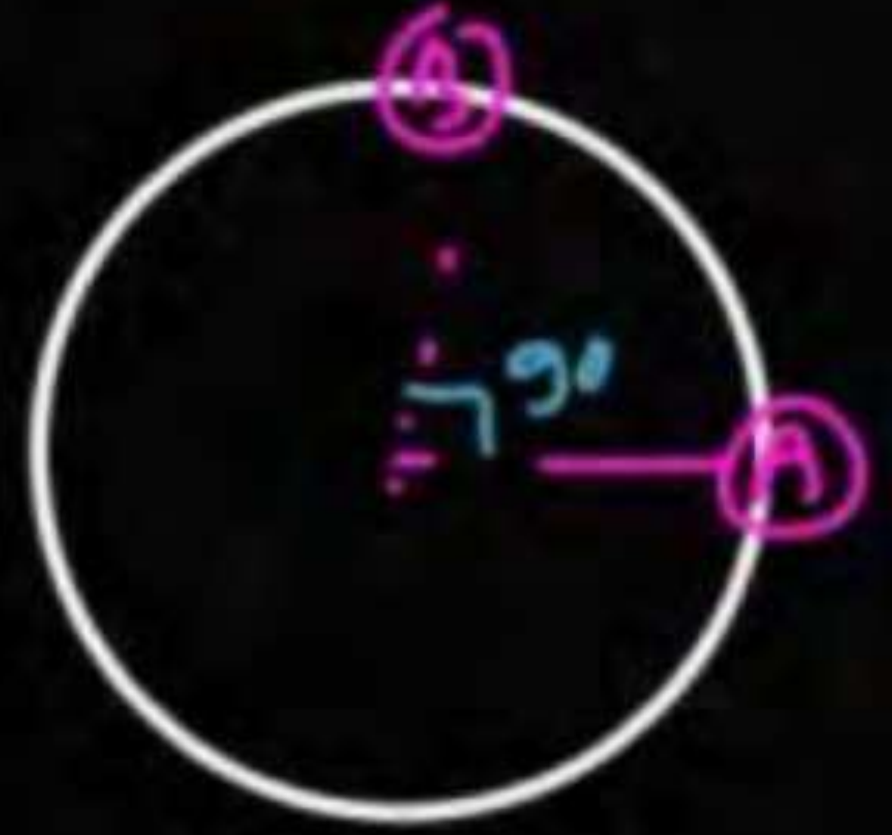
$360 \longrightarrow T$

$1 \longrightarrow \frac{T}{360}$

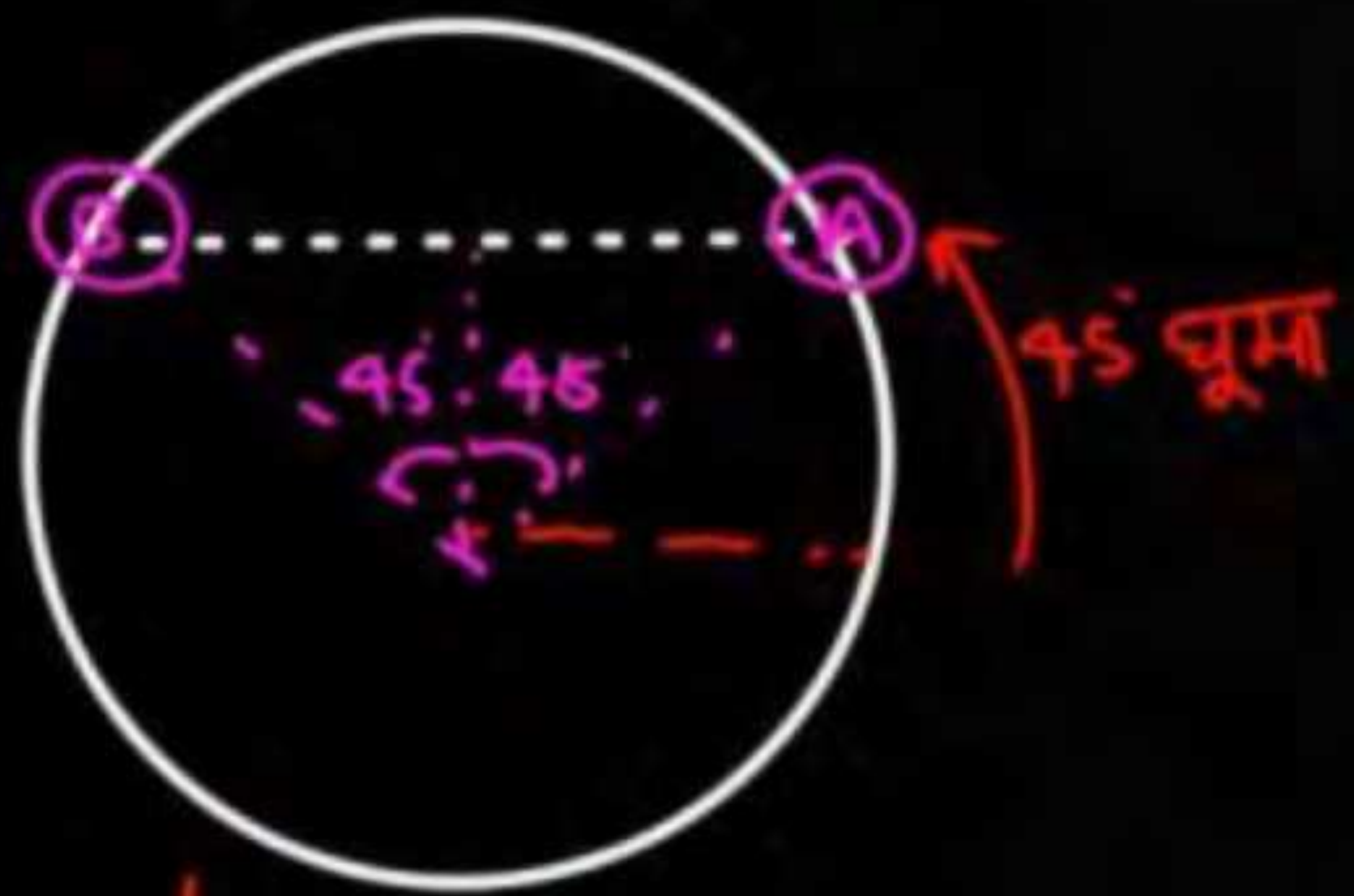
$150 \longrightarrow \frac{T}{360} \times 150$

Q $x_1 = A\sin\omega t$ (A)

$x_2 = A\sin(\omega t + 90^\circ)$ (B)

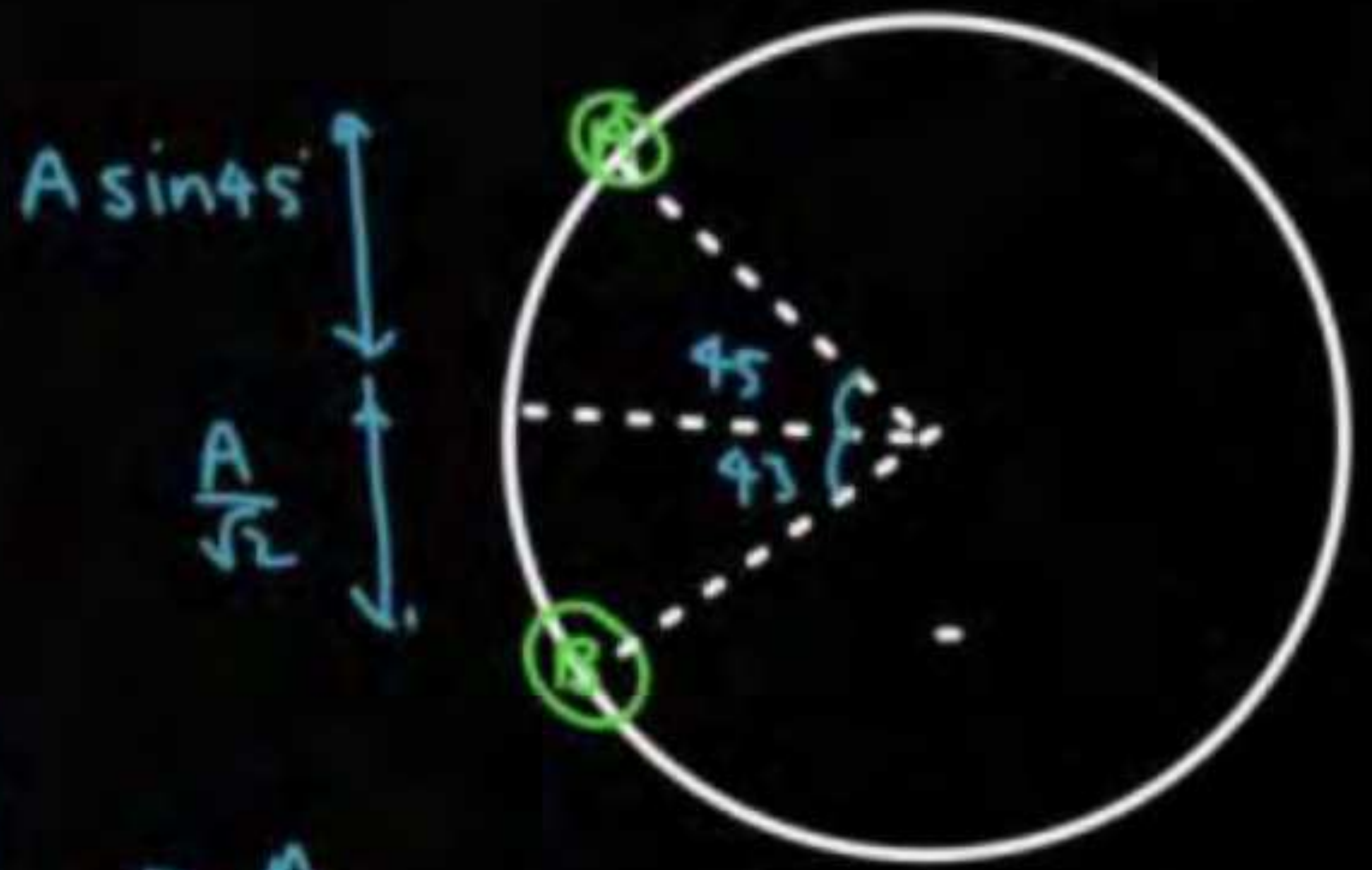


① When they will meet



$t = T/8$

② max sep^r



$$(\text{gap})_{max} = \frac{A}{\sqrt{2}} + \frac{A}{\sqrt{2}} = A\sqrt{2}$$

Superposition of SHM

$$\vec{F_1} = -k\vec{x_1} \longrightarrow x_1 = A_1 \sin \omega t$$

$$\vec{F_2} = -k\vec{x_2} \longrightarrow x_2 = A_2 \sin(\omega t + \phi)$$

$$x_{net} = A_{net} \sin(\omega t + \theta)$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\vec{F}_{net} = \vec{F_1} + \vec{F_2} = -k(\vec{x}_1 + \vec{x}_2)$$

$$\boxed{\vec{F}_{net} = -k\vec{x}_{net}}$$

$x_1 = A_1 \sin\omega t$

$x_2 = A_2 \sin(\omega t + \phi)$

$x_{net} = x_1 + x_2$

$x_{net} = A_{net} \sin(\omega t + \theta)$

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

phase diff.

write

you may skip this in notes

maths

$x_1 = A_1 \sin\omega t$

$x_2 = A_2 \sin(\omega t + \phi)$

$x_{net} = x_1 + x_2$

$x_{net} = A_1 \sin\omega t + A_2 \sin(\omega t + \phi)$

$= A_1 \sin\omega t + A_2 \sin\omega t \cos\phi + A_2 \cos\omega t \sin\phi$

$x_{net} = (A_1 + A_2\cos\phi)\sin\omega t + A_2 \sin\phi \cos\omega t$

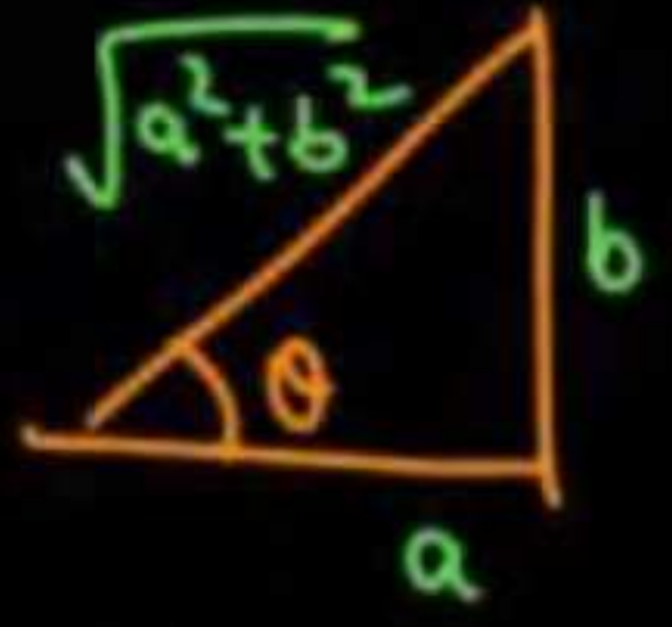
$x_{net} = a \sin\omega t + b\cos\omega t$

$x_{net} = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\sin\omega t + \frac{b}{\sqrt{a^2+b^2}}\cos\omega t\right)$

$x_{net} = \sqrt{a^2+b^2}\,\sin(\omega t + \theta)$

$x_{net} = A_{net}\sin(\omega t + \theta)$

$\tan\theta = \frac{b}{a} = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$



$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$$A_{net} = \sqrt{a^2 + b^2}$$

$$= \sqrt{(A_1 + A_2\cos\phi)^2 + (A_2\sin\phi)^2}$$

$$= \sqrt{A_1^2 + A_2^2\cos^2\phi + 2A_1A_2\cos\phi + A_2^2\sin^2\phi}$$

$$\boxed{A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}}$$

Q

$$x_1 = A\sin\omega t$$
$$x_2 = A\sin(\omega t + 60^\circ)$$

$$x_{net} = x_1 + x_2 = A_{net}\sin(\omega t + \theta)$$

$$A_{net} = \sqrt{A^2 + A^2 + 2\cdot A\cdot A\cdot\cos 60^\circ}$$
$$= A\sqrt{3}$$

$$\tan\theta = \frac{A\sin 60}{A + A\cos 60} = \frac{\sqrt{3}/2}{1+\frac{1}{2}} = \frac{1}{\sqrt{3}}$$

$$\boxed{\theta = 30^\circ}$$

Q

$$x_1 = A\sin\omega t$$
$$x_2 = A\sqrt{3}\cos\omega t = A\sqrt{3}\sin(\omega t + 90^\circ)$$

$$x_{net} = ?$$

$$A_{net} = \sqrt{A^2 + (A\sqrt{3})^2 + 2\cdot A\cdot A\sqrt{3}\cos 90^\circ}$$
$$= 2A$$

$$\tan\theta = \frac{A\sqrt{3}\sin 90}{A + A\sqrt{3}\cos 90^\circ} = \sqrt{3}$$

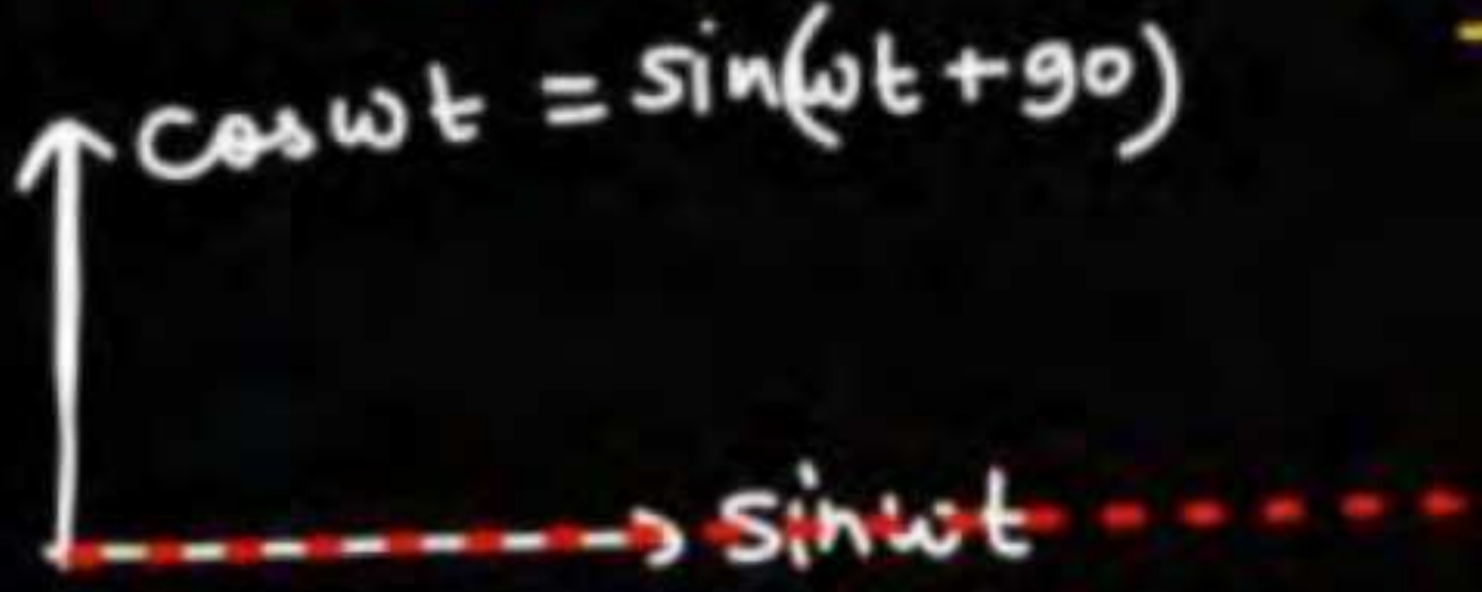
$$\theta = 60^\circ$$

$$A_{net} = 2A\sin(\omega t + 60^\circ)$$

Q

$$x_1 = A\sin\omega t$$
$$x_2 = A\sin(\omega t + 60^\circ)$$

$$x_{net} = x_1 + x_2 = A_{net}\sin(\omega t + \theta)$$



$$A_{net} = \sqrt{A^2 + A^2 + 2AA\cos 60^\circ}$$

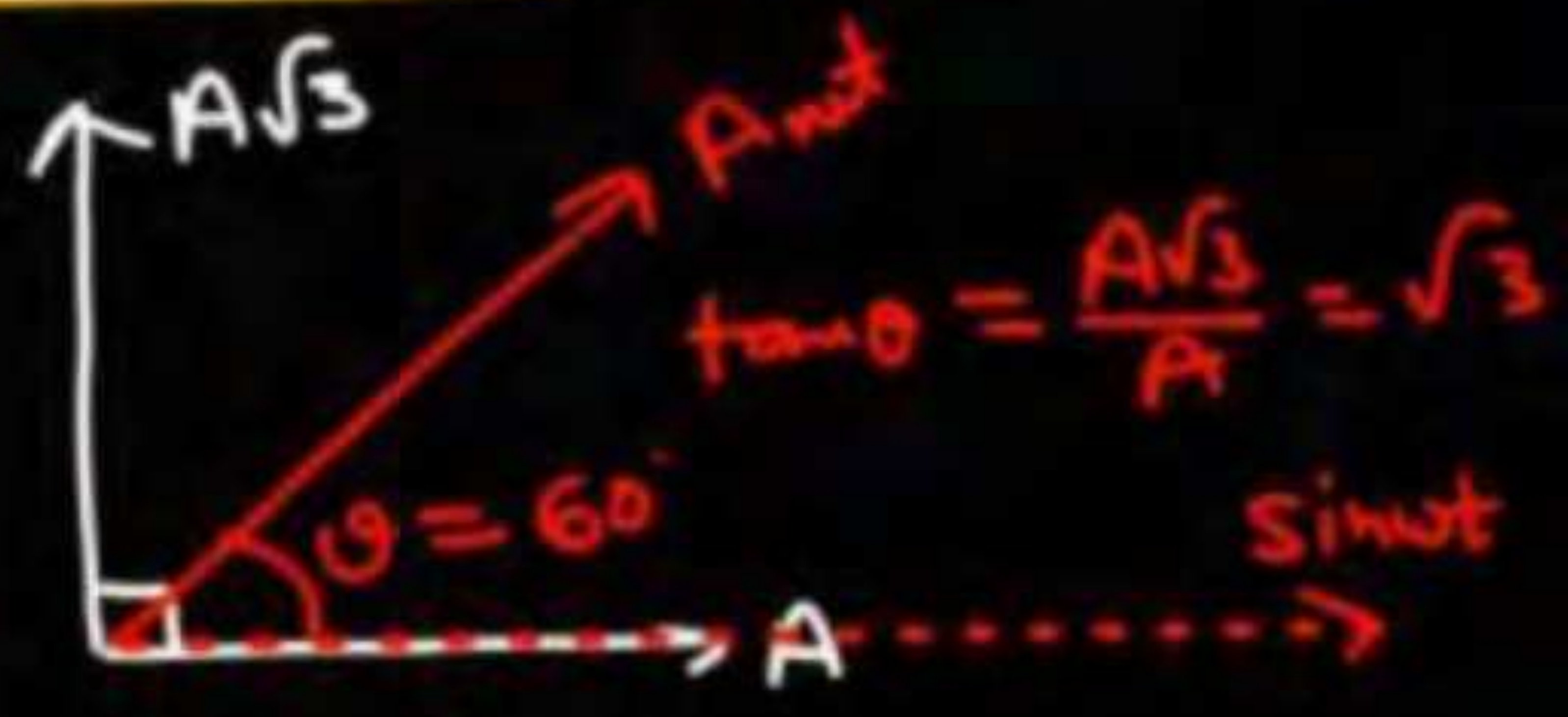
$$A_{net} = A\sqrt{3}$$

$$\boxed{x_{net} = A\sqrt{3}\sin(\omega t + 30^\circ)}$$

Q

$$x_1 = A\sin\omega t$$
$$x_2 = A\sqrt{3}\cos\omega t = A\sqrt{3}\sin(\omega t + 90^\circ)$$

$$x_{net} = ?$$



$$\tan\theta = \frac{A\sqrt{3}}{A} = \sqrt{3}$$

$$A_{net} = \sqrt{A^2 + (A\sqrt{3})^2} = 2A$$

$$x_{net} = 2A\sin(\omega t + 60^\circ)$$

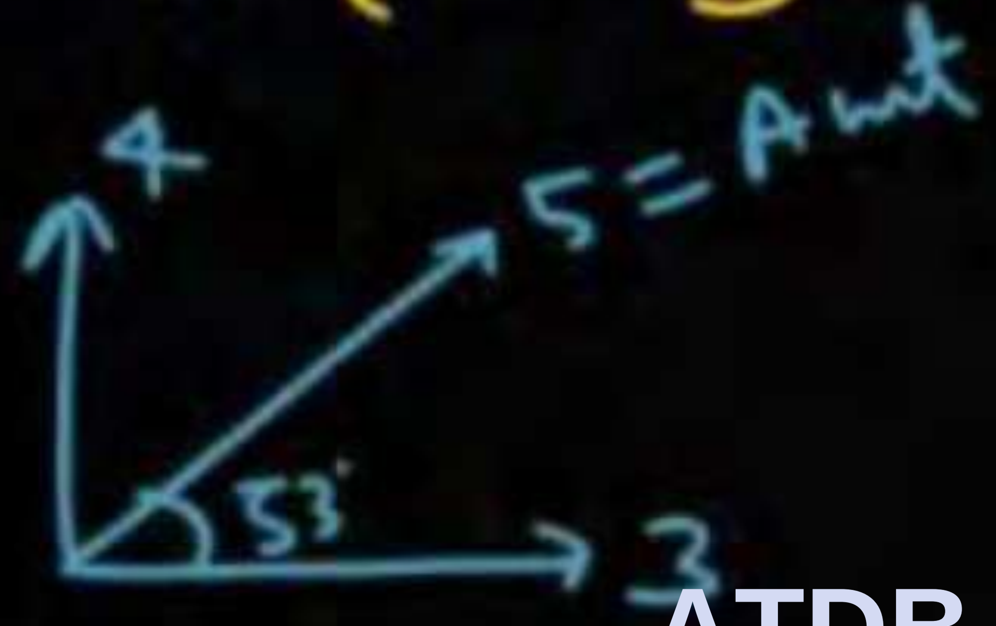
Q

$$x_1 = 3\sin\omega t$$

$$x_2 = 4\sin(\omega t + 90^\circ)$$

$$x_{net} = 3\sin\omega t + 4\sin(\omega t + 90^\circ)$$

$$x_{net} = 5\sin(\omega t + 53^\circ)$$



Q

$$x_1 = 10\sqrt{2}\sin(\omega t + 45^\circ)$$

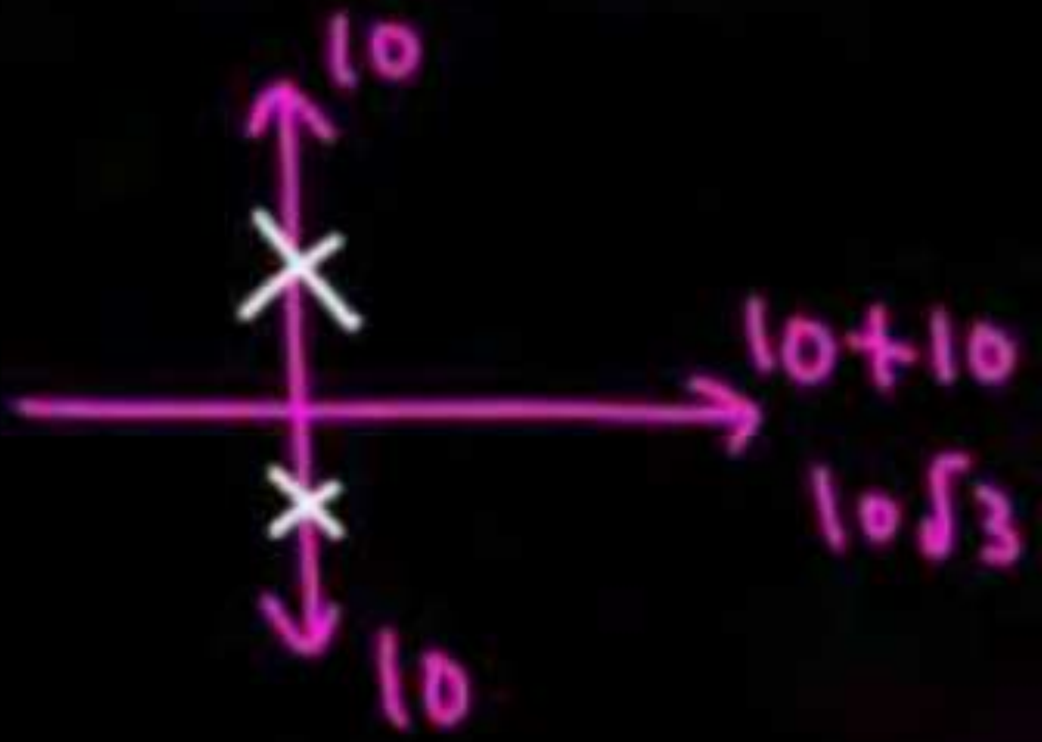
$$x_2 = 10\sin\omega t$$

$$x_3 = 20\sin(\omega t - 30^\circ)$$

find $x_1 + x_2 + x_3 = ?$

$$A_{net} = 20 + 10\sqrt{3}$$

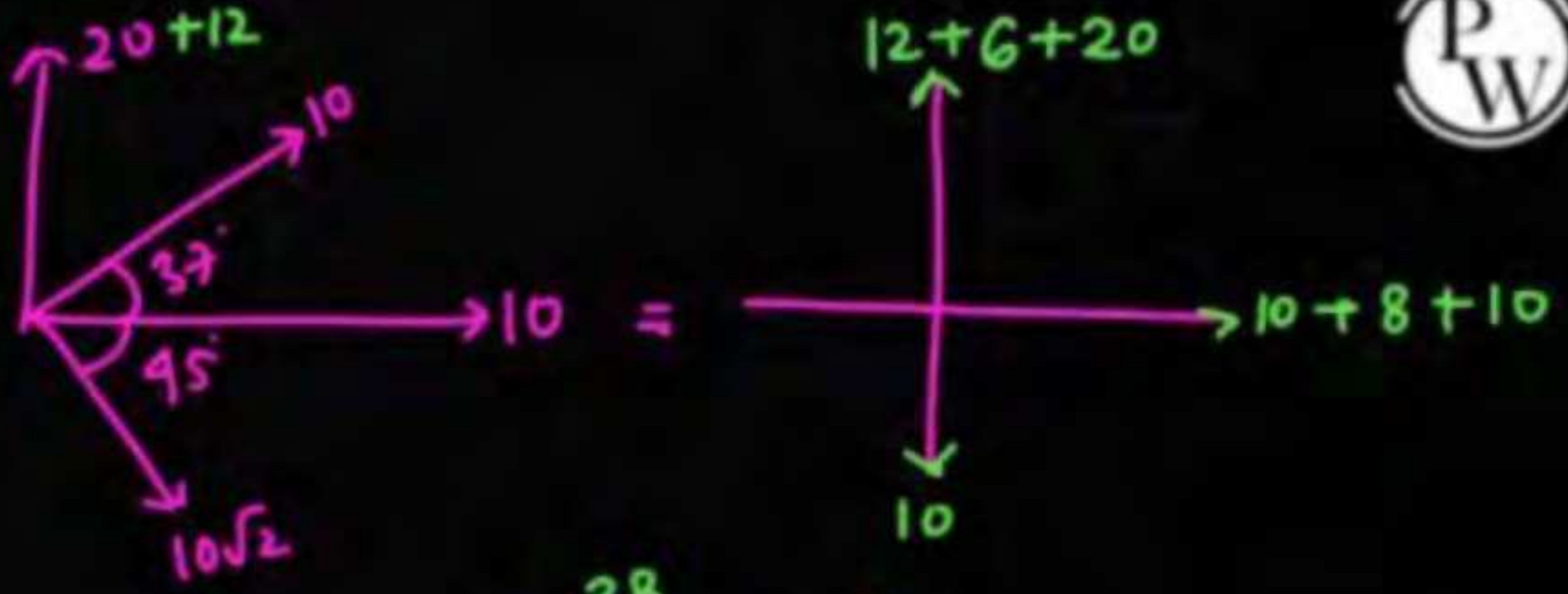
$$\boxed{x_{net} = (20 + 10\sqrt{3})\sin(\omega t + 0^\circ)}$$



Q

$$x_1 = 10 \sin \omega t$$
$$x_2 = 20 \sin(\omega t + 90^\circ)$$
$$x_3 = 10 \sin(\omega t + 37^\circ)$$
$$x_4 = 10\sqrt{2} \sin(\omega t - 45^\circ)$$
$$x_5 = 12 \sin(\omega t + 90)$$

$$x_{net} = 28\sqrt{2} \sin(\omega t + 45^\circ)$$

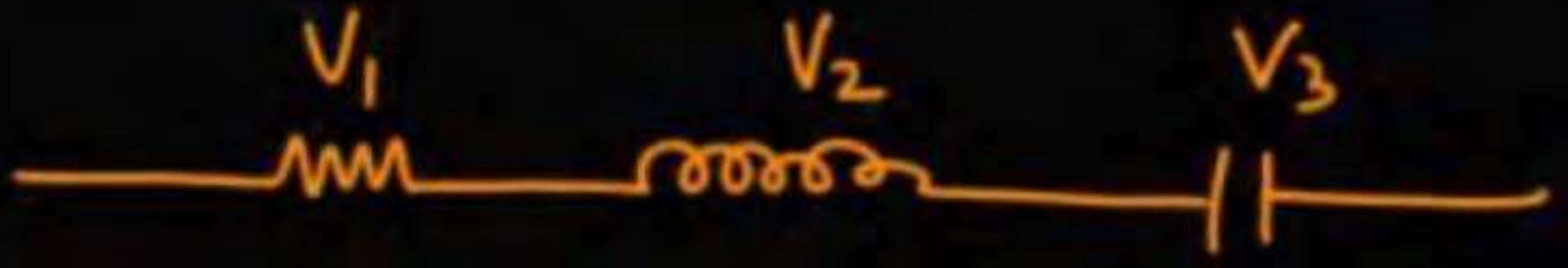


Q

$$V_1 = 10\sin\omega t$$

$$V_2 = 20\sin(\omega t + 90^\circ)$$

$$V_3 = 30\sin(\omega t - 90^\circ)$$



$$V_1 + V_2 + V_3 =$$

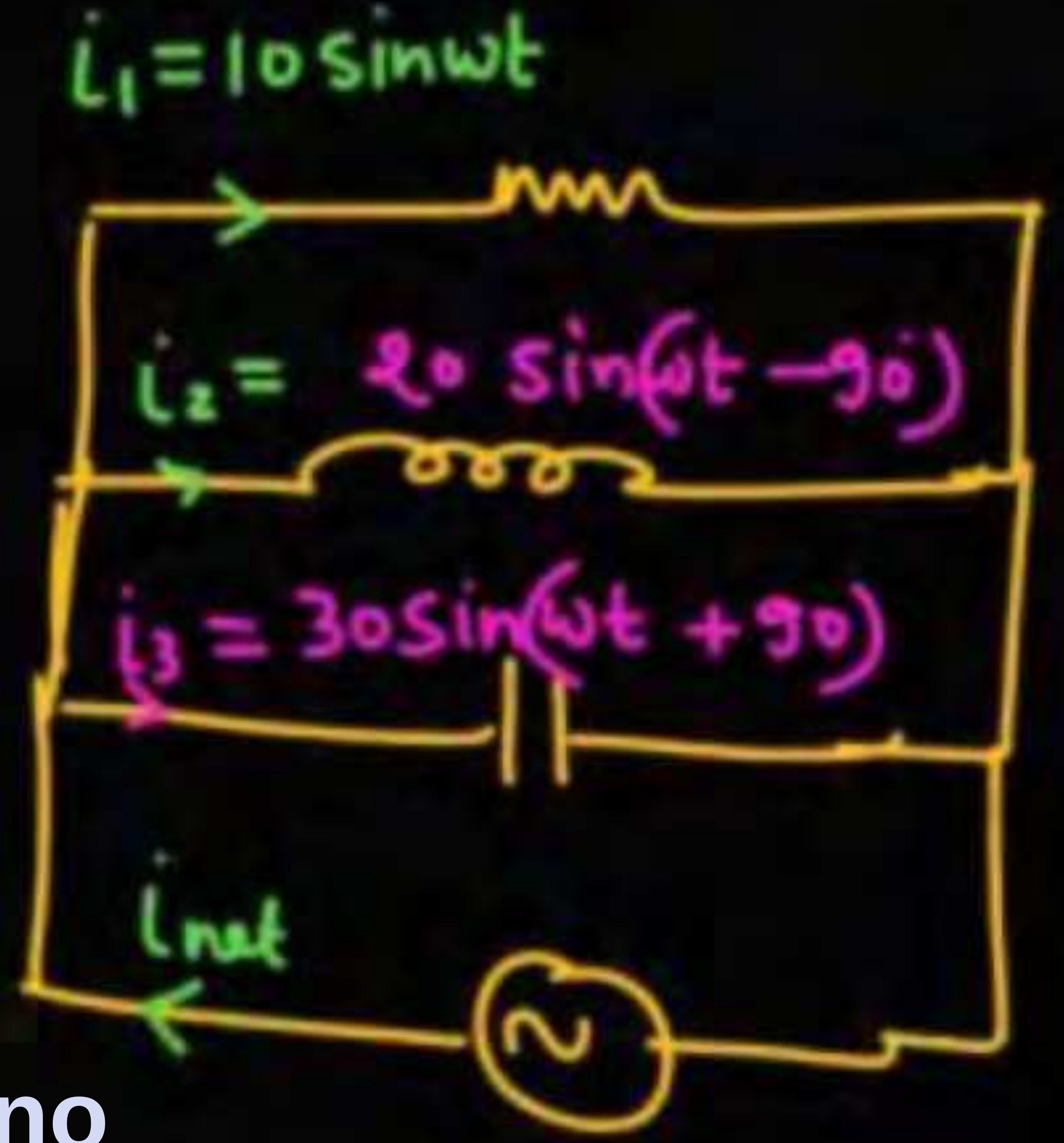
$$= 10\sqrt{2}\sin(\omega t - 45^\circ)$$

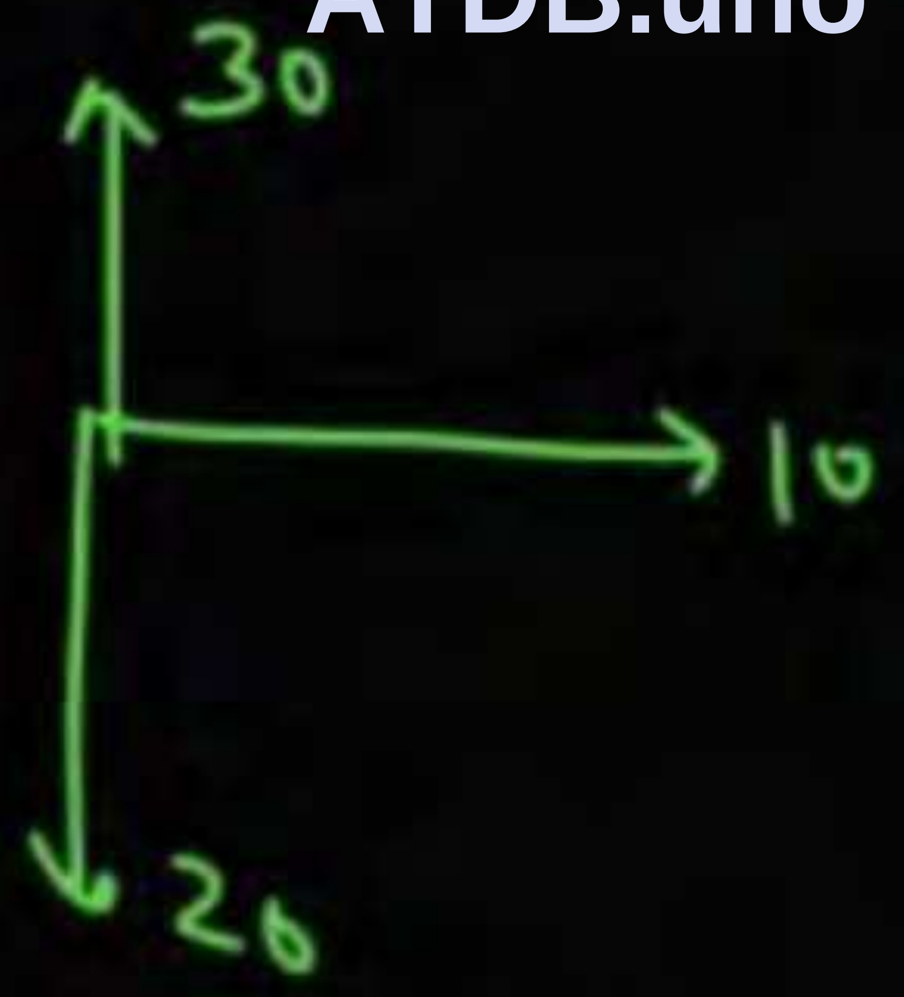
Q

$$i_{net} = i_1 + i_2 + i_3$$

$$= 10\sin\omega t + 20\sin(\omega t - 90^\circ) + 30\sin(\omega t + 90^\circ)$$

$$= 10\sqrt{2}\sin(\omega t + 45^\circ)$$





Homework

- Revise full SHM
- module → Parikshit JA ⇒ 10, 17, 18, 19-23, 27,
  PYQ → All


THANK
YOU