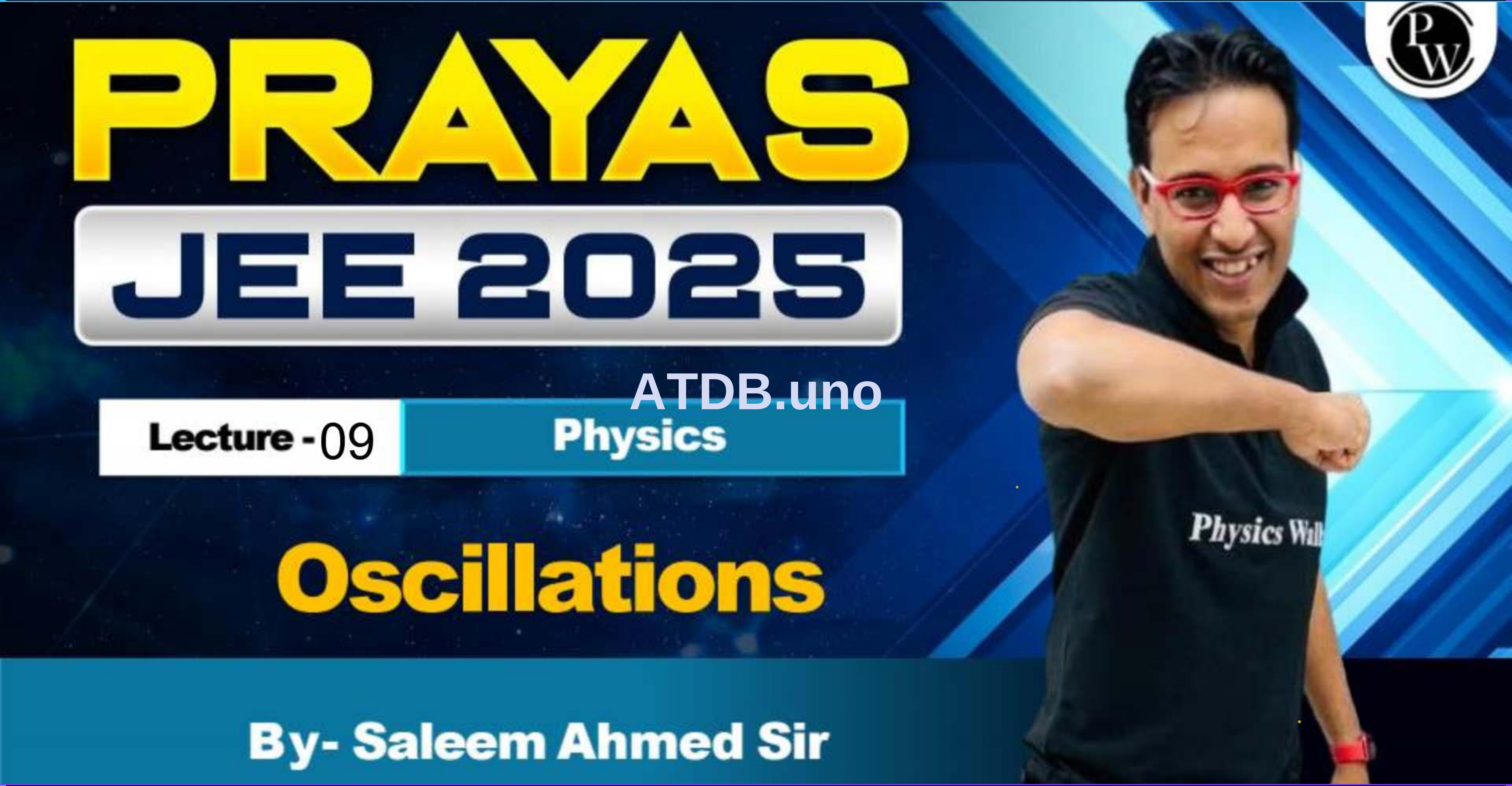

PRAYAS
JEE 2025
Lecture - 09
Physics
Oscillations
By- Saleem Ahmed Sir
ATDB.uno
Physics Wall

Topics *to be covered*

Time period of SHM In Various Cases.

1

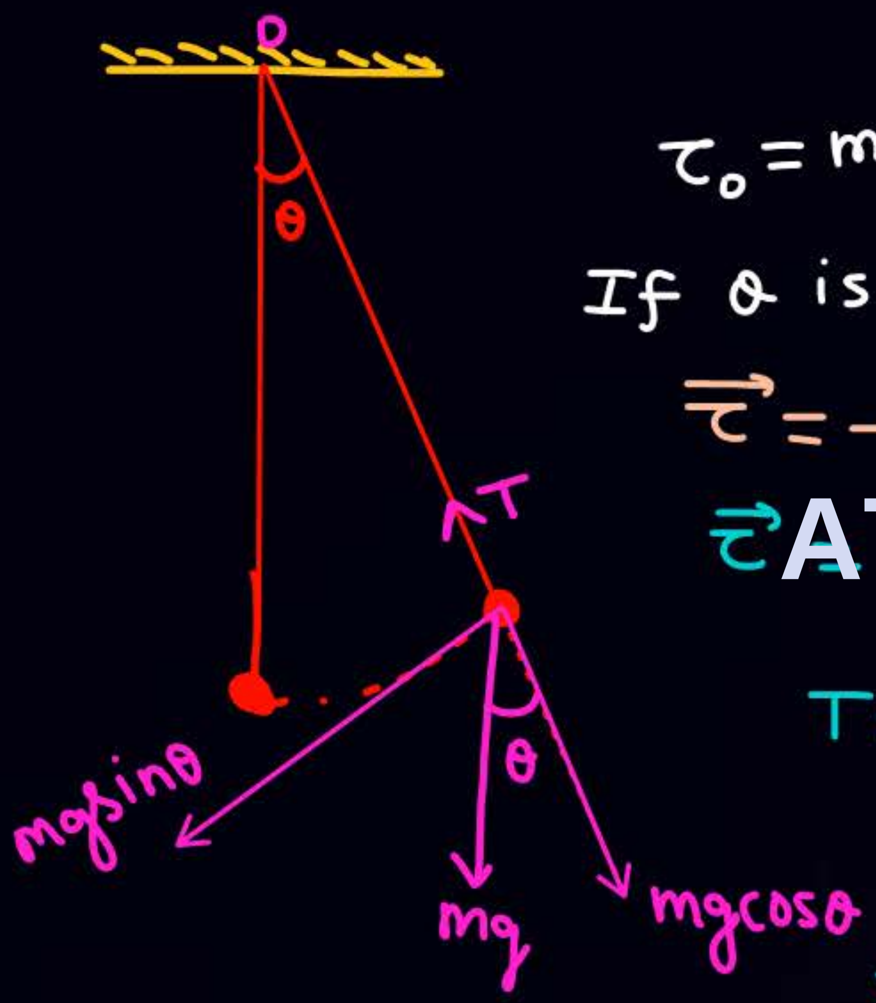
2

3

4

Simple pendulum - $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

Second pendulum $T = 2\ sec$, $l = 1m$



$T = 2\pi\sqrt{\frac{I}{mgd_c}}$

$T = 2\pi\sqrt{\frac{mL^2}{mgL}}$

$T = 2\pi\sqrt{\frac{L}{g}}$

$\tau_0 = mg\sin\theta \cdot L$

If θ is very small $\sin\theta \simeq \theta$

$\vec{\tau} = -mgL\vec{\theta}$

$\vec{\tau} = -k\vec{\theta}$

$T = 2\pi\sqrt{\frac{I}{K}} = 2\pi\sqrt{\frac{mL^2}{mgL}}$

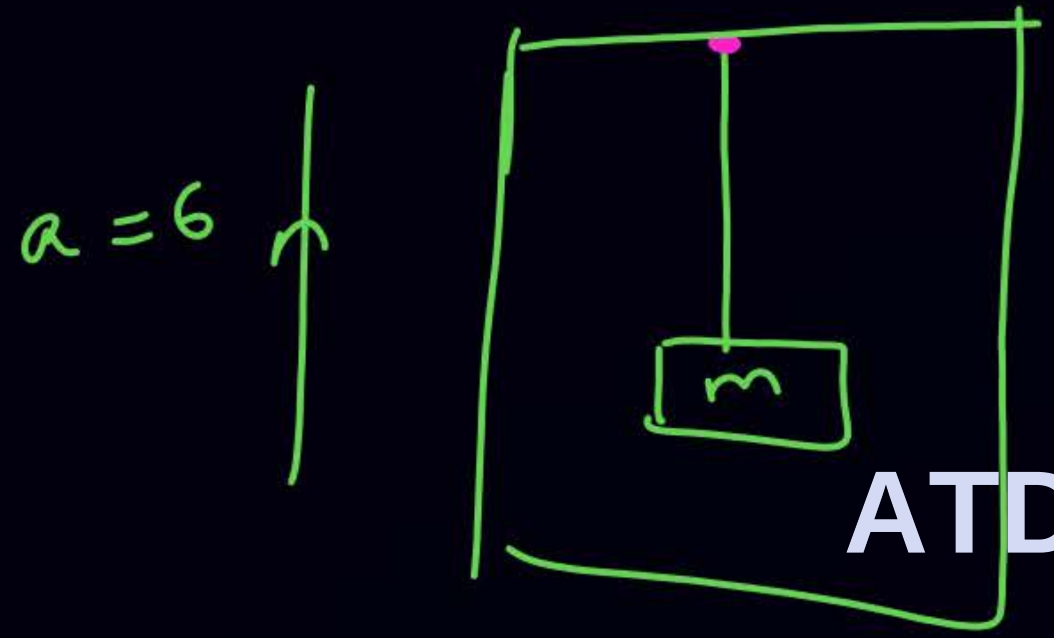
$T = 2\pi\sqrt{\frac{L}{g}}$

$\sin\theta \simeq \tan\theta \simeq \frac{x}{L}$

$F_{net} = mg\sin\theta \simeq mg\frac{x}{L}$

$\vec{F}_{net} = -\frac{mg}{L}\vec{x}$

$T = 2\pi\sqrt{\frac{m}{mg/L}} = 2\pi\sqrt{\frac{L}{g}}$



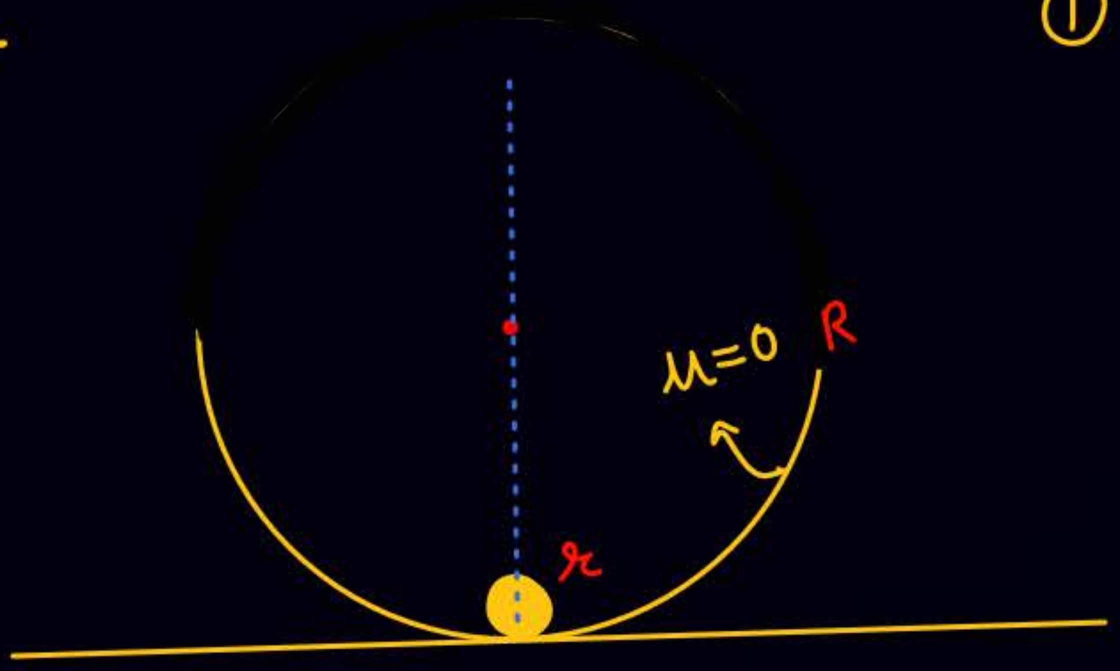
$$y = 3t^2$$

$$v_y = 6t$$

$$a_y = 6$$

$$T = 2\pi\sqrt{\frac{l}{g+a}} = 2\pi\sqrt{\frac{l}{16}}$$

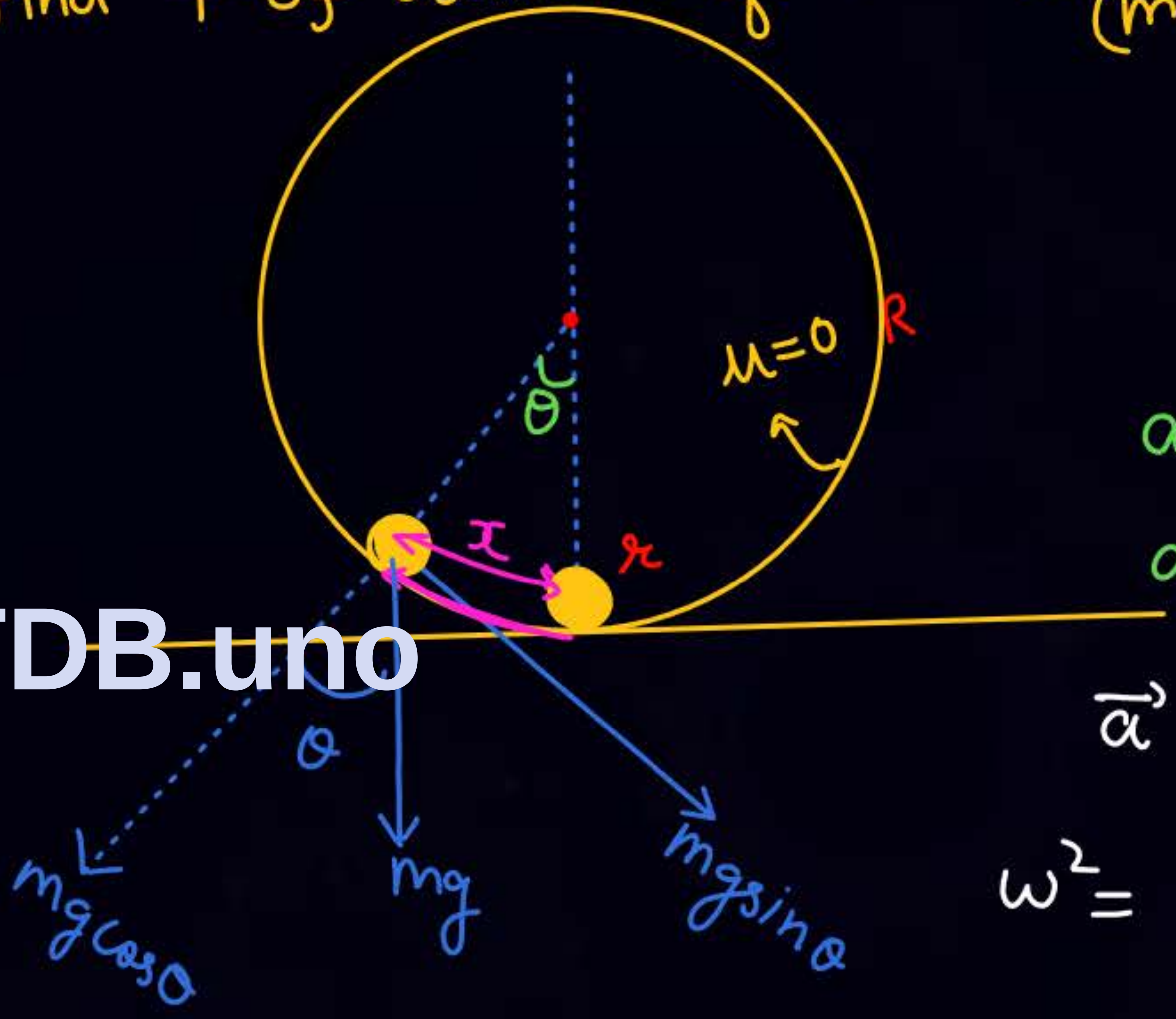
Q



If ball is very small
$r << R$

$T = 2\pi\sqrt{\frac{R}{g}}$

(1) Inner surface of hemisphere is smooth
find T of oscillation of ball. (solid sphere) (m, r)



$a = g\sin\theta$

$a = g.\frac{x}{R-r}$

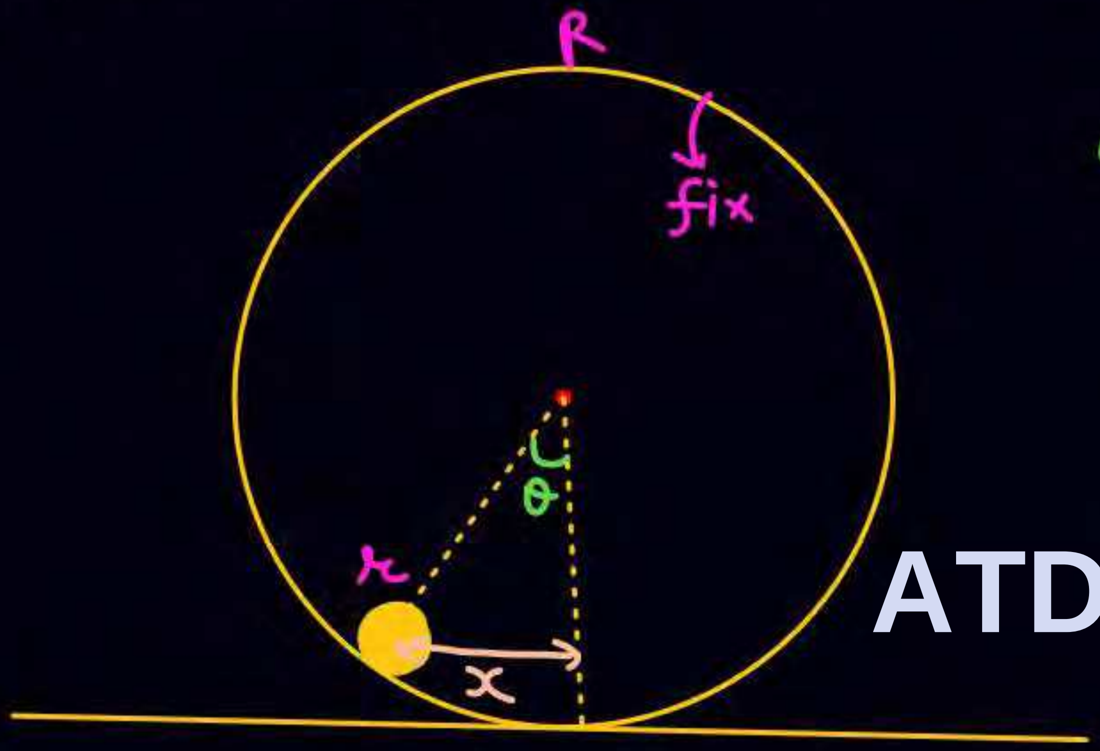
$\vec{a} = -\frac{g}{R-r}\vec{x}$

$\omega^2 = \frac{g}{R-r}$

$T = 2\pi\sqrt{\frac{R-r}{g}}$

Repeat the above prob.

(b) $\mu \neq 0$, inner surface have sufficient friction for rolling.



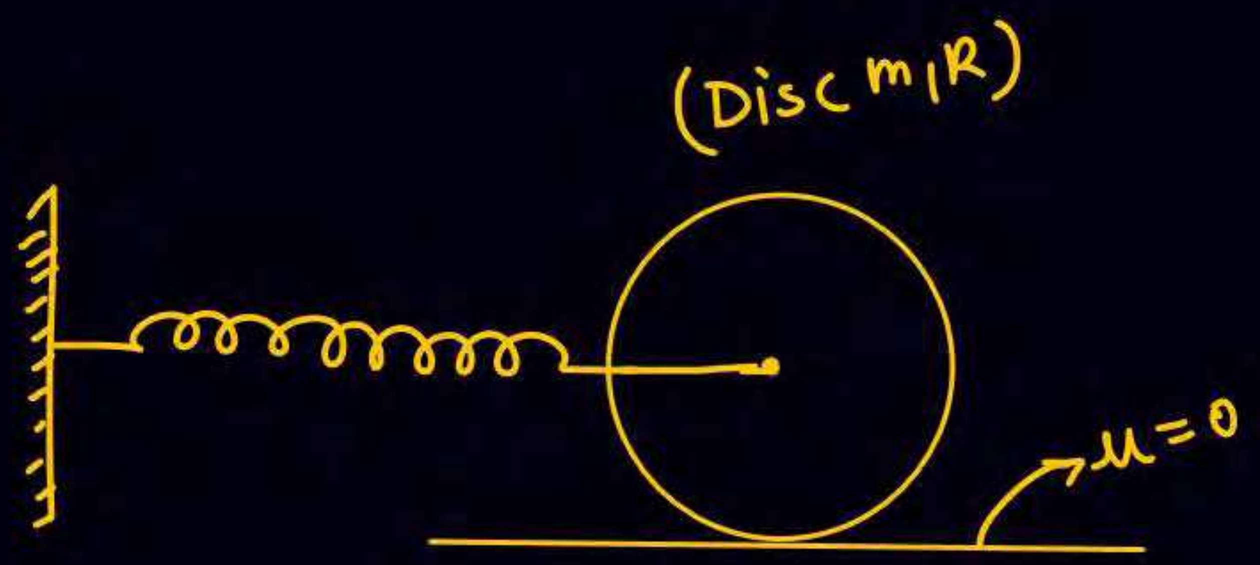
$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{\frac{2}{5}mR^2}{mR^2}} = \frac{5}{7}g\sin\theta$$

$$a = \frac{5}{7}g\frac{x}{R-r}$$

$$\vec{a} = -\frac{5g}{7(R-r)}\vec{x}$$

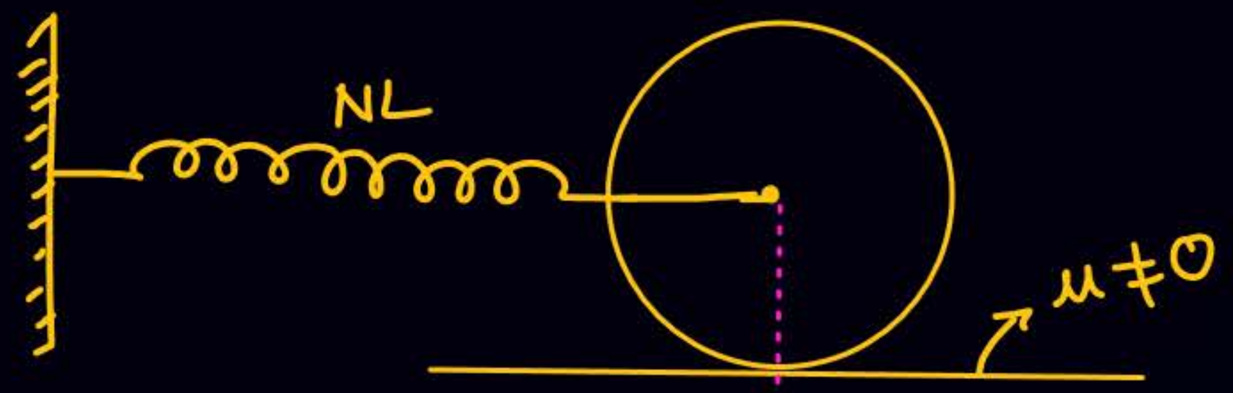
$$T = 2\pi\sqrt{\frac{\frac{7}{5}(R-r)}{g}}$$

Q



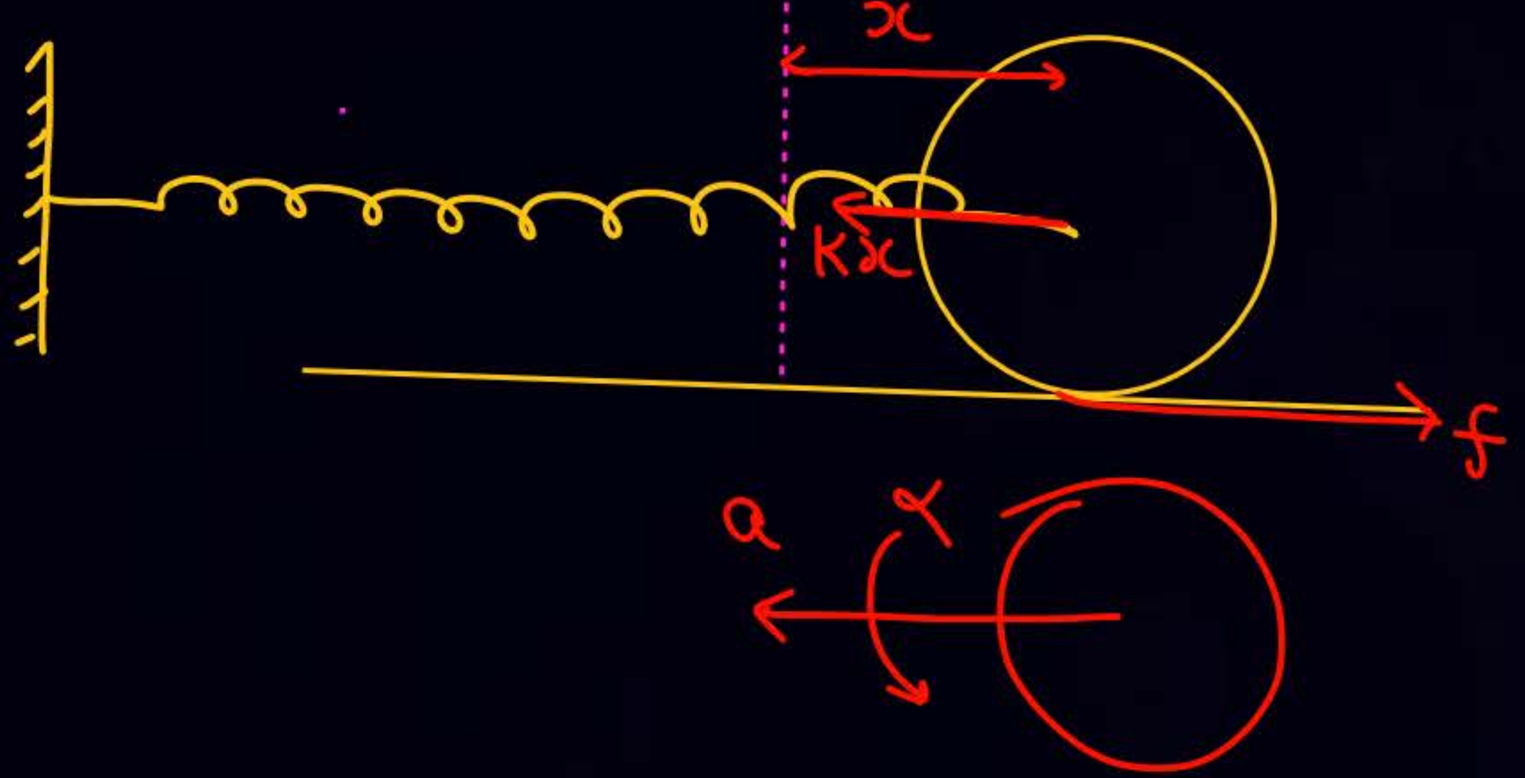
$$T = 2\pi\sqrt{\frac{m}{K}}$$

Q (Disc m, R)



friction is sufficient for pure rolling find T of oscillation for small displacement.

Sol^n



$$kx - f = ma$$

$$f \cdot R = I\alpha = \frac{mR^2}{2} \cdot \frac{a}{R}$$

$$f = \frac{ma}{2}$$

$$kx - \frac{ma}{2} = ma$$

$$kx = \frac{3ma}{2}$$

$$a = \frac{2K}{3m} x$$

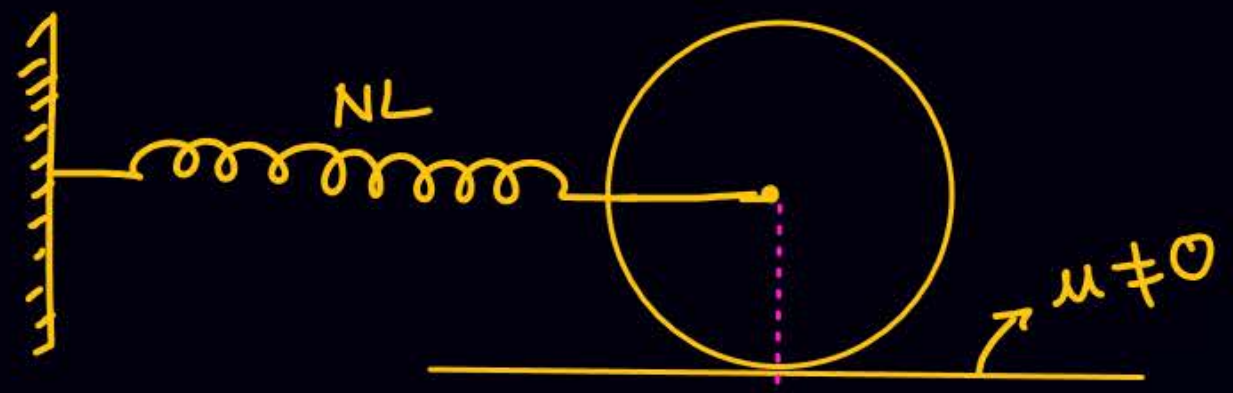
$$\vec{a} = -\frac{2K}{3m} \vec{x}$$

$$\vec{a} = -\omega^2 \vec{x}$$

$$\omega = \sqrt{\frac{2K}{3m}}$$

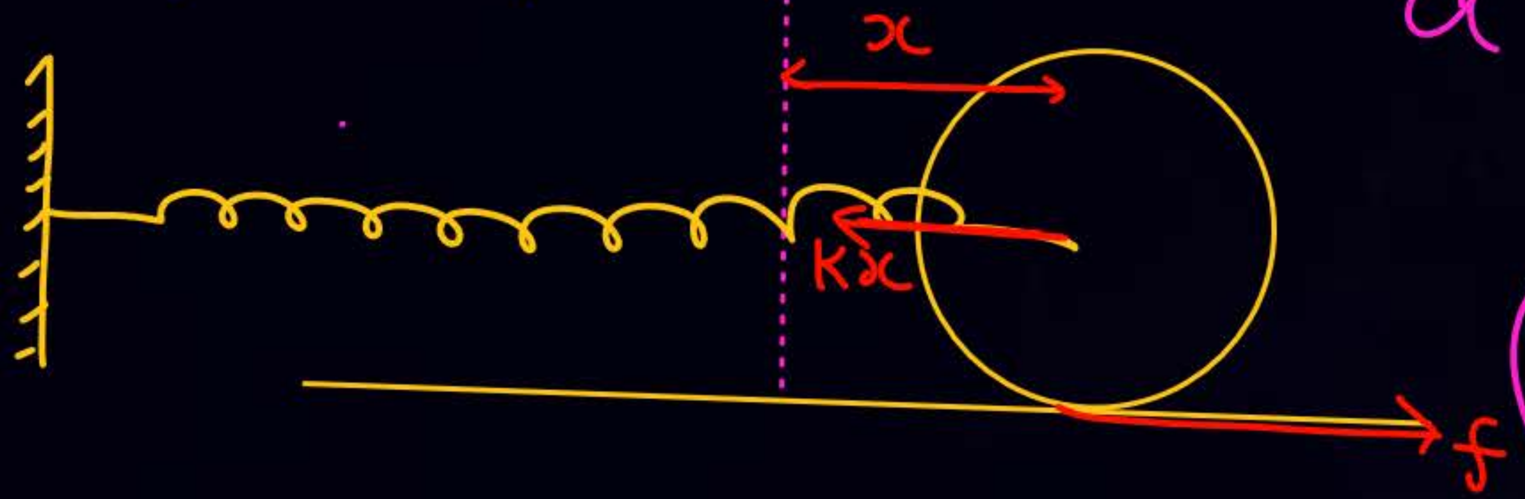
$$T = 2\pi \sqrt{\frac{3m}{2K}}$$

Q (Disc m, R)



friction is sufficient for pure rolling find T of oscillation for small displacement.

Sol^n



$$\frac{d}{dx}\left(\frac{1}{2}Kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{mR^2}{2}\cdot\frac{v^2}{R^2}\right) = 0$$

$$\frac{K}{2}2x + \frac{3}{4}m\left(2v\frac{dv}{dx}\right) = 0$$

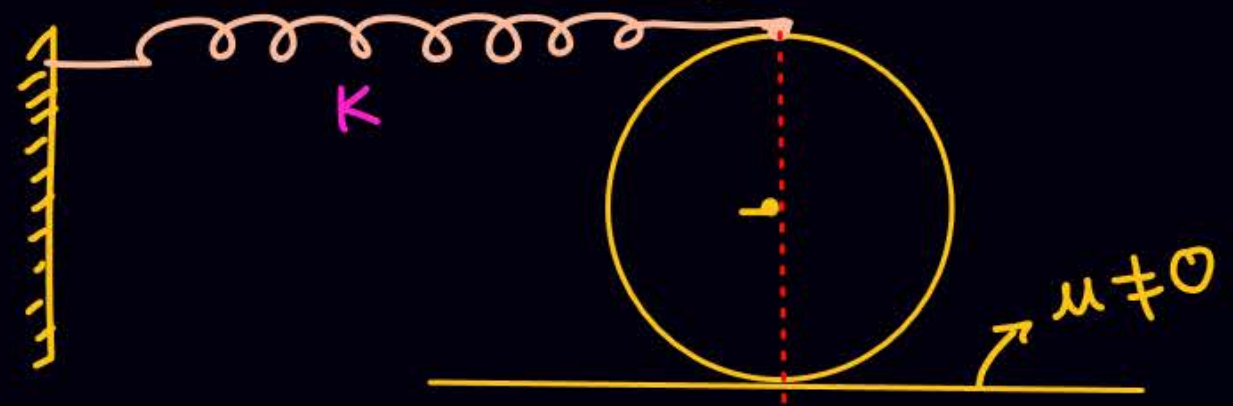
$$Kx = -\frac{3}{2}ma$$

$$a = -\frac{2Kx}{3m}$$

$$T = 2\pi\sqrt{\frac{3m}{2K}}$$

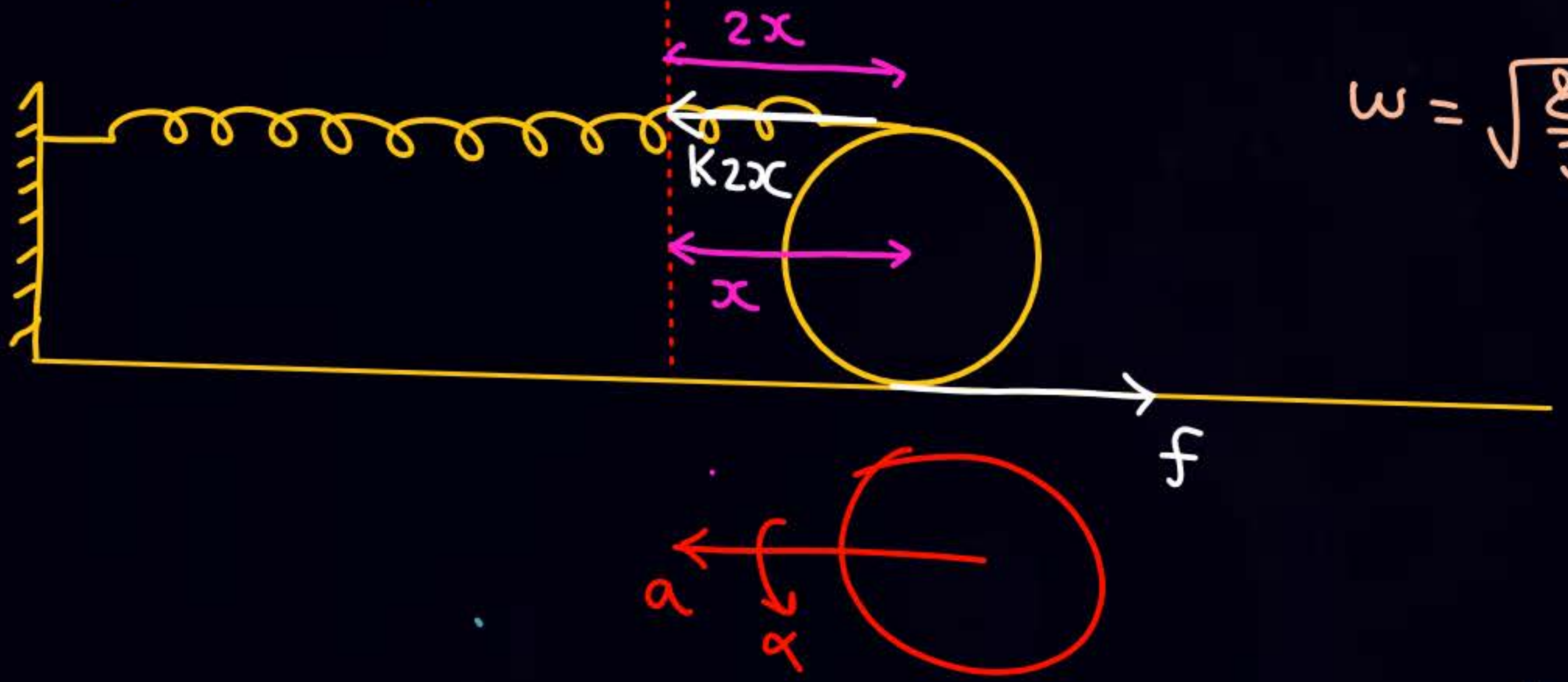
$$T = 2\pi\sqrt{\frac{3m}{2K}}$$

Q



friction is sufficient for pure rolling
find T of oscillation for small displacement.

Sol^n



$2kx - f = ma$ —— ①

$2kxR + fR = I\alpha = \frac{mR^2}{2}\cdot\frac{a}{R}$

$2kx + f = \frac{ma}{2}$ —— ②

$4kx = \frac{3ma}{2}$

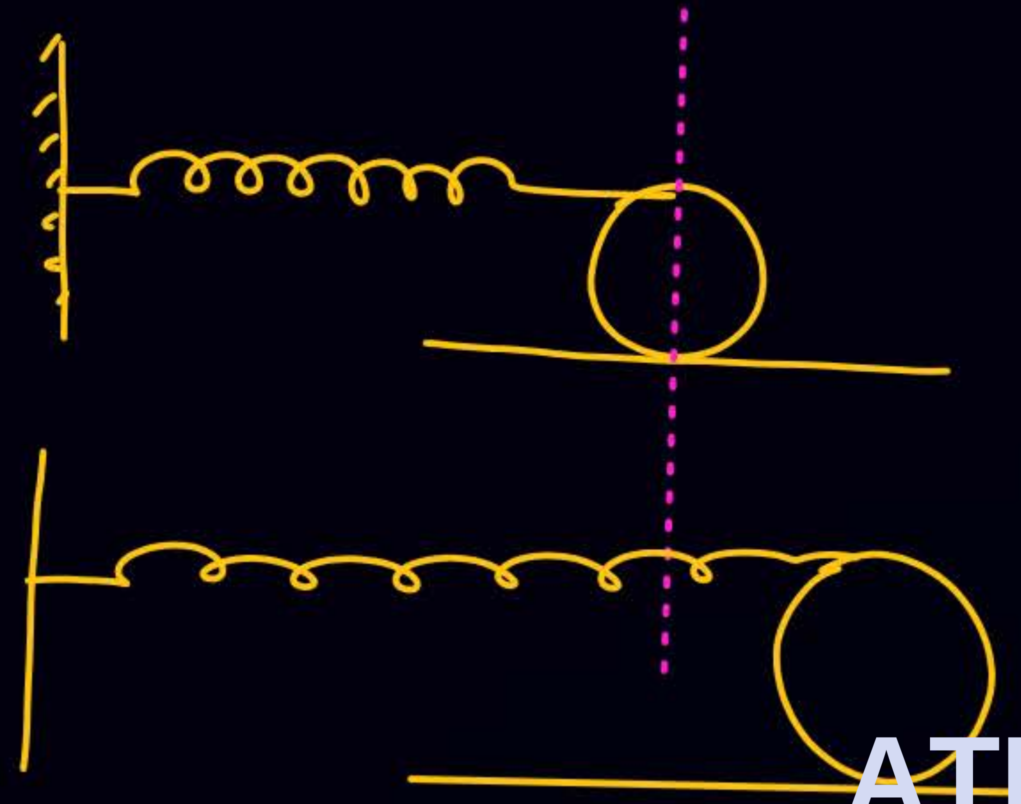
$\vec{a} = -\frac{8k}{3m}\vec{x}$

$\omega = \sqrt{\frac{8k}{3m}}$

$$T = 2\pi\sqrt{\frac{3m}{8k}}$$

(*)

M-2



$$T.E = \frac{1}{2}K(2x)^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$T.E = 2Kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{mR^2}{2}\cdot\frac{V^2}{R^2}$$

$$T.E = 2Kx^2 + \frac{3}{4}mv^2 = \text{const}$$

$$\frac{d(T.E)}{dx} = 2K\,2x + \frac{3}{4}m\,2v\frac{dv}{dx} = 0$$

$$4Kx + \frac{3}{2}ma = 0$$

$$\vec{a} = -\frac{8K}{3m}\vec{x}$$

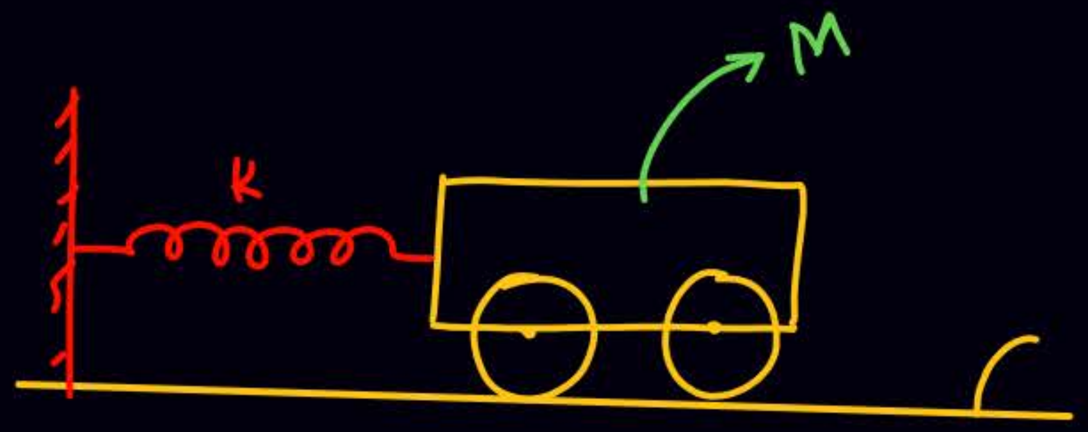
$$\omega = \sqrt{\frac{8K}{3m}} \qquad \boxed{T = 2\pi\sqrt{\frac{3m}{8K}}}$$

$$\frac{d(T.E)}{dt} = 2K\,2x\frac{dx}{dt} + \frac{3}{4}m\,2v\frac{dv}{dt} = 0$$

$$4Kx = -\frac{3}{2}ma$$

$$\vec{a} = -\frac{8K\vec{x}}{3m}$$

Q



friction is sufficient for pure rolling

$M \longrightarrow$ mass of box

wheel $\Rightarrow$ Disc $\equiv (m, R)$

find time period for small oscillation.

Sol^n

$$T.E = \frac{1}{2}Kx^2 + \frac{1}{2}MV^2 + \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)\times 4$$

$$= \frac{1}{2}Kx^2 + \frac{1}{2}MV^2 + \left(\frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\frac{V^2}{R^2}\right)\times 4$$

$$T.E = \frac{1}{2}Kx^2 + \frac{1}{2}MV^2 + 3mV^2$$

$$\frac{d(T.E)}{dx} = 0 = \frac{K}{2}2x + \frac{1}{2}M\,2v.\frac{dv}{dx} + 3m\,2v\frac{dv}{dx} = 0$$

$$Kx + Ma + 6ma = 0$$

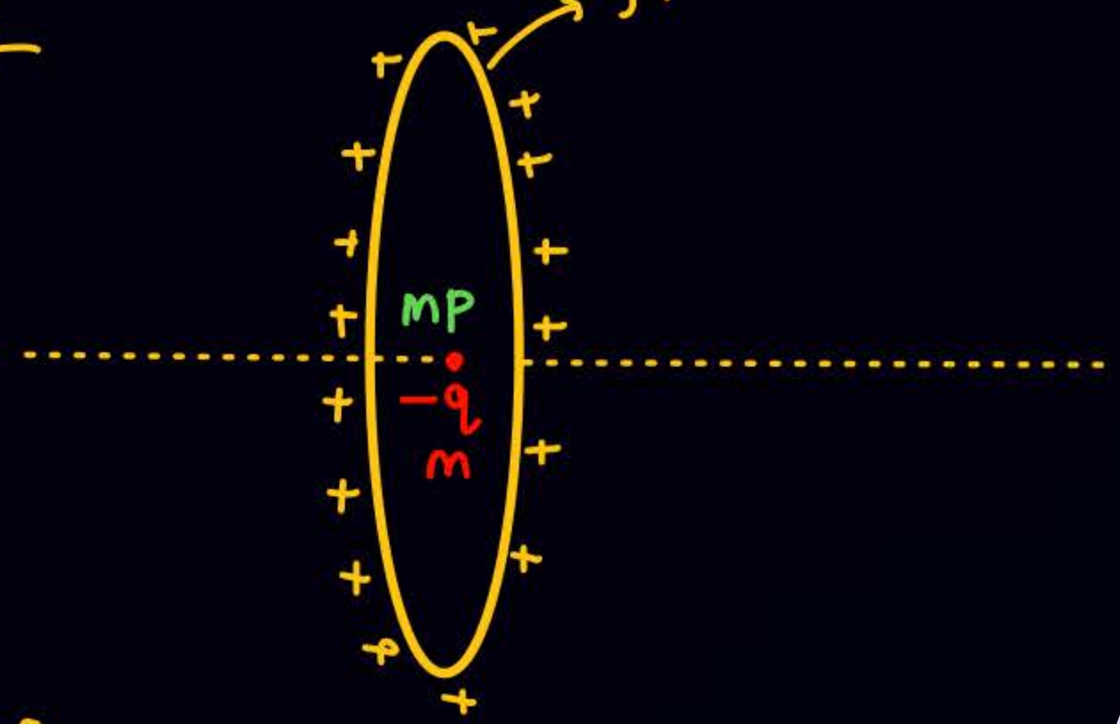
$$a(M+6m) = -Kx$$

$$\vec{a} = \left(\frac{-K}{M+6m}\right)\vec{x}$$

$$\omega = \sqrt{\frac{K}{M+6m}}$$

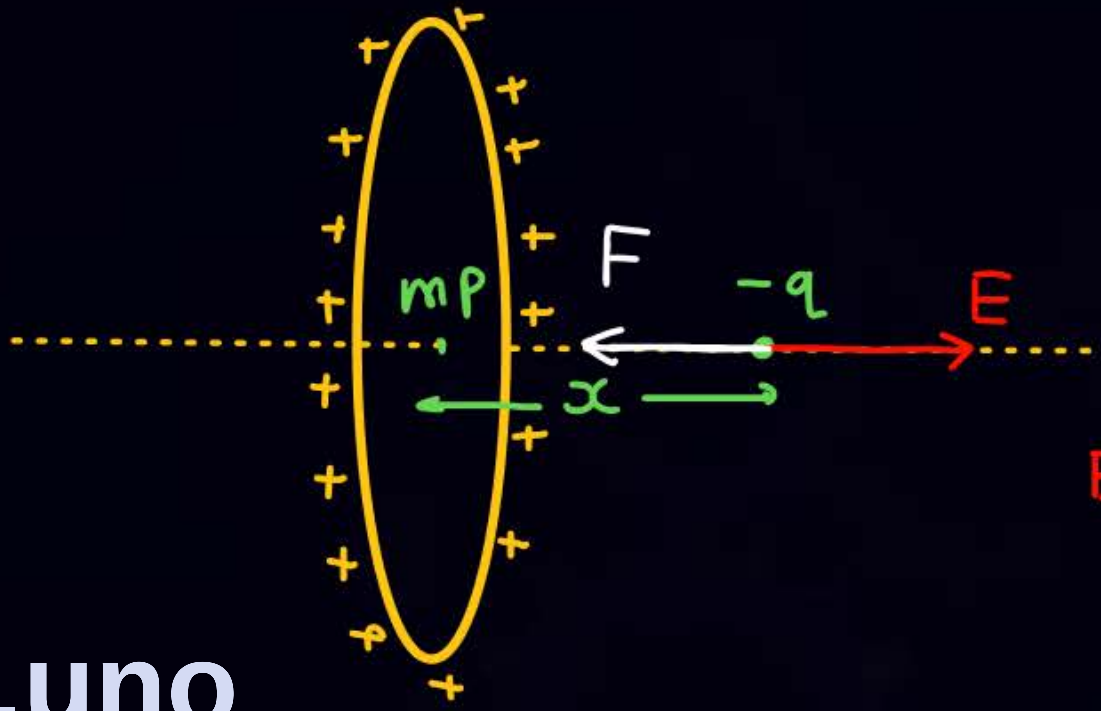
$$\boxed{T = 2\pi\sqrt{\frac{M+6m}{K}}}$$

Q



for small oscillation of (−q, m) find time period

Sol^n



$$\vec{F} = q\vec{E}$$

$$E = \frac{KQx}{(R^2+x^2)^{3/2}}$$

$$F = qE = \frac{qKQx}{(R^2+x^2)^{3/2}}$$

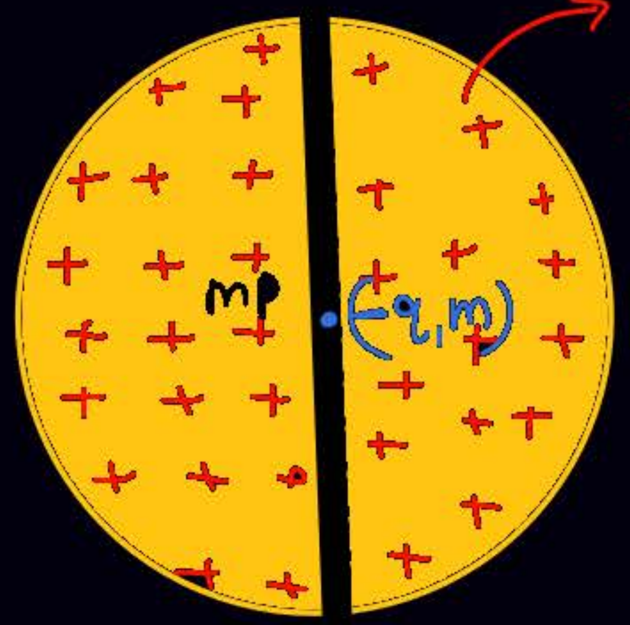
$$\vec{F} = -\frac{KQq\,\vec{x}}{(R^2+x^2)^{3/2}}$$

If $x << R$

$$\vec{F} = -\frac{KQq}{R^3}\vec{x}$$

$$T = 2\pi\sqrt{\frac{mR^3}{KQq}}$$

Q



find T of oscillation if (−q, m) is displaced

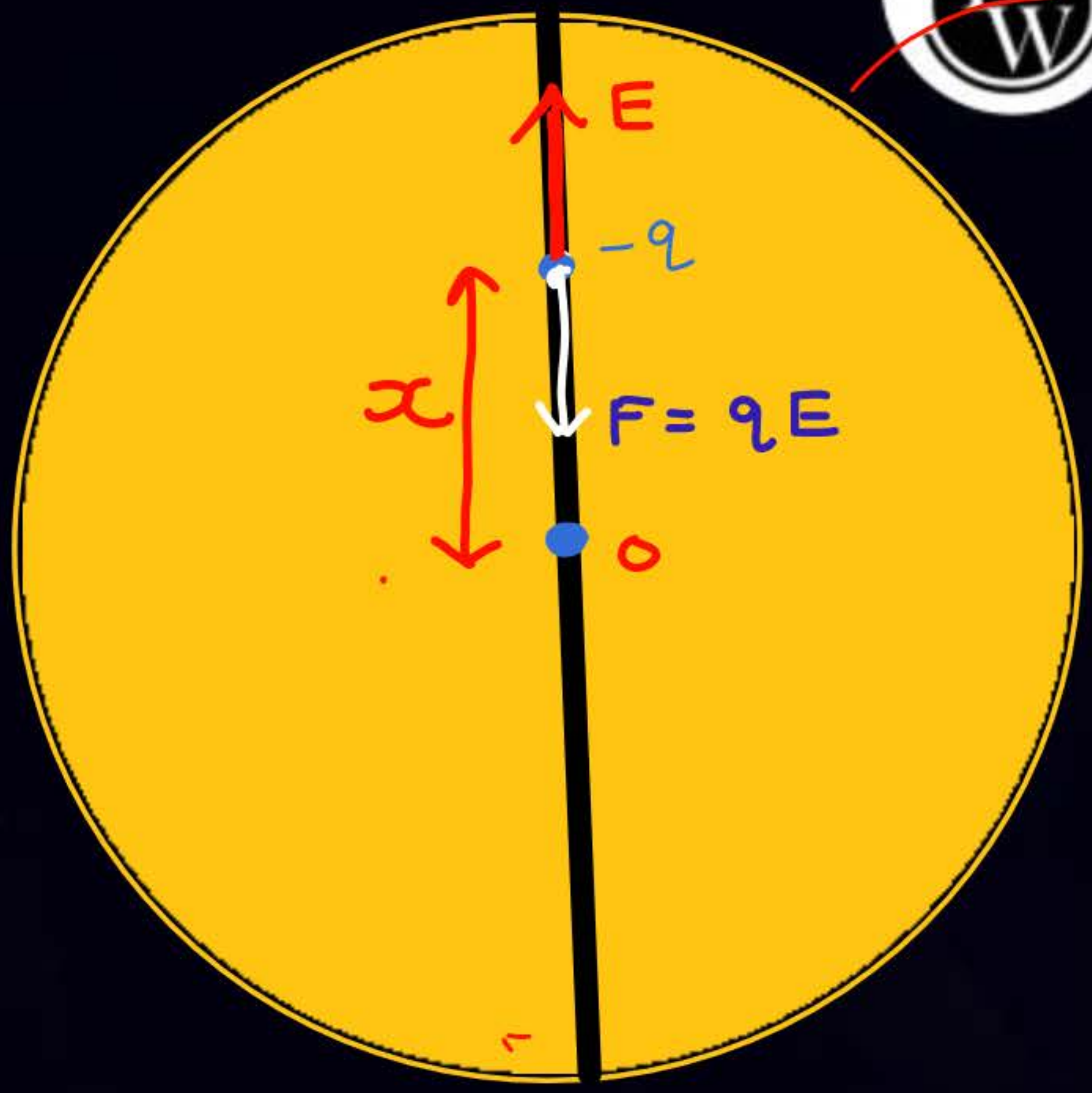
Sol^n

$F_{net} = qE$

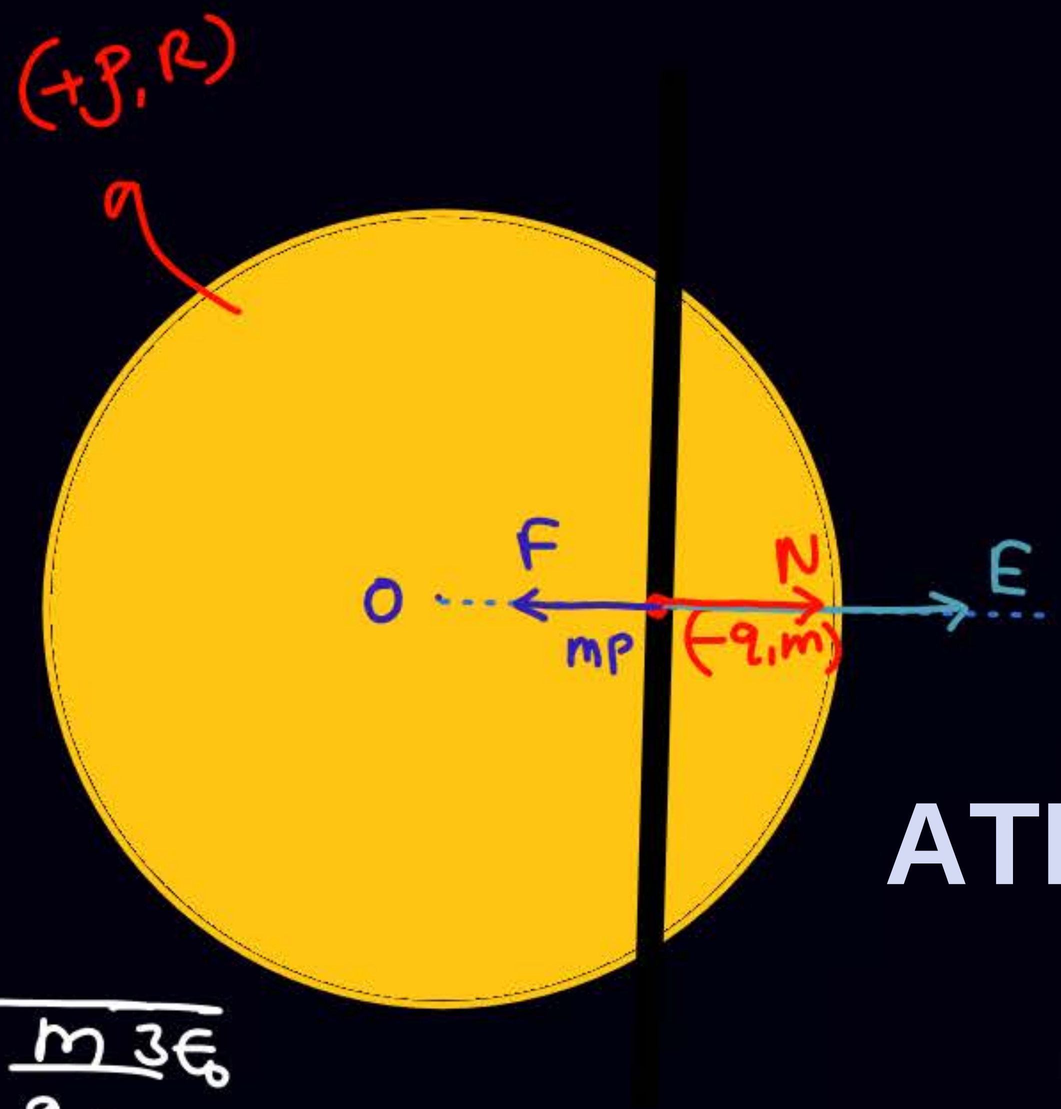
$\vec{E} = \frac{\rho \vec{r}}{3\epsilon_0}$

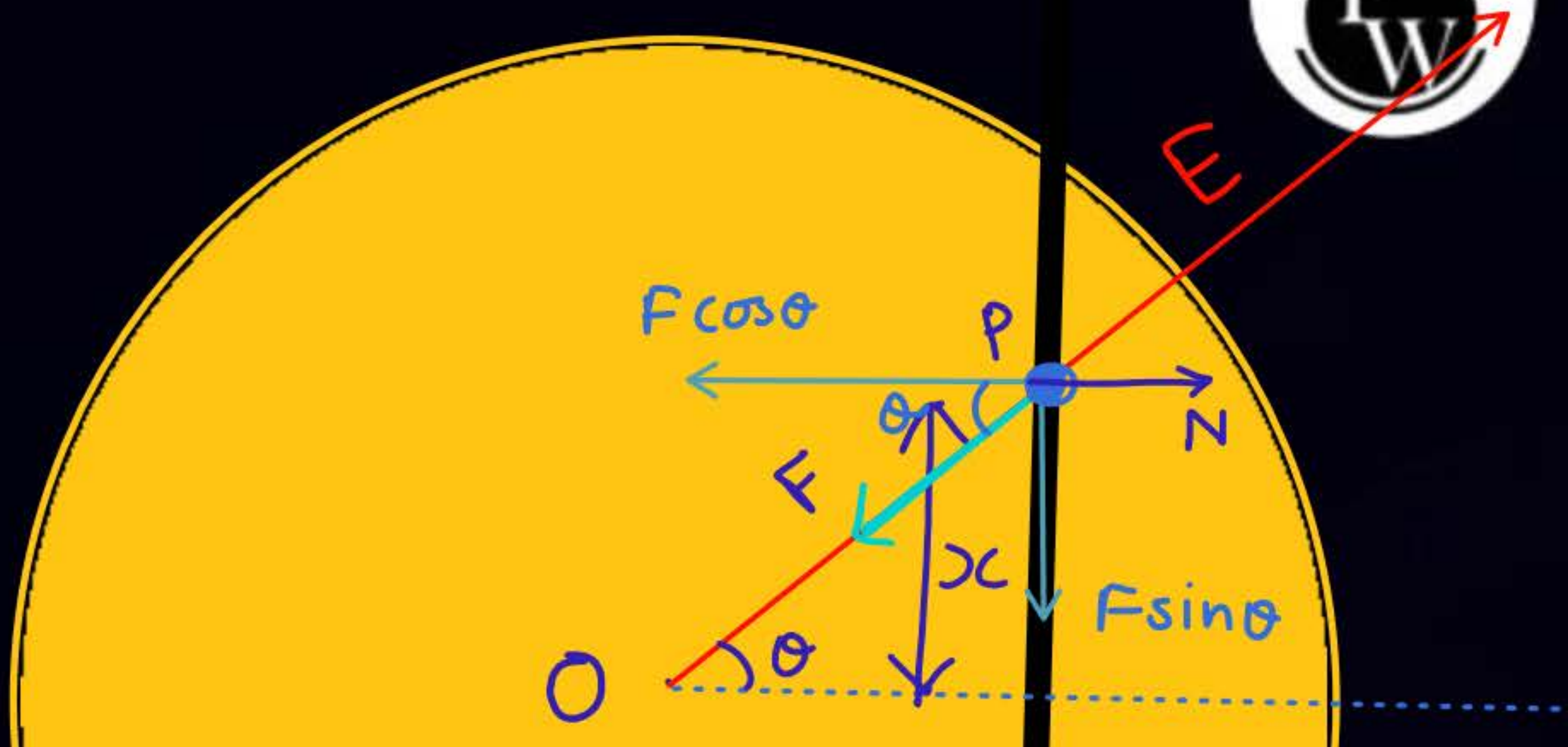
$\vec{F} = -\frac{q\rho \vec{x}}{3\epsilon_0}$

$T = 2\pi\sqrt{\frac{m\,3\epsilon_0}{q\rho}}$



Q





$F_{net} = F\sin\theta$ (नीचे)

$= qE\sin\theta = \dfrac{q\rho(OP)}{3\epsilon_0}\sin\theta$

$F_{net} = \dfrac{q\rho}{3\epsilon_0}(OP\sin\theta)$

$F_{net} = \dfrac{q\rho}{3\epsilon_0}x$

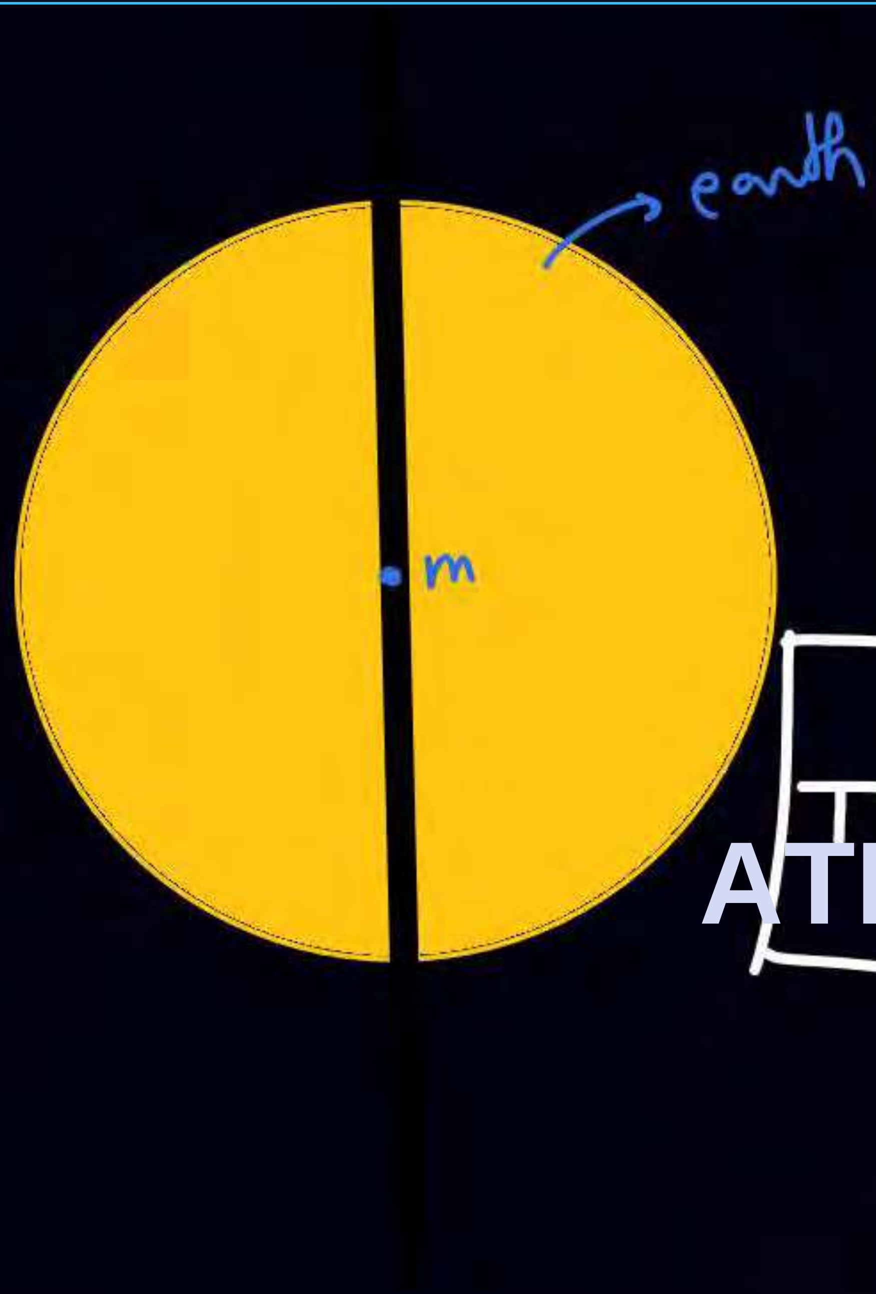
$\vec{F} = -\dfrac{q\rho}{3\epsilon_0}\vec{x}$

$T = 2\pi\sqrt{\dfrac{m\,3\epsilon_0}{q\rho}}$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

$$g = \frac{GM}{R^2}$$

Q



$T = ?$

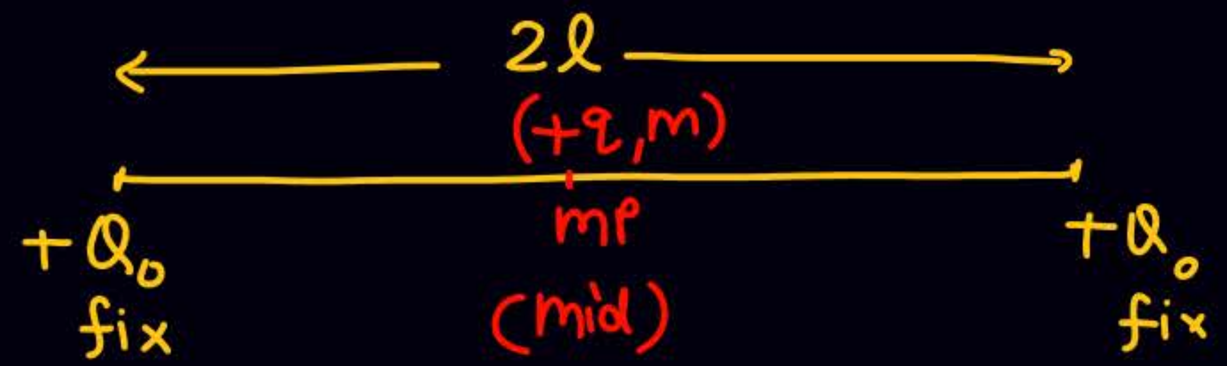
$G \rightarrow \frac{1}{4\pi\epsilon_0}$

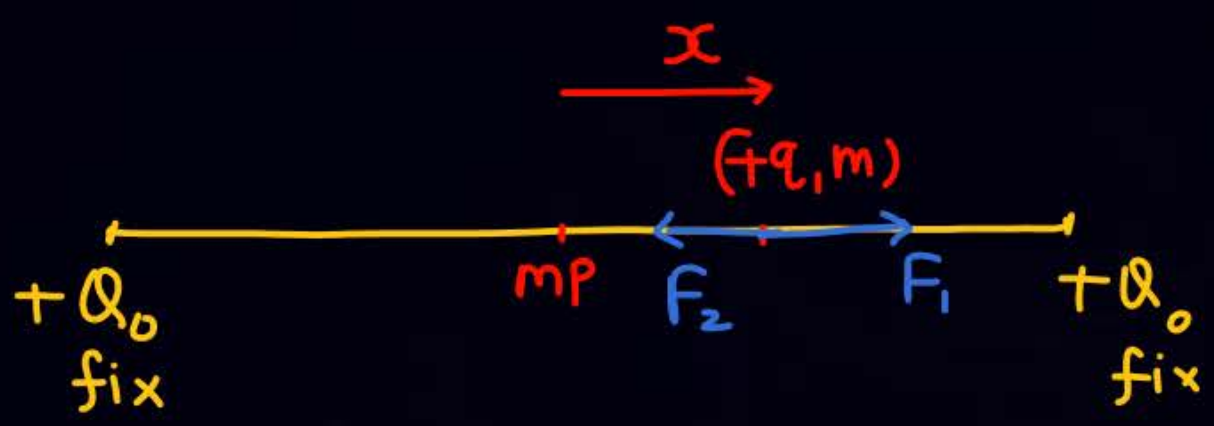
$q \rightarrow m$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

$$T = 2\pi\sqrt{\frac{\ldots}{4KQ_0 q}}$$

Q





find T of oscillation if (+q, m) is slightly displaced along

$$F_{net} = \frac{KQ_0 q}{(\ell + x)^2} - \frac{KQ_0 q}{(\ell - x)^2}$$

$$\vec{F}_{net} = KQ_0 q\left[\frac{(\ell - x)^2 - (\ell + x)^2}{(\ell + x)^2(\ell - x)^2}\right]$$

$$= KQ_0 q\left[\frac{2\ell \cdot (-2x)}{(\ell^2 - x^2)^2}\right]$$

$$F_{net} = -\frac{KQ_0 q\, 4\ell x}{(\ell^2 - x^2)^2}$$

SHM ✗

$x << \ell$

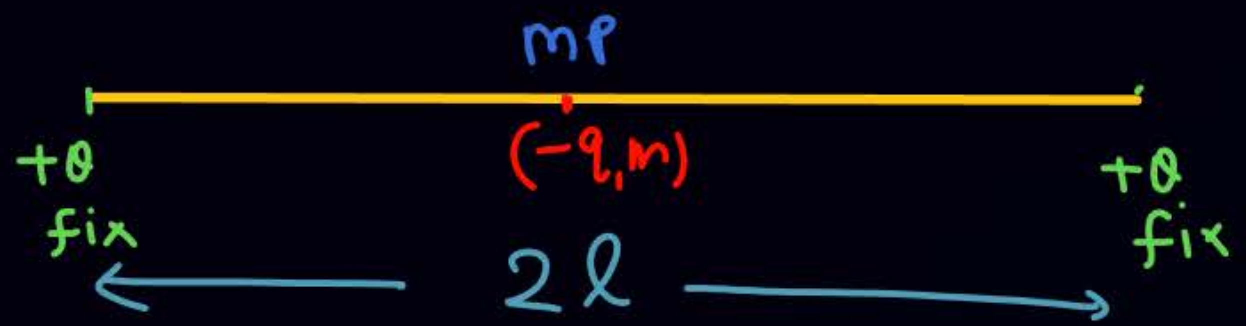
$$\vec{F} = -\frac{4KQ_0 q}{\ell^3}\vec{x}$$

$$\frac{\omega^2_{Axis} = \frac{4KQ_0q}{ml^3}}{\omega^2_{equit.} = \frac{2KQ_0q}{ml^3}}$$

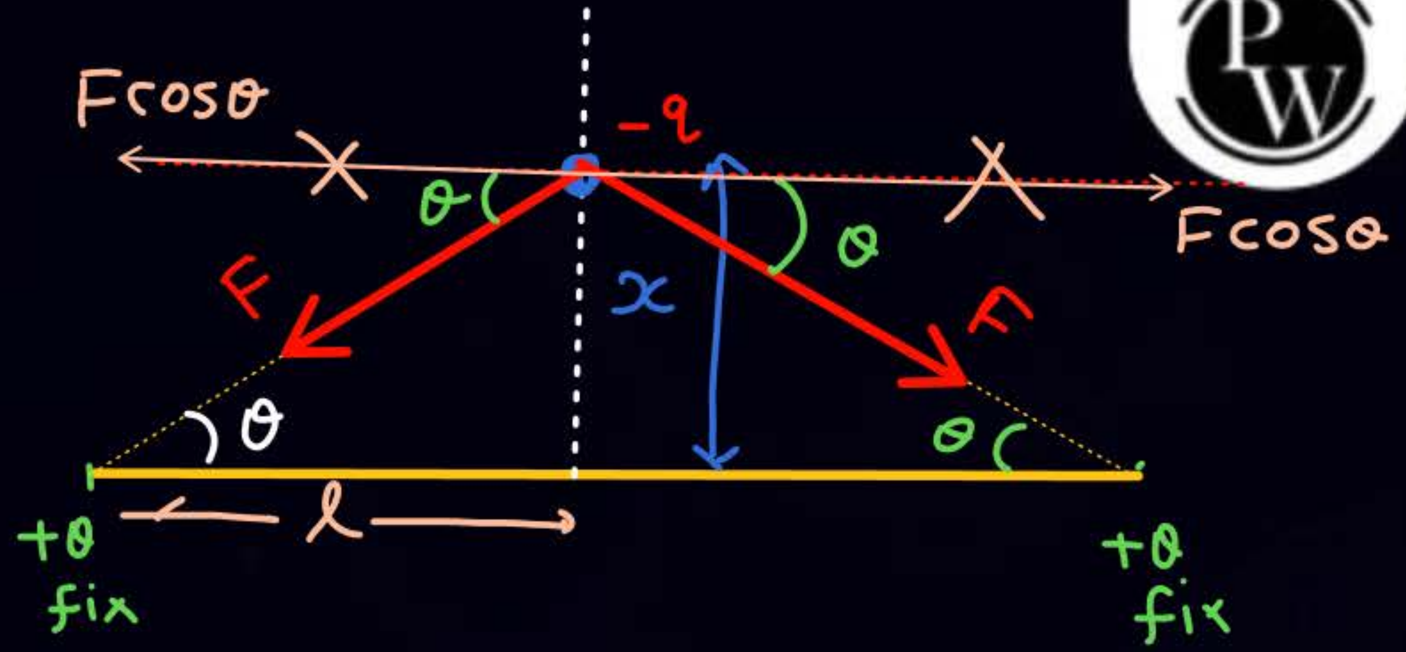
$$\frac{T_{equi}}{T_{Axis}} = \sqrt{2}$$

$$\frac{\omega_{Axis}}{\omega_{equi}} = \sqrt{2}$$

Q



find T if (-q, m) is displaced slightly along y-Axis



$$F_{net} = 2F\sin\theta \; (नीचे)$$

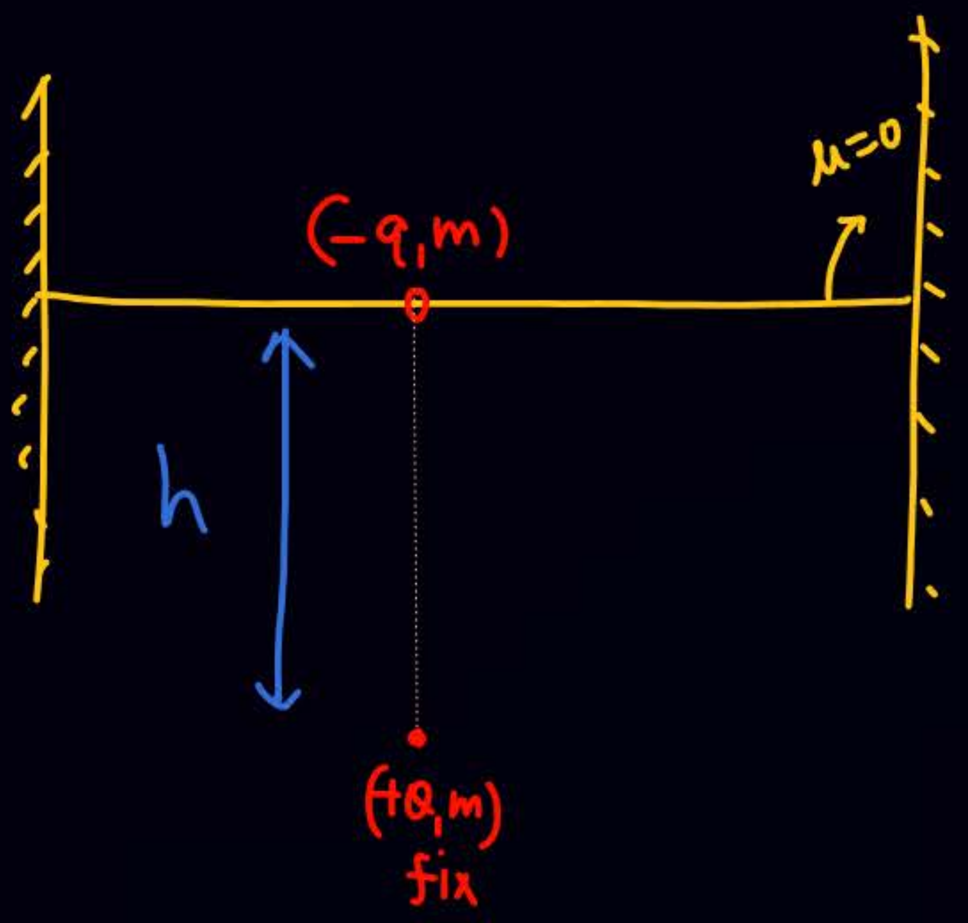
$$= \frac{2KQq}{(l^2+x^2)} \times \frac{x}{\sqrt{l^2+x^2}} = \frac{2KQqx}{(l^2+x^2)^{3/2}}$$

$(x << l)$

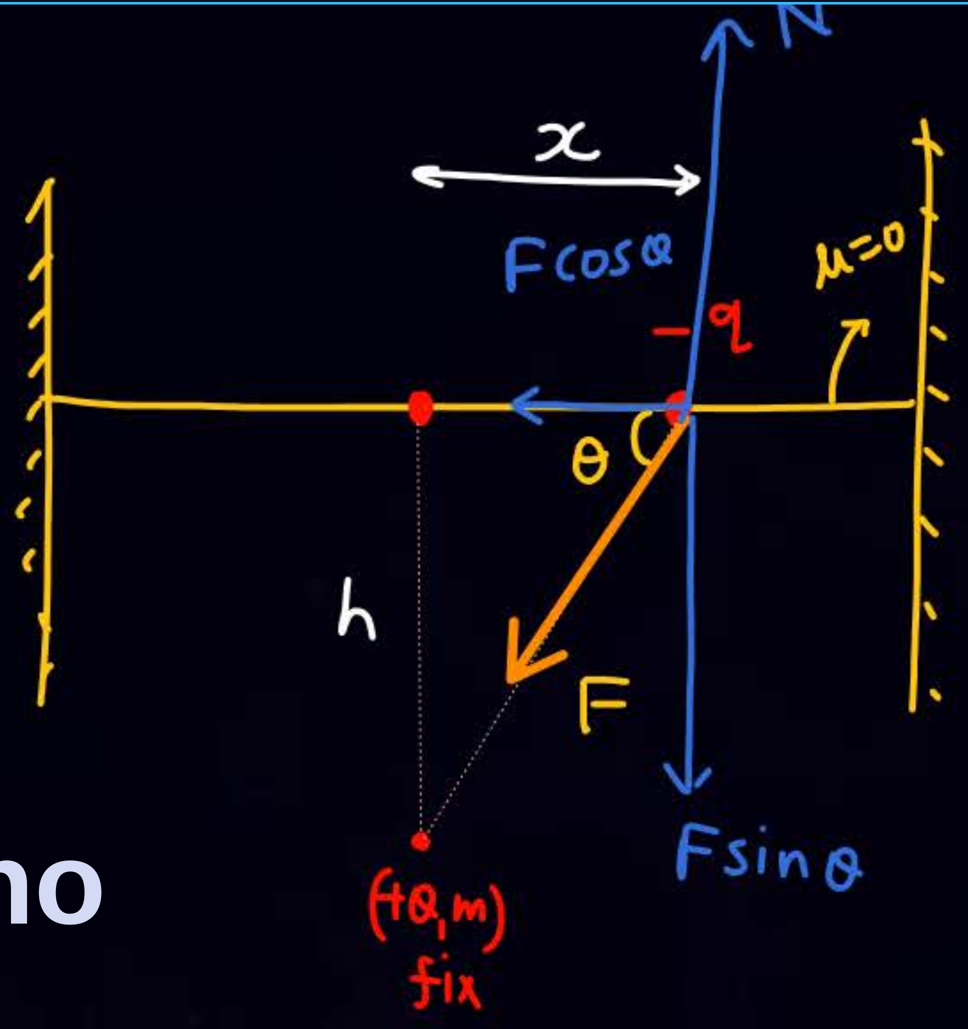
$$\vec{F}_{net} = -\left(\frac{2KQq}{l^3}\right)\vec{x}$$

$$T = 2\pi\sqrt{\frac{ml^3}{2KQq}}$$

Q



Q

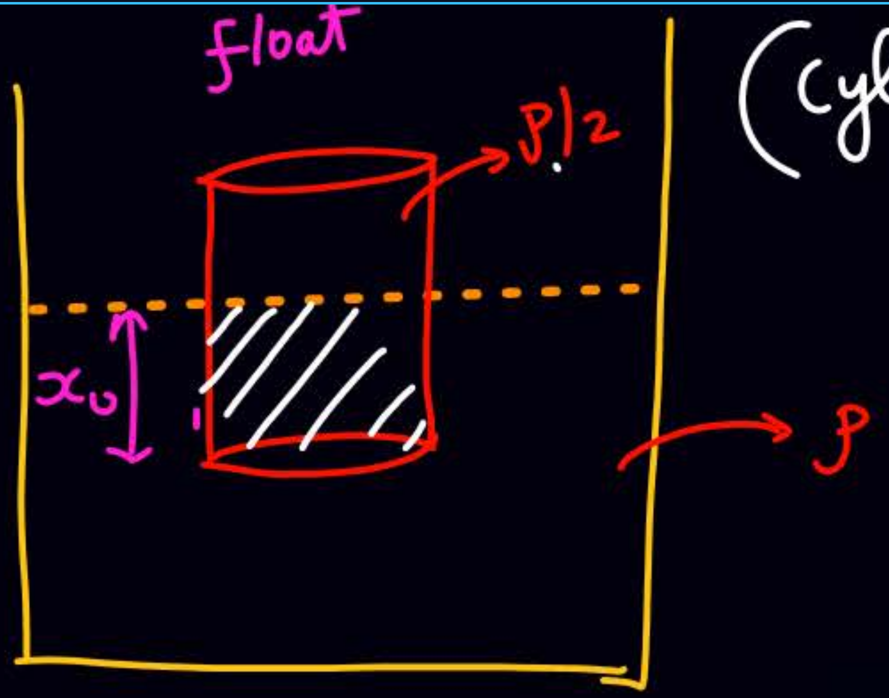


$$F_{net} = F\cos\theta = \frac{KQq}{\left(\sqrt{h^2+x^2}\right)^2}\cdot\frac{x}{\sqrt{h^2+x^2}}$$

$$F = \frac{KQq}{(h^2+x^2)^{3/2}}\,x$$

If $x \ll h$: $\vec{F} = \frac{-KQq}{h^3}\vec{x}$

Q (cylinder $\rho/2$, L,)

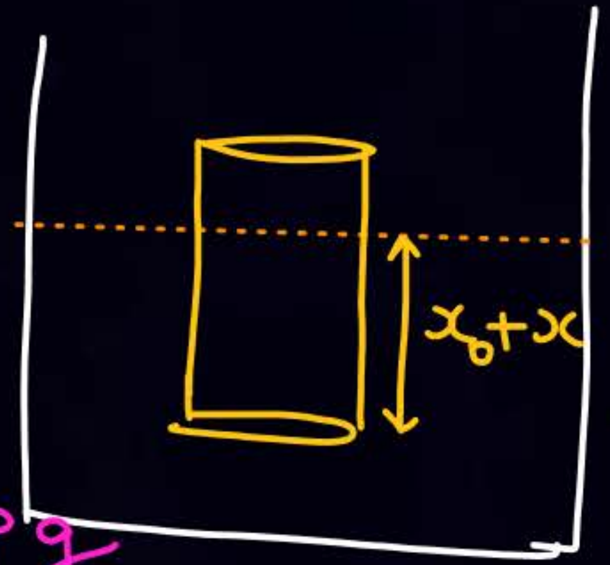


$$B = V_d \rho_\ell g$$

If cylinder is pushed down by x & release $(x < l/2)$

$T = ?$

$m_{body} = \rho\, vol^n_{body}$



$F_{net} = 0$

$mg = B = V_d \cdot \rho_\ell g$

$mg = AL\frac{\rho}{2} g = A x_0 \rho g$

$$x_0 = L/2$$

$F_{net} = mg - B$

$F_{net} = mg - A(x_0 + x)\rho g$

$F_{net} = mg - A x_0 \rho g - A x \rho g$

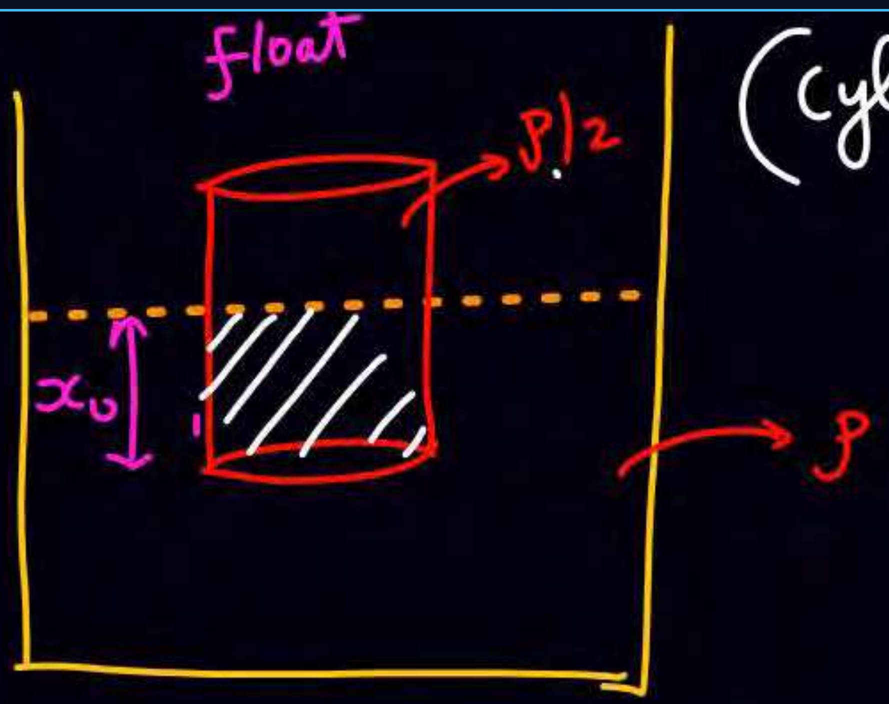
$$F_{net} = -A\rho g x$$

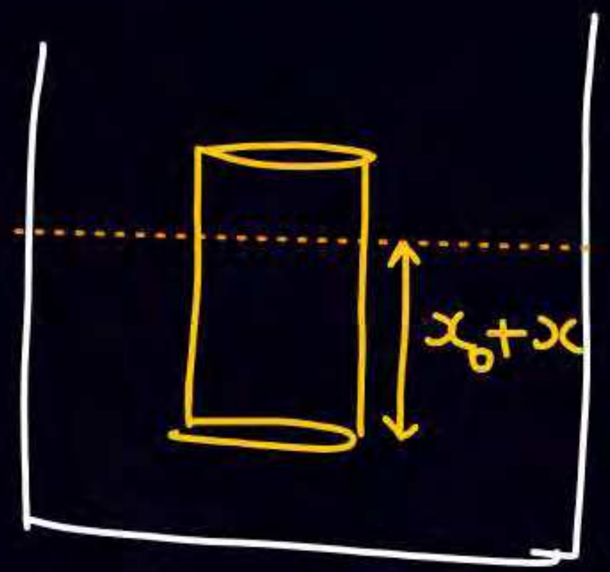
$T = 2\pi\sqrt{\frac{m}{A\rho g}}$

$T = 2\pi\sqrt{\frac{AL\rho_{body}}{A\rho_{liq.} g}}$

$T = 2\pi\sqrt{\frac{L}{g}\frac{\rho_{body}}{\rho_{liq}}}$

Q (Cylinder $\rho/2$, L)

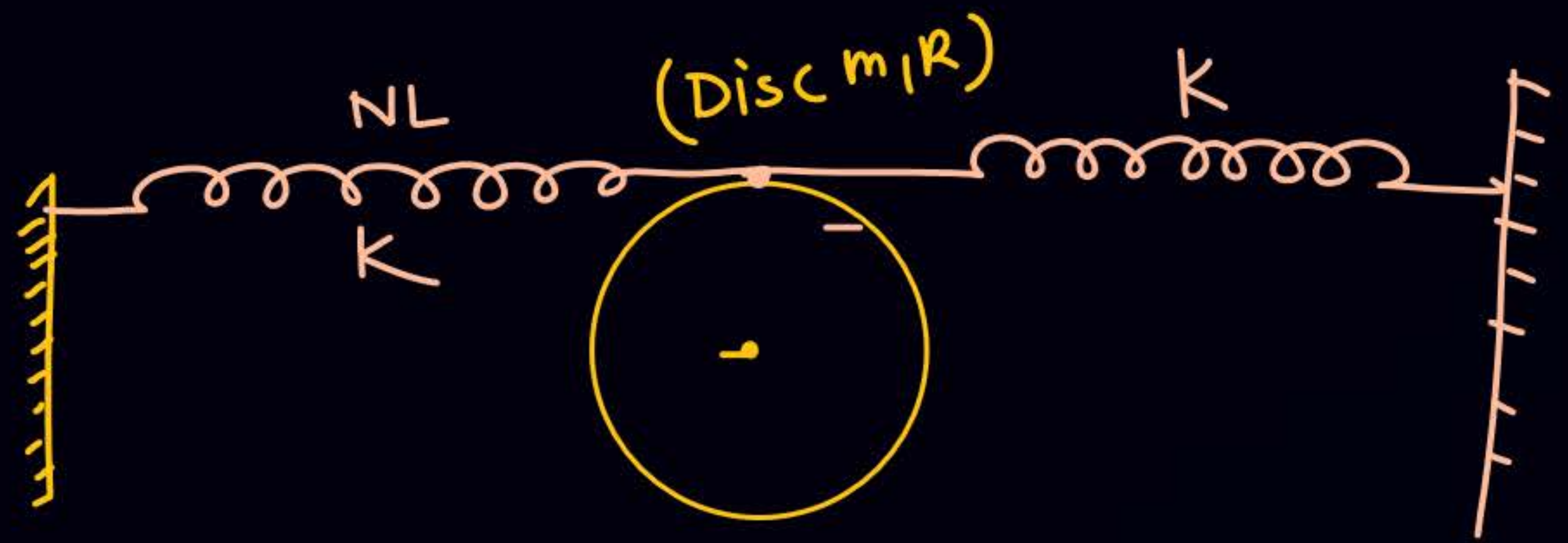




$$F_{net} = B_{extra}$$

$$F_{net} = A x \rho_l g$$

Q



$K_{eq} = 2K$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$2 = 2\pi\sqrt{\frac{l}{g}}$$

$$1 = \pi^2\frac{l}{g}$$

$$g = \pi^2 l$$

KTG & thermodynamics.

Assumption

- Random
- → All the collision are elastic.
- → Effect of gravity neglected
- → Size of molecule is very small as compare to volⁿ of container.
- → No interaction
- → P.E. = 0 , Internal Energy ⇒ In the form of K.E.
- → Real gas behave like ideal gas at high tem & low pressure.

- A gas which follows all gas laws and gas equation at every possible temperature and pressure is known as ideal or perfect gas.
- Volume of gas molecules is negligible as compared to volume of container so volume of gas = volume of container (Except 0K)
- No intermoleculer force act between gas molecules.
- Potential energy of ideal gas is zero so internal energy of ideal gas is perfectly translational K.E. of gas. It is directly proportional to absolute temperature.

  So, internal energy depends only and only on its temperature.

  $$E_{trans} \propto T$$

- All real gases behaves as ideal gas at high temperature and low pressure.
- Gas molecule have point mass and negligible volume and velocity is very high (10^7 cm/s). That's why there is no effect of gravity on them.

Very-Very-Very Imp

Proof Not important

JEE main 24

Average Velocity $= \dfrac{\vec{V_1}+\vec{V_2}+\vec{V_3}+\dots}{n} = 0$

(Rem)

V_{rms} = Root mean Square Velocity $= \sqrt{\dfrac{V_1^2+V_2^2+V_3^2+\dots}{n}} = \sqrt{\dfrac{3RT}{M}} \propto \sqrt{T}$

Avrg Speed $= \dfrac{V_1+V_2+V_3+\dots}{n} = \sqrt{\dfrac{8}{\pi}\dfrac{RT}{M}}$

$R = \dfrac{25}{3}\ \dfrac{J}{K\,mol}$

$R = 8.33\ J/Kmol$

most probable speed $= V_{mp} = \sqrt{\dfrac{2RT}{M}} \propto \sqrt{T}$

T → kelvin

M → molar mass

$V_{rms} >$ Avrg speed $> V_{mp}$

DIFFERENT SPEEDS OF GAS MOLECULES

- **Average velocity**

Because molecules are in random motion in all possible direction in all possible velocity. Therefore, the average velocity of the gas in molecules in container is zero. $<\vec{v}> = \frac{\vec{v}_1 + \vec{v}_2 + \ldots\ldots\ldots\vec{v}_N}{N} = 0$

rms speed of molecules $v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$

Mean speed of molecules :By maxwell's velocity distribution law v_M or $<|\vec{v}|> = v_{mean}$

$$<|\vec{v}|> = v_{mean} = \frac{|\vec{v}_1| + |\vec{v}_2| + \ldots\ldots\ldots|\vec{v}_n|}{N} = \sqrt{\frac{8}{\pi}\frac{P}{\rho}} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8kT}{\pi m}} = 1.59\sqrt{\frac{kT}{m}}$$

Most probable speed of molecules (v_{mp})

At a given temperature, the speed to which maximum number of molecules belongs is called as most probable speed (v_{mp}) $\quad v_{mp} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$

Q. Speed of ten gas particle are given as 1, 0, 2, 4, 6, 3, 6, 8, 6, 4

$$\text{Avrg speed} = \frac{1+0+2+4+6+3+6+8+6+4}{10} = 4$$

$$V_{rms} = \sqrt{\frac{1^2+0^2+2^2+4^2+6^2+3^2+6^2+8^2+6^2+4^2}{10}}$$

$$V_{\text{most prob. speed}} = 6$$

DOf (Degree of freedom)

* No. of different ways in which a molecule (or atom) can exhibit energy.

monoatomic ⟶ Dof = 3, (v_x, v_y, v_z)

Diatomic ⟶ Dof = 5 ($v_x, v_y, v_z, \omega_x, \omega_y$)

3 ⟶ translation
2 ⟶ Rotational

At high temp = Dof = 3 + 2 + 2
(3 → transl., 2 → Rot, 2 → Vibration)

Triatomic → 6 (Nonlinear)

3+3 → Rotational ($\omega_x, \omega_y, \omega_z$)

↳ Translation (v_x, v_y, v_z) H_2O, SO_2.

Linear (like diatomic) → 3 + 2 = 5

Ex. CO_2, HCN

3 → $v_x\ v_y\ v_z$ (Translation)

2 → (ω_x, ω_y Rot.)

Maxwell equipartition Law of Energy.

Energy corresponding to each dof of an ideal gas molecule $= \frac{KT}{2}$

$\Rightarrow$ Energy of one molecule of a gas with dof 'f' is equal to $= f\frac{KT}{2}$

$\therefore$ $\boxed{K = \frac{R}{N_A}}$

$$U = \frac{fKT}{2} = \frac{Rf}{N_A}\frac{T}{2} \longrightarrow 1 \text{ molecule}$$

So for 1 mole $\Rightarrow U = N_A \cdot \left(\frac{Rf}{N_A}\frac{T}{2}\right) = \frac{Rf\,T}{2}$

for n mole $U = n\left(\frac{RfT}{2}\right)$

degree of freedom

$$* \quad U = \frac{nfRT}{2} \Rightarrow T\uparrow, U\uparrow, \Delta U > 0$$

$$T\downarrow, U\downarrow, \Delta U < 0$$

$$\Delta U = \frac{nfR\,\Delta T}{2}$$

Home Work

- DPP
- module PYQ (Solve)


THANK
YOU
ATDB.uno