PRAYAS

JEE 2025

Lecture - 03

Physics

Waves

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Topics to be covered

1. Energy & power analysis in wave
2. Equation of wave.
3.
4.

Question

Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is 16×10^{11} Nm^{-2}, the extension of wire over its natural length is: [JEE Mains 2020]

$\frac{T}{A} = Y \frac{\Delta l}{l}$ $\Delta l = \frac{Tl}{AY} = \frac{\mu v^2 l}{AY}$

$v = \sqrt{\frac{T}{\mu}}$

$T = \mu v^2$

A 0.02 mm

B 0.04 mm

C 0.03 mm

D 0.01 mm

Ans. (C)

Question

$20.6 \times 10^3 \equiv 5.15 \times 10^3$

A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T, the velocity changed to v/2. The value of T is close to:

[JEE Mains 2020]

Check ques

$v = \sqrt{\frac{T}{\mu}}$

$v/2$

$T/4$

(A) 10.2×10^2 N

(B) 10.2×10^2 N

(C) 10.2×10^2 N

(D) 10.2×10^2 N

Ans. (B)

Which of the following equations represents a travelling wave? [JEE Mains 2021]

(A) $y = A \sin(15x - 2t)$

(B) $y = Ae^{-x^2}(vt + \theta)$ ✗

(C) $y = Ae^{x}\cos(\omega t - \theta)$ ✗

(D) $y = A \sin x \cos\omega t$ ≡ standing wave

Ans. (A)

Question

The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be _____%. [JEE Mains 2021]

$v = \sqrt{\frac{T}{\mu}}$

$v = \frac{T^{1/2}}{\mu^{1/2}}$

$\frac{\Delta V}{V} = \frac{1}{2}\frac{\Delta T}{T}$

$= \frac{1}{2} \times 4 = 2\%$

Question

The speed of a transverse wave passing through a string of length 50 cm and mass 10 g is 60 ms^{-1}. The area of cross-section of the wire is 2.0 mm^2 and its Young's modulus is 1.2 × 1011 Nm^{-2}. The extension of the wire over its natural length due to its tension will be $x \times 10^{-5}$ m. The value off x is _____. [JEE Mains 2022]

l
m
v
A
Y

Ans. 15

Question

The equation of wave is given by $Y = 10^{-2}\sin 2\pi\left(160t - 0.5x + \frac{\pi}{4}\right)$, where x and Y are in m and *t* is s. The speed of the wave is _____ km h^{-1} **(11 April 2023 - Shift 1)**

$$\frac{w}{k} = \frac{2\pi \times 160}{2\pi \times .5} = \frac{1600}{5} = 320$$

Ans : (1152)

Question

The equation of wave is given by $Y = 10^{-2}\sin 2\pi\left(160t - 0.5x + \frac{\pi}{4}\right)$, where x and Y are in m and *t* is s. The speed of the wave is _____ km h^{-1} **(11 April 2023 - Shift 1)**

Ans : (1152)

Question

A travelling wave is described by the equation $y(x, t) = [0.05 \sin (8x - 4t)]m$.
The velocity of the wave is : [all the quantities are in SI unit]

(24 January 2023 - Shift 1)

A. $4\ ms^{-1}$

B. $2\ ms^{-1}$

C. $0.5\ ms^{-1}$

D. $8\ ms^{-1}$

Ans : (C)

Question

In the wave equation, $y = 0.5\sin\frac{2\pi}{\lambda}(400t - x)m$ the velocity of the wave will be :

[JEE Mains 2022]

A $200\ \text{m/s}$

B $20\sqrt{2}\text{m/s}$

C $400\ \text{m/s}$

D $400\sqrt{2}\ \text{m/s}$

Ans. (C)

Question

A steel wire with mass per unit length $7.0 \times 10^{-3} \text{kg m}^{-1}$ is under tension of 70 N. The speed of transverse waves in the wire will be : **(01 February 2023 - Shift 1)**

$\sqrt{\frac{T}{\mu}}$

(A) $200\ \pi$ m/s

(B) 100 m/s

(C) 10 m/s

(D) 50 m/s

Ans : (B)

Power transmission in Travelling wave

2 min
Not important all

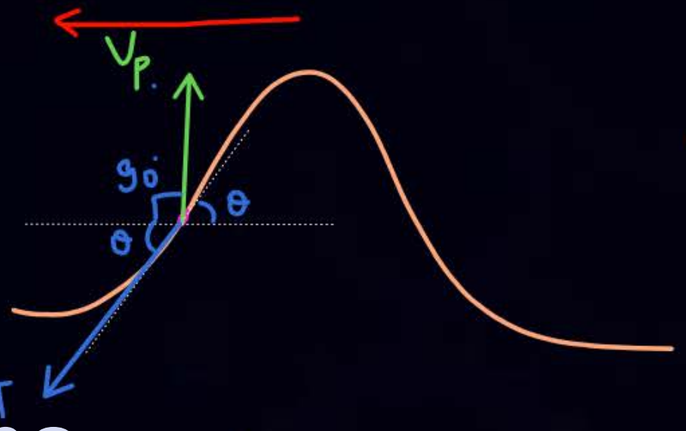
$$P = \vec{F}\cdot\vec{v} = Tv\cos(90+\theta)$$

$$P = -Tv\sin\theta$$

If θ is very small

$$P = -Tv\tan\theta$$

$$\boxed{P = -T\left(\frac{\partial y}{\partial t}\right)\left(\frac{\partial y}{\partial x}\right)}$$



$$V_p = -V_w \text{ slope}$$

$$= - - +$$

$$V_p > 0$$

$$y = A\sin(\omega t - kx + \phi)$$

$$P = +T\,A\omega\cos(\omega t - kx + \phi)\cdot Ak\cos(\omega t - kx + \phi)$$

$$P = TA^2\omega k\cos^2(\omega t - kx + \phi)$$

$$P = \mu A^2 V^2 \omega k \cos^2(\omega t - kx + \phi)$$

$$P = \mu A^2 V.V. \omega k \cos^2(\omega t - kx + \phi)$$

$$P = \mu A^2 V \frac{\omega}{k} \omega k \cos^2(\omega t - kx + \phi)$$

$$P = \mu A^2 \omega^2 \cos^2(\omega t - kx + \phi) V_{wave}$$

$$P = \mu A^2 \omega^2 \cos^2(\omega t - kx + \phi) V$$

$$\langle P \rangle = \frac{1}{2} \mu A^2 \omega^2 V$$

$$P = \mu A^2 \omega^2 \cos^2(\omega t - kx) \cdot V_w$$

Results

* $P = -T \frac{\partial y}{\partial x} \cdot \frac{\partial y}{\partial t} = -T(\text{slope})(V_p)$

* $\langle P \rangle = \frac{1}{2} \mu A^2 \omega^2 V_w$

* $\langle \text{Intensity} \rangle = \frac{\frac{1}{2} \mu A^2 \omega^2 V_w}{\text{Area}}$

* $\frac{dU}{dx} = \frac{1}{2} T (\text{slope})^2$

* $\frac{dK}{dx} = \frac{dU}{dx} = \frac{1}{2} \mu A^2 \omega^2 \omega^2 (\omega t - kx + \phi)$

Total energy for one wavelength of travelling

$\frac{1}{2} \mu A^2 \omega^2 \lambda$

$\text{Intensity} \propto A^2 \omega^2$

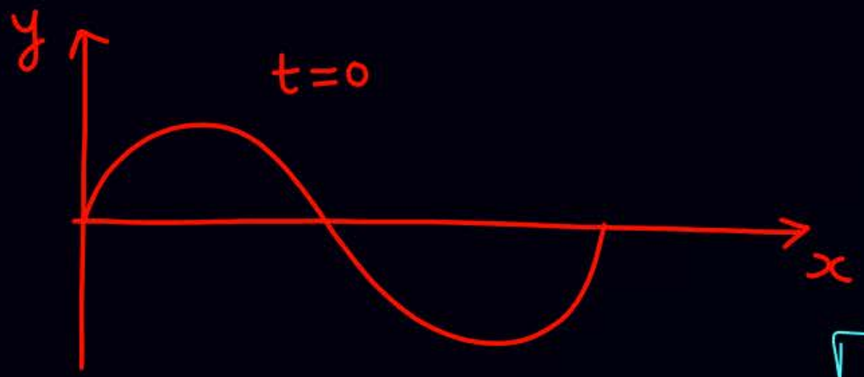
$\langle P \rangle \propto A^2 \omega^2$

$V_w = \frac{\omega}{k} = \frac{2\pi/T}{2\pi/\lambda}$

$V_w = \frac{\lambda}{T} = \lambda f$

$C = f\lambda$

Q If wave is moving along +x-Axis and at $t=0$, $y = A\sin kx$. Find eqⁿ of wave



Soln

$y = A \sin k(x - V_w t)$ Ans

$y = A \sin(kx - kV_w t)$

$y = A \sin(kx - \omega t)$

$\omega = V_w k$

Replace $x \xrightarrow{\text{by}} x - V_w t$

Q If wave is moving along +x Axis with speed 2m/s at t=0 its eqn is given as

(a) $y = 10 \sin(2\pi x + \pi/6)$

find eqn of wave

Soln

$x \longrightarrow x - V_w t$

$x \longrightarrow x - 2t$

$y = 10 \sin[2\pi(x-2t) + \pi/6]$

$y = 10 \sin[2\pi x - 4\pi t + \pi/6]$

(b) $y = \frac{1}{x^2+2}$, (At t=0)

$x \longrightarrow x - V_w t$

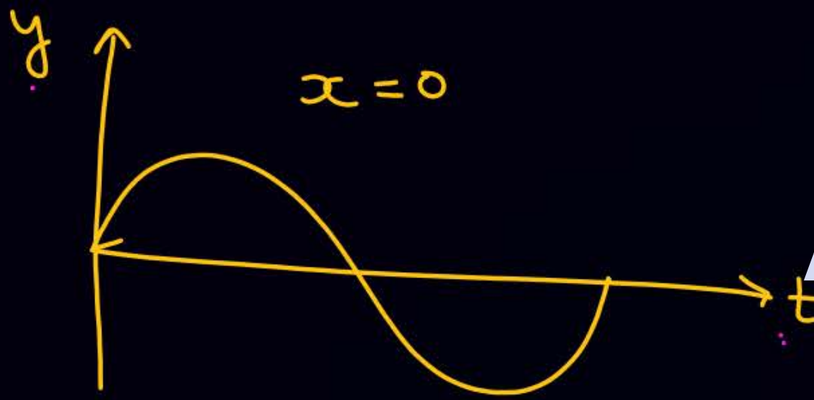
$x \longrightarrow x - 2t$

$y = \frac{1}{(x-2t)^2 + 2}$

(c) $y = 10\, e^{-[x^2+3]}$

$y = 10\, e^{-[(x-2t)^2+3]}$

Q If wave is moving along +x dirⁿ such that for $x=0$, relation b/w y & t is given as $y = A\sin\omega t$. find eqⁿ of wave.



Solⁿ $t \longrightarrow t - \frac{x}{v}$

$$y = A\sin\omega\left(t - \frac{x}{v}\right)$$

$$= A\sin\left(\omega t - \frac{\omega}{v}x\right)$$

$$= A\sin(\omega t - kx)$$

$y = f(x,t)$

$y = f(x,t)$

$y = f\left(x, t - \frac{x}{v}\right) = f(x,t)$

$t \longrightarrow t - \frac{x}{v_w}$

Q If wave is moving along +x Axis with speed 2m/s at $x=0$ its eqn is given as

(a) $y = 10 \sin(2\pi t + \pi/6)$

find eqn of wave

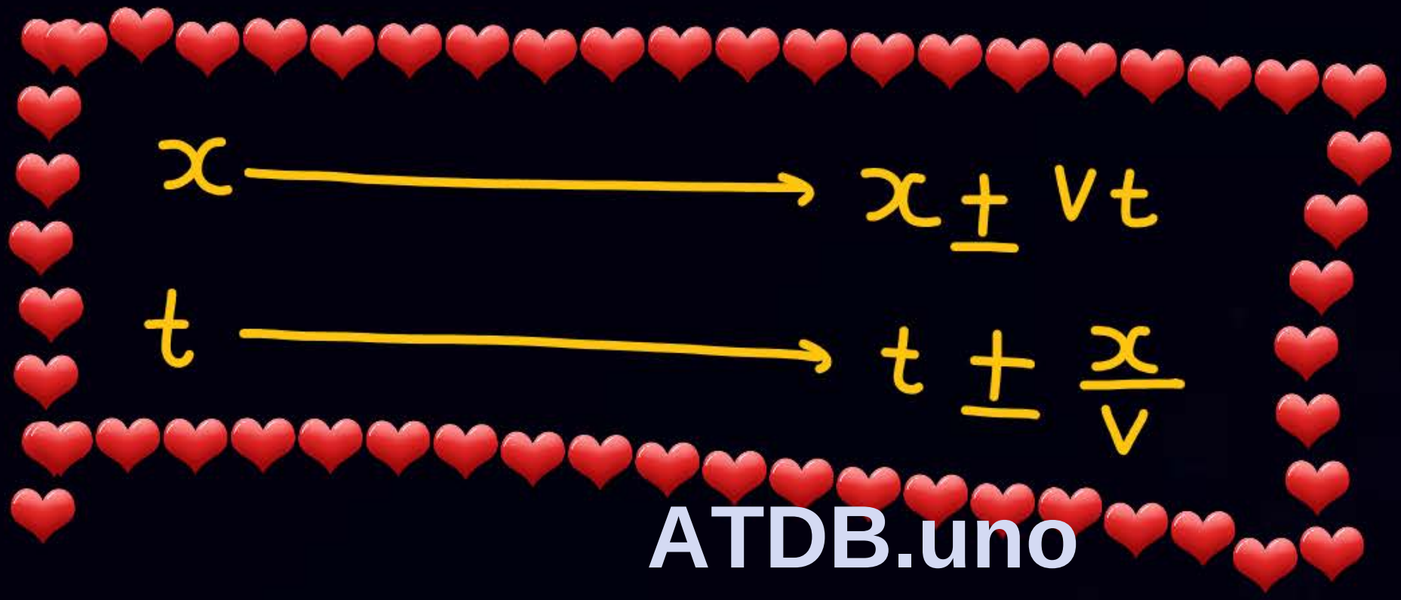
Soln $t \longrightarrow t - \frac{x}{2}$

$$y = 10 \sin\left[2\pi\left(t - \frac{x}{2}\right) + \pi/6\right]$$

(b) $y = \frac{1}{t^2+2}$, (At $x=0$)

$$t \longrightarrow t - \frac{x}{v_w}$$

$$y = \frac{1}{\left(t - \frac{x}{2}\right)^2 + 2}$$



Q At $t=0$, a transverse wave in wire is described by function $y = \frac{6}{x^2+3}$.
write the function (x,t) that described the this pulse if wave is travelling along $+x$ axis with speed 4.5 m/s

Q A wave is travelling with speed 2 m/s along $+x$-axis such that displacement of particle at $x=0$ is given by relation $y = \frac{2}{t^2+1}$. find eq^n of wave

Sol^n

$$y = \frac{6}{(x-4.5t)^2+3}$$

$x \longrightarrow x-4.5t$

$$y = \frac{2}{\left(t-\frac{x}{2}\right)^2+1}$$

$t \longrightarrow t-\frac{x}{2}$

$$P = -T \frac{\partial y}{\partial x} \frac{\partial y}{\partial t}$$

$$\langle P \rangle = \frac{1}{2} \mu A^2 \omega^2 V_w$$

$$\langle I \rangle = \frac{\langle P \rangle}{\text{Area}} = \frac{1}{2} \rho A^2 \omega^2 V_w$$

$$x \longrightarrow x \pm V_w t$$

$$t \longrightarrow t \pm \frac{x}{V_w}$$

Q If for $x=0$ particle y V_s t relation is given as

$$y = \frac{1}{t^2+5}$$

, wave is moving along +x axis with speed 3 m/s

find eqn of wave

Soln

$$t \longrightarrow t - \frac{x}{V_w}$$

$$y = \frac{1}{\left(t - \frac{x}{3}\right)^2 + 5}$$

(b) if wave is moving along -x Axis

$$t \longrightarrow t + \frac{x}{V_w}$$

$$y = \frac{1}{\left(t + \frac{x}{3}\right)^2 + 5}$$

Q A wave is moving along $+x$ axis with speed 4 m/s such that at $t=0$, relation b/w y & x given as

$$y = \frac{1}{x^2+3}$$

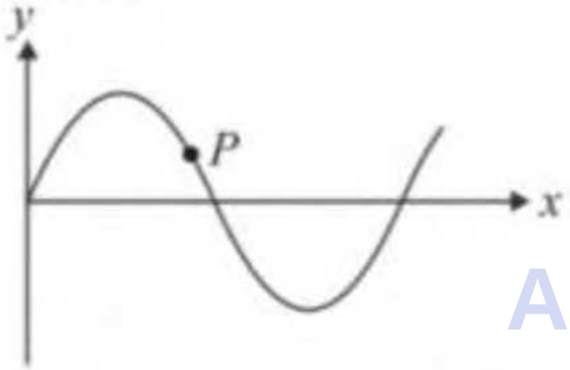
find eq of wave

Sol^n

$x \longrightarrow x-4t$

$$y = \frac{1}{(x-4t)^2+3}$$

Q. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is **(IIT-JEE 2008)**



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s

(b) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s

(c) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

(d) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

Home Work

- DPP-01(must)
- Module ⇒ Prarambh ⇒ 1, 2, 5, 6,

Prabal ⇒ 3, 5, 6,

Parikshit ⇒ (All) ⇒ 4

PYQ ⇒ 1, 2, 5, , 7, 8, 9,