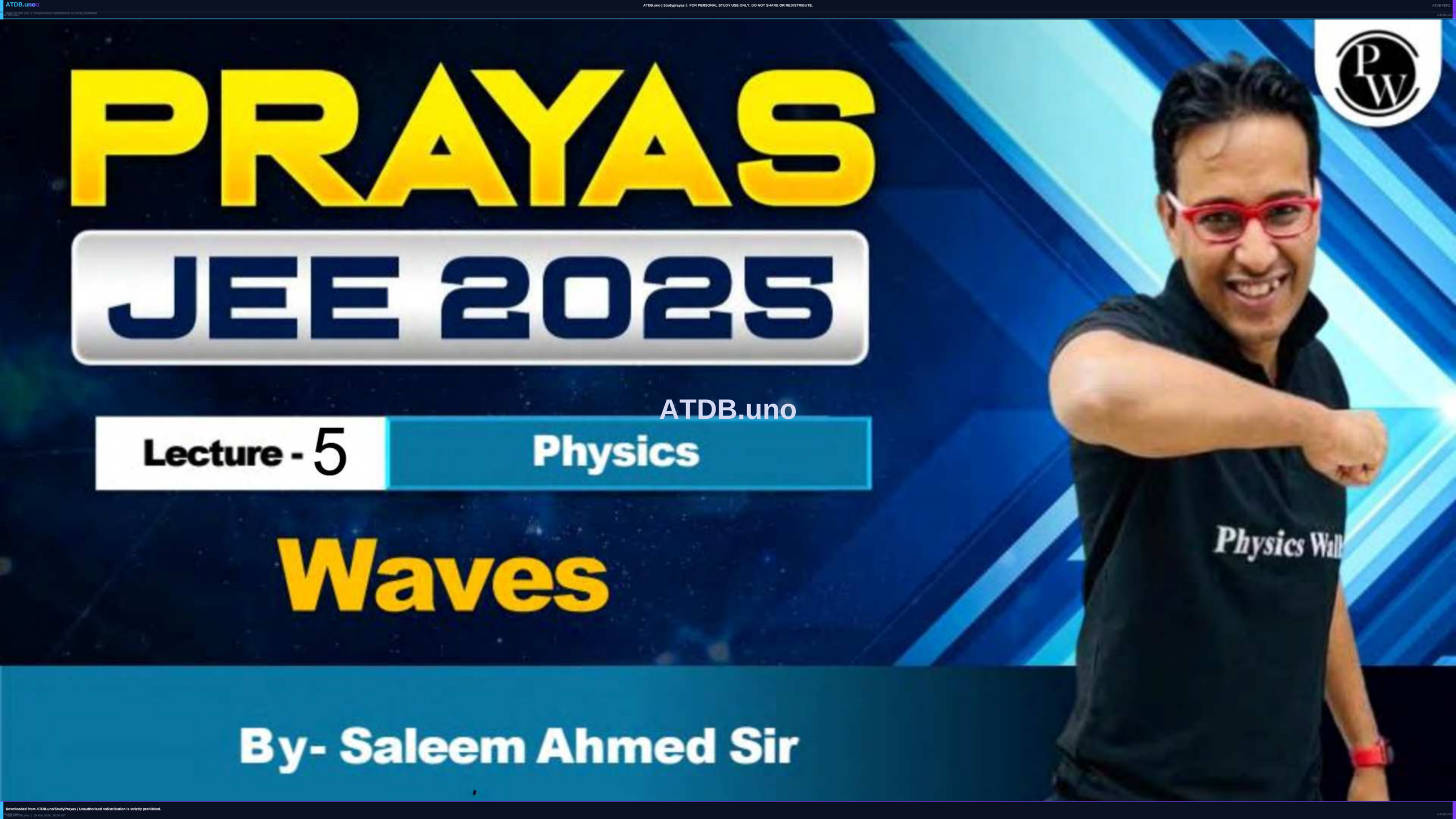
PRAYAS

JEE 2025

Lecture - 5

Physics

Waves



By- Saleem Ahmed Sir

Topics *to be covered*

1. Superposition of Wave

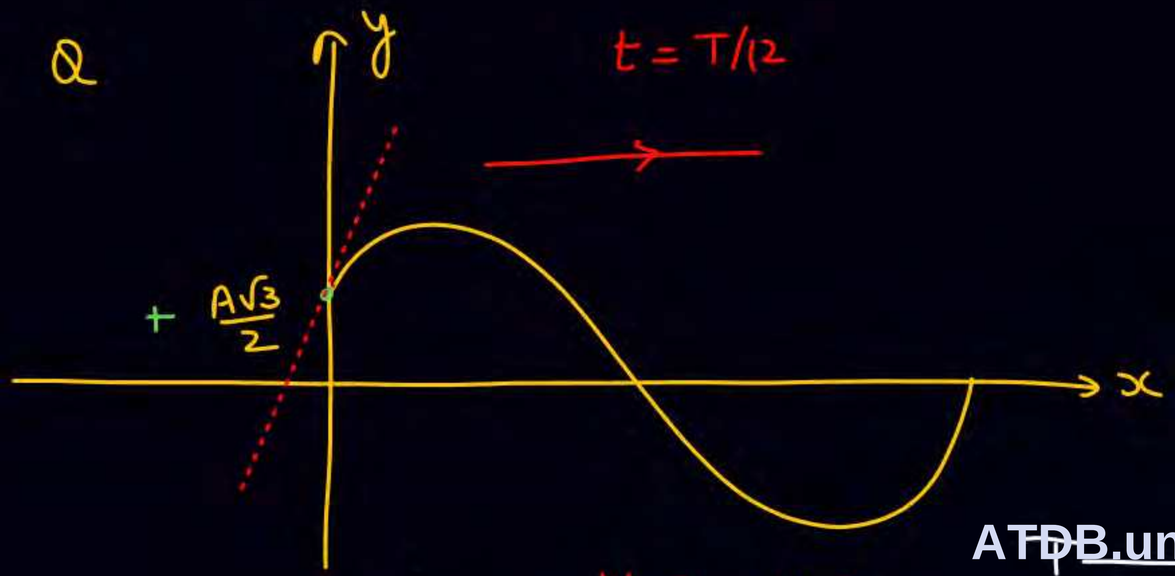
2. Reflection of Wave

3. Reflection from fixed End

4.

$$y = A\sin(\omega t - kx + 120^\circ - 30^\circ)$$

Q



$V_p = - + +$

ϕ → 60, 120

$360^\circ \longrightarrow T$

$30^\circ \longrightarrow T/12$

$y = A\sin(\omega t - kx + \phi)$

$x = 0,\ t = T/12,\ y = \frac{A\sqrt{3}}{2}$

$$\frac{A\sqrt{3}}{2} = A\sin\left(\frac{2\pi}{T}\cdot\frac{T}{12} - 0 + \phi\right)$$

$$\frac{\sqrt{3}}{2} = \sin(30 + \phi)$$

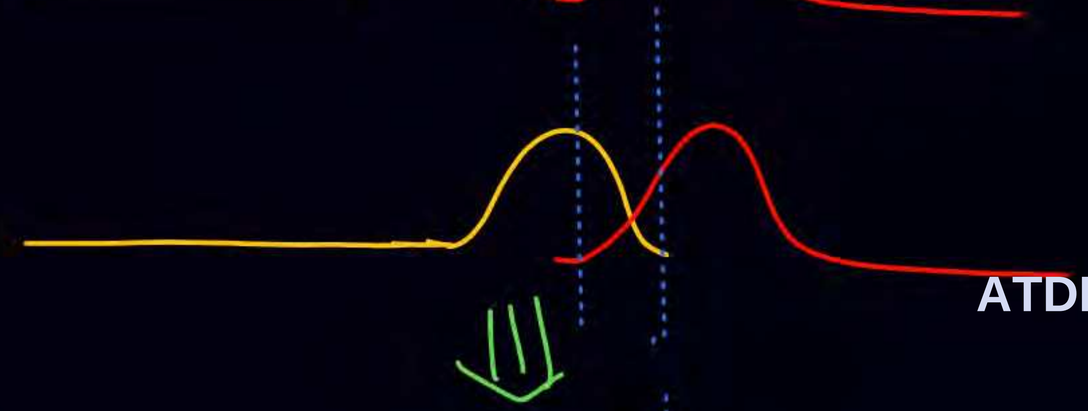
$30 + \phi$ → 60, 120

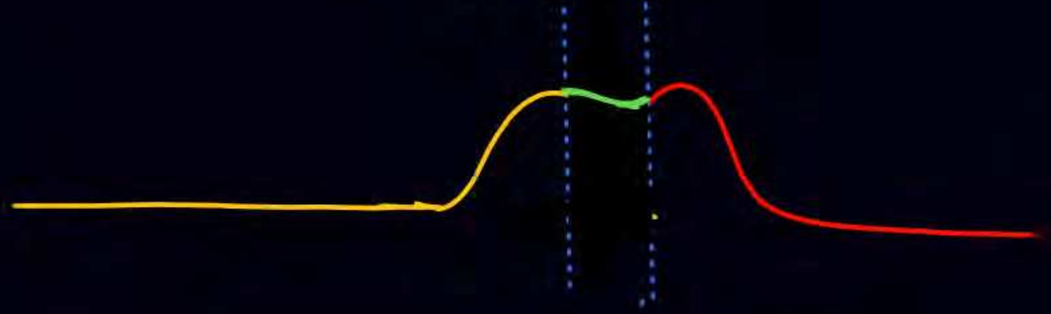
ϕ → 30, 90

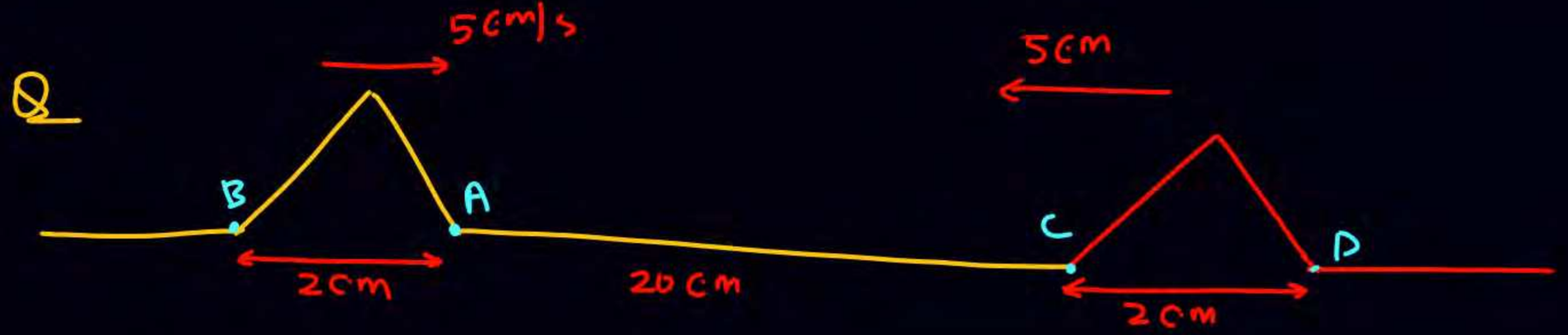
$V_p = - ++$; $V_p < 0$

$\frac{dy}{dt} < 0$ ✓

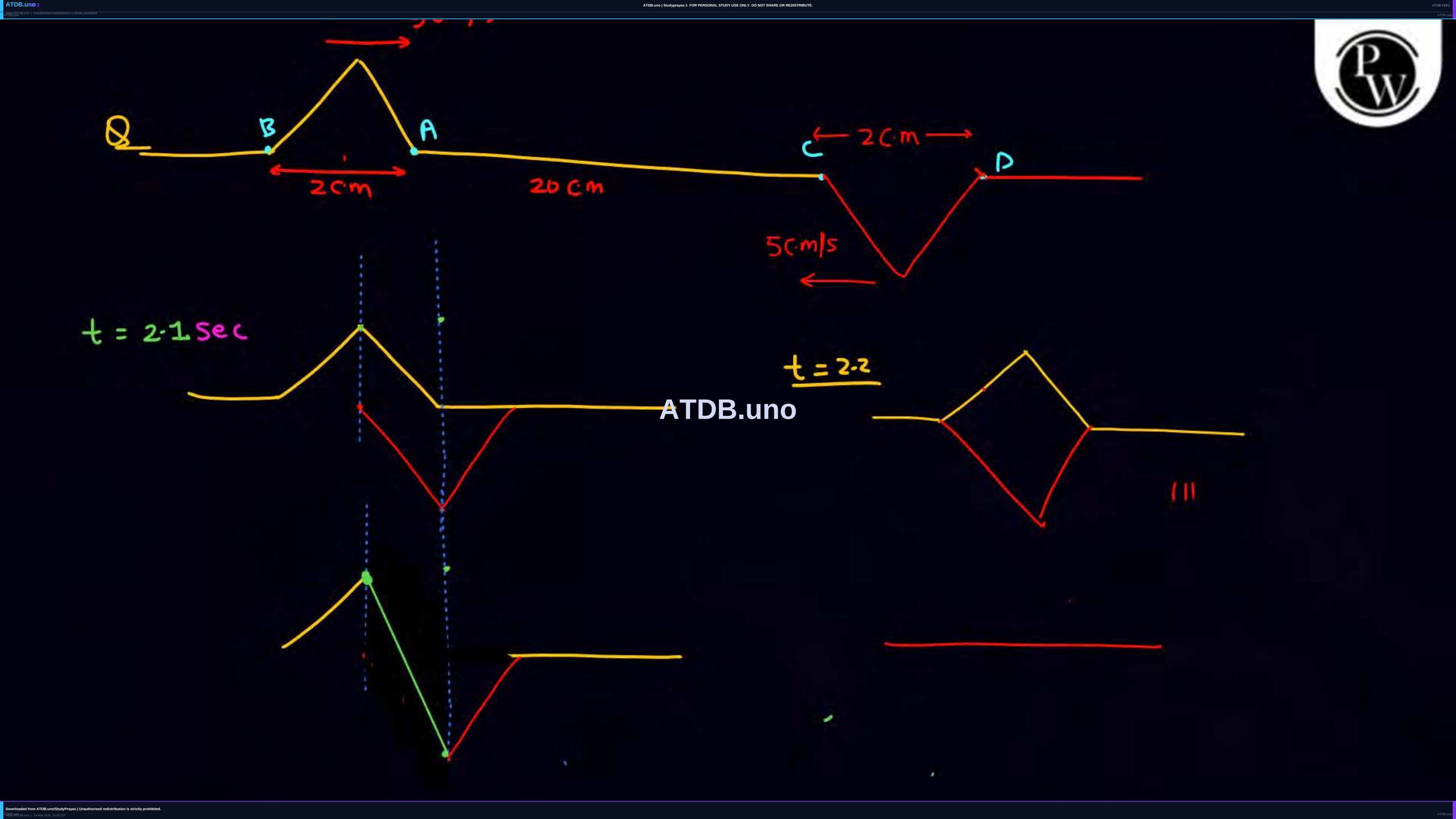
$y_{net} = y_1 + y_2$

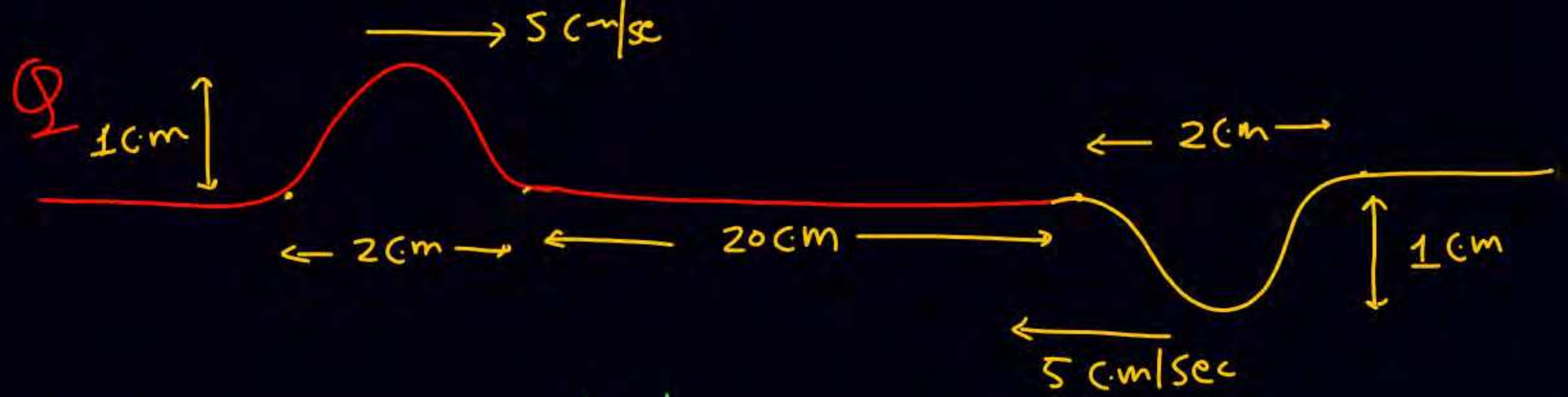


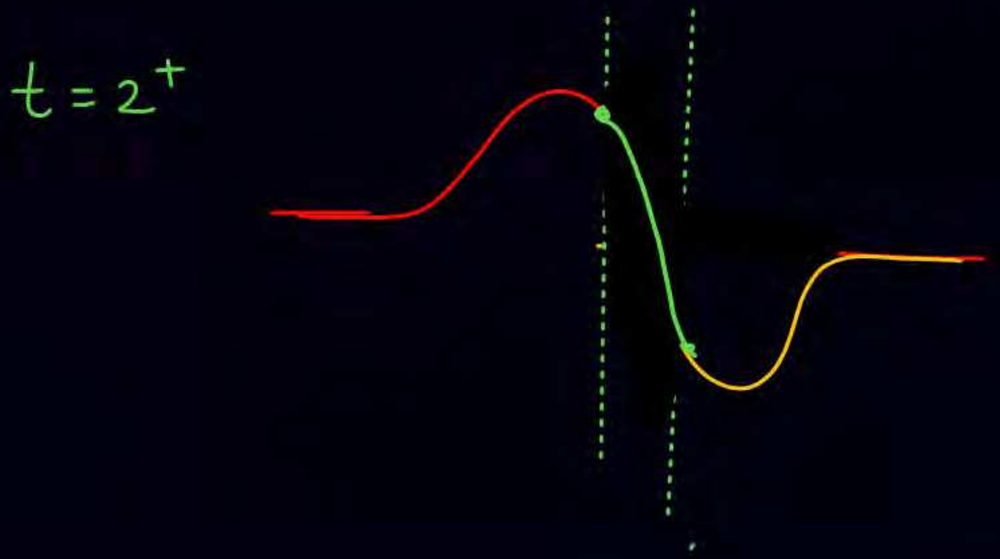




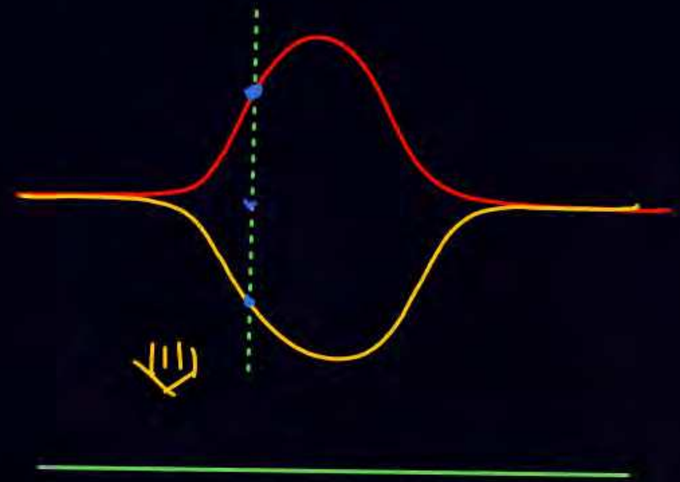
$t = 2.1\,\text{Sec}$



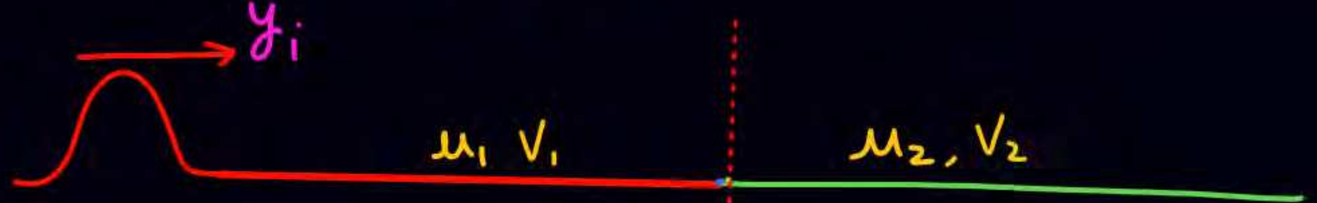




t = 2·2 sec

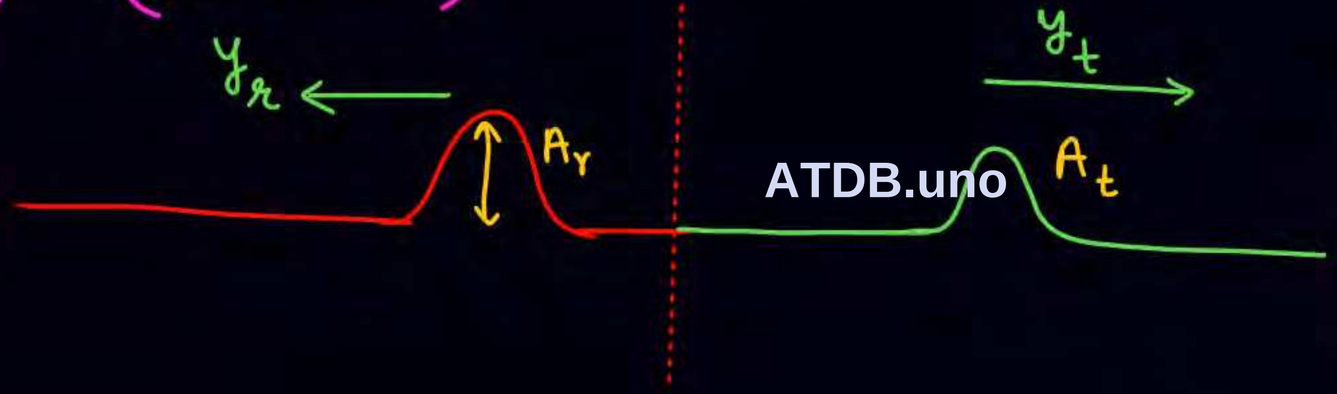


Reflection & Refraction of wave



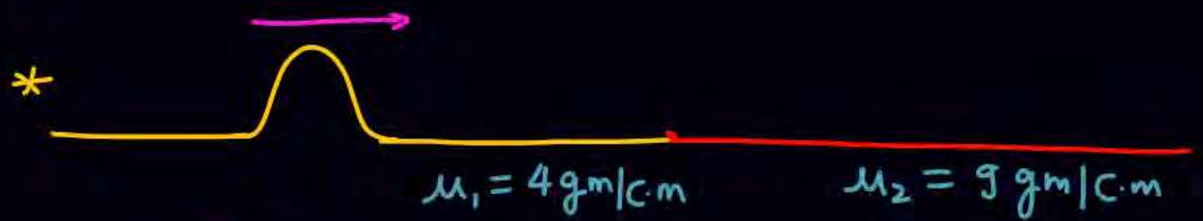
$$V_w = \sqrt{\frac{T}{\mu}} \rightarrow \text{same}$$

$$y_i = A_i \sin(\omega t - kx + \phi)$$



$$A_t = \frac{2V_2}{V_1+V_1} A_i$$

$$A_r = \frac{V_2-V_1}{V_2+V_1} A_i$$

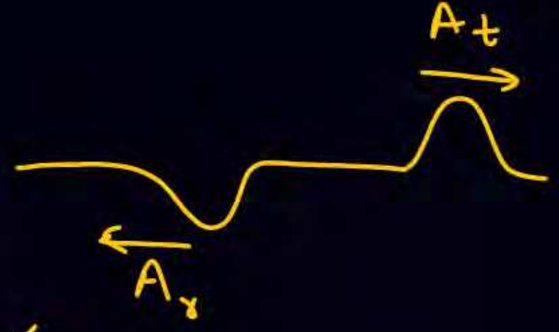


(3)

(4)

$$y_i = 10 \sin(4\pi t - 2\pi x)$$

(1) $$A_t = \frac{2V_2}{V_1 + V_2} A_i = \frac{2\sqrt{T/\mu_2}}{\sqrt{\frac{T}{\mu_1}} + \sqrt{\frac{T}{\mu_2}}} A_i = \frac{2\sqrt{\frac{T}{9}}}{\sqrt{\frac{T}{4}} + \sqrt{\frac{T}{9}}} \times 10 = 8$$



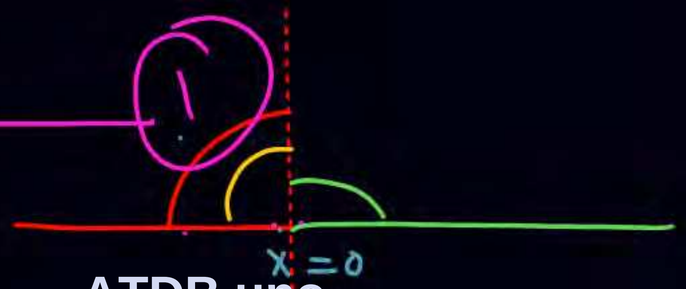
(2) $$A_r = \frac{V_2 - V_1}{V_2 + V_1} A_i = \frac{\sqrt{\frac{T}{9}} - \sqrt{\frac{T}{4}}}{\sqrt{\frac{T}{9}} + \sqrt{\frac{T}{4}}} \times 10 = \frac{\frac{1}{3} - \frac{1}{2}}{\frac{1}{3} + \frac{1}{2}} \times 10 = -2$$

$$\langle P_i \rangle = \langle P_r \rangle + \langle P_t \rangle$$

$$\frac{1}{2}\mu_1 A_i^2 \omega^2 V_1 = \frac{1}{2}\mu_1 A_r^2 \omega^2 V_1 + \frac{1}{2}\mu_2 A_t^2 \omega^2 V_2$$

$$A_i^2 - A_r^2 = \sqrt{\frac{\mu_2}{\mu_1}}\, A_t^2 \quad \text{— (1)}$$

$$V = \sqrt{\frac{T}{\mu}}$$



$$(y)_{x=0^-} = (y)_{x=0^+}$$

$$A_i + A_r = A_t \quad \text{— (2)}$$

Q If 64% of power of incident wave is transmitted. find A_t & A_r. (If $A_i^0 = 10\,m$)

Soln

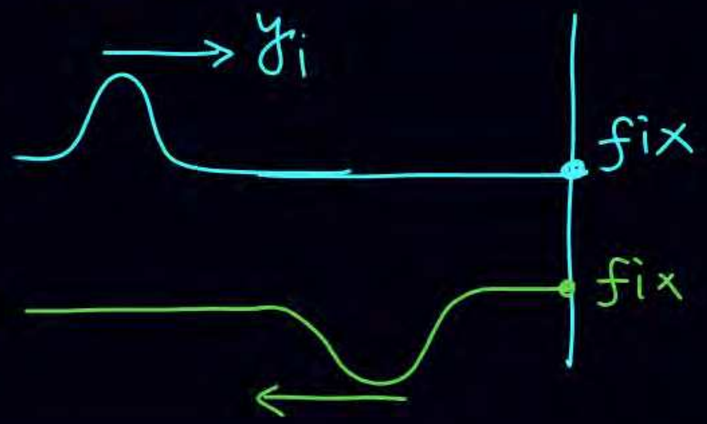
$$A_r = \pm 6$$

$$A_i^0 + A_r = A_t$$

(1) $10 + 6 = A_t = \underline{16}$

(2) $10 - 6 = A_t = \underline{4}$

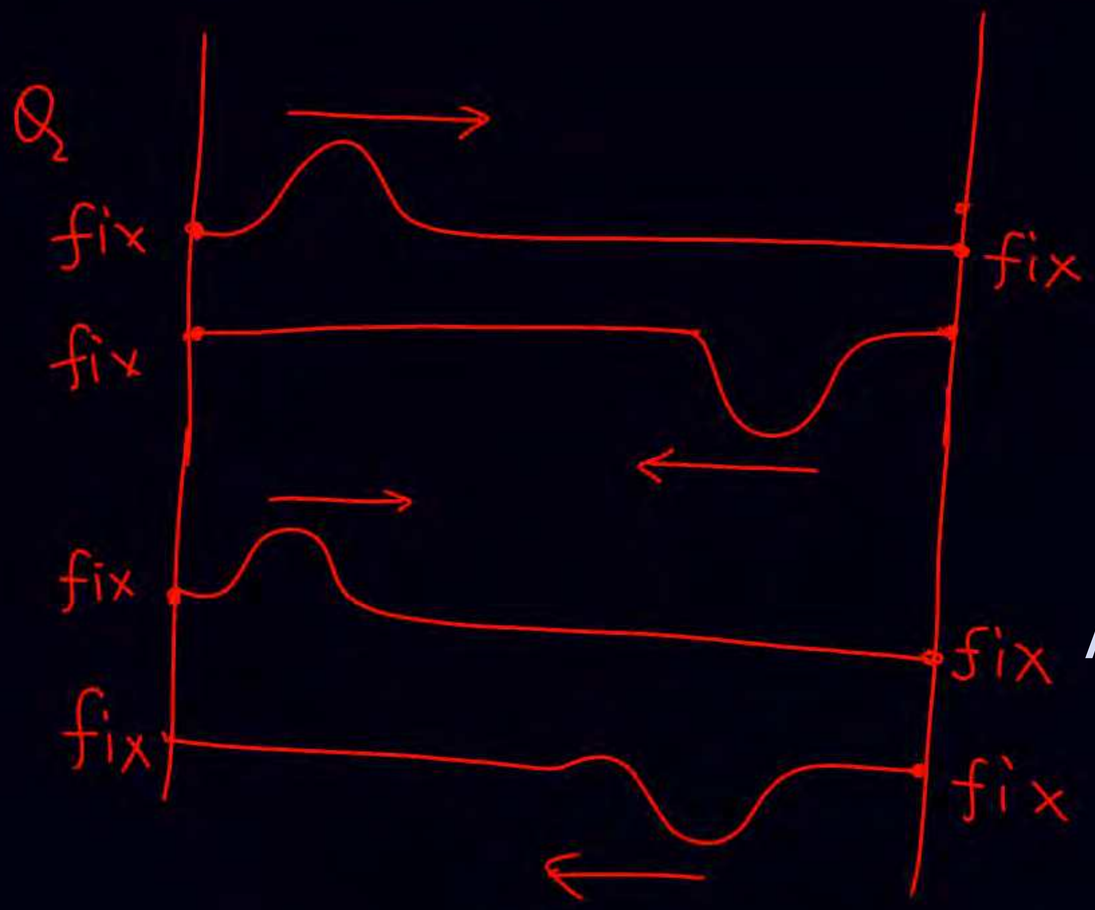
Reflection from fixed end



$$y_i = A \sin(\omega t - kx)$$

$$y_r = A \sin(\omega t + kx + \pi)$$

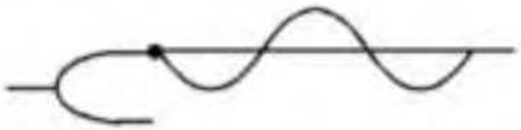
Reflection from free end



ATDB.uno

12. The prong of a electrically operated tuning fork is connected to a long string of $\mu = 1$ kg/m and tension 25N. The maximum velocity of the prong is 1 cm/s, then the average power needed to drive the prong is:

किसी विद्युत चालित स्वरित्र की भुजा को एक ऐसी लम्बी रस्सी के साथ जोड़ दिया जाता है जिसके लिए $\mu = 1$ kg/m तथा तनाव 25 N है। यदि भुजा का अधिकतम वेग 1 cm/s हो तो इसे चलाने के लिए आवश्यक औसत शक्ति होगी :-



$V_{max} = A\omega$

(A) 5×10^{-4} W (B) 2.5×10^{-4} W (C) 1×10^{-4} W (D) 10^{-3} W

Ans. (B)

$$\langle P \rangle = \frac{1}{2} \mu A^2 \omega^2 \sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2} \times 1 \times (10^{-2})^2 \times \sqrt{\frac{25}{1}}$$

17. A string consists of two parts attached at $x = 0$. The right part of the string ($x > 0$) has mass μ_r per unit length and the left part of the string ($x < 0$) has mass μ_l per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure, what is the amplitude of the wave that is transmitted to the right part of the string ?

एक रस्सी दो भागों से मिलकर बनी है जो $x = 0$ पर जुड़े हैं। रस्सी का दाँया भाग ($x > 0$) के प्रति इकाई लम्बाई का द्रव्यमान μ_r तथा बाह्य भाग ($x < 0$) के लिए μ_l है। रस्सी में तनाव का मान T है। यदि इकाई आयाम वाली एक तरंग चित्रानुसार रस्सी के बाँये भाग के अनुदिश गति करे तो रस्सी के दाँये भाग में पारगमित होने वाली तरंग का आयाम होगा :-

$$\frac{2V_2}{V_1+V_2}A_i = \frac{2V_2/V_1}{1+V_2/V_1} \longrightarrow \frac{\mu_l \quad \mu_r}{}$$

(A) 1

(B) $\dfrac{2}{1+\sqrt{\mu_l/\mu_r}}$

(C) $\dfrac{2\sqrt{\mu_l/\mu_r}}{1+\sqrt{\mu_l/\mu_r}}$

(D) $\dfrac{\sqrt{\mu_l/\mu_r}-1}{\sqrt{\mu_l/\mu_r}+1}$

Ans. (C)

1. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave in 0.08m and 2.0 s respectively then α and β in appropriate units are **[AIEEE - 2008]**

x-अक्ष के अनुदिश गति कर रही एक तरंग का समीकरण $y(x, t) = 0.005 \cos(\alpha x - \beta t)$ द्वारा प्रदर्शित किया गया है। यदि तरंग की तरंगदैर्ध्य तथा आवर्तकाल क्रमश: 0.08m तथा 2.0 s हो तो α तथा β के मान होगें

(1) $\alpha = 25.00\pi, \beta = \pi$

(2) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$

(3) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$

(4) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

Ans. (1)

7. A uniform string of length 20m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is :-

(take $g = 10\ ms^{-2}$)

20m लम्बाई की एकसमान डोरी को एक दृढ़ आधार से लटकाया गया है। इसके निचले सिरे से एक सूक्ष्म तरंग-स्पंद चालित होता है। ऊपर आधार तक पहुँचने में लगने वाला समय है :- **[JEE-Main-2016]**

($g = 10\ ms^{-2}$ लें)

(1) $\sqrt{2}\,s$ (2) $2\pi\sqrt{2}\,s$ (3) $2s$ (4) $2\sqrt{2}\,s$

Ans. (4)

14. A transverse periodic wave on a string with a linear mass density of 0.200 kg/m is described by the following equation

$$y = 0.05 \sin(420t - 21.0\,x)$$

where x and y are in metres and t is in seconds.

The tension in the string is equal to :

एक आवर्ती अनुप्रस्थ तरंग एक डोरी जिसका रेखीय द्रव्यमान घनत्व 0.200 kg/m है, पर निम्न समीकरण द्वारा व्यक्त की जाती है :

$$y = 0.05 \sin(420t - 21.0\,x)$$

जहाँ x व y मीटर में है एवं t सैकण्ड में है। डोरी में तनाव है :

(A) 32 N (B) 42 N (C) 66 N (D) 80 N

Ans. (D)

13. A wire is 4 m long and has a mass 0.2 kg. The wire is kept horizontally. A transverse pulse is generated by plucking one end of the taut (tight) wire. The pulse makes four trips back and forth along the cord in 0.8 sec. The tension is the cord will be :-

एक 4 m लम्बे तार का द्रव्यमान 0.2 kg है। तार को क्षैतिजत: रखा जाता है। तने हुये तार के एक सिरे को झटका देकर एक अनुप्रस्थ स्पंद उत्पन्न किया जाता है। यह स्पंद तार के अनुदिश 0.8 sec में चार बार आगे-पीछे गति करता है। तार में तनाव होगा:-

(A) 80 N (B) 160 N (C) 240 N (D) 320 N

Ans. (A)

3. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to :

द्रव्यमान M वाली एक भारी गेंद को कार की छत से द्रव्यमान m (m << M) वाली एक हल्की रस्सी से लटकाया जाता है। जब कार विराम में होती है तो रस्सी में उत्पन्न अनुप्रस्थ तरंगों की चाल 60 ms^{-1} होती है। जब कार का त्वरण a होता है तब तरंग चाल 60.5 ms^{-1} तक बढ़ जाती है तो गुरूत्वीय त्वरण g के पदों में a का निकटतम मान होगा :-

[JEE-Main-2019_Jan]

(1) $\frac{g}{5}$ (2) $\frac{g}{20}$ (3) $\frac{g}{10}$ (4) $\frac{g}{30}$

Ans. (1)

Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave ?

245 Hz आवृत्ति की कोई ध्वनि तरंग धनात्मक x-अक्ष के अनुदिश 300 ms^{-1} की चाल से गमन कर रही है। इस तरंग का प्रत्येक कण 6 cm की कुल दूरी का दोलन करता है। इस प्रगामी तरंग के लिए गणितीय व्यंजक क्या होगा ?

[JEE-Main-2021_March]

(1) $Y(x,t) = 0.03\,[\sin 5.1\,x - (0.2 \times 10^3)t]$

(2) $Y(x,t) = 0.06\,[\sin 5.1\,x - (1.5 \times 10^3)t]$

(3) $Y(x,t) = 0.06\,[\sin 0.8\,x - (0.5 \times 10^3)t]$

(4) $Y(x,t) = 0.03\,[\sin 5.1\,x - (1.5 \times 10^3)t]$

ns. (4)

12. A transverse wave is represented by $y = 2\sin(\omega t - kx)$ cm. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity, will be:

एक अनुप्रस्थ तरंग समीकरण $y = 2\sin(\omega t - kx)$ cm द्वारा प्रदर्शित है। उस तरंगदैर्ध्य का मान (cm में) ज्ञात कीजिए, जिस पर तरंग वेग, कण के अधिकतम वेग के बराबर होगा। **[JEE-Main-2022_July]**

(A) 4π (B) 2π (C) π (D) 2

Ans. (A)

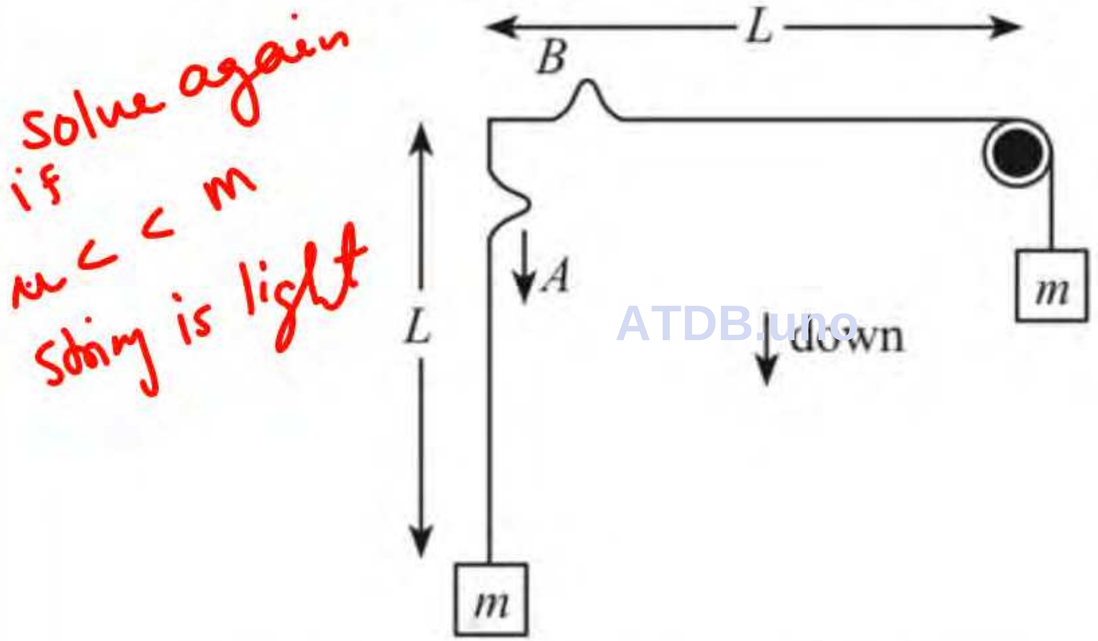
The speed of a transverse wave passing through a string of length 50 cm and mass 10 g is 60 ms^{-1}. The area of cross-section of the wire is 2.0 mm^2 and its Young's modulus is 1.2×10^{11} Nm^{-2}. The extension of the wire over its natural length due to its tension will be $x \times 10^{-5}$ m. The value of x is_____.

एक 50 cm लम्बी एवं 10 g द्रव्यमान की रस्सी पर चलने वाली अनुप्रस्थ तरंग की चाल 60 ms^{-1} है। तार का अनुप्रस्थ क्षेत्राफल 2.0 mm^2 और इसका यंग गुणांक 1.2×10^{11} Nm^{-2} है। तन्यता के कारण इसकी वास्तविक लम्बाई में हुई वृद्धि $x \times 10^{-5}$ m है। x का मान है _____.

[JEE-Main-2022 July]

Ans. (15)

28. Two strings with identical mass per unit length μ are drawn below. Both are stretched by means of an identical weight of mass m hanging on each string as shown. At time $t = 0$ an identical wave pulse is flipped onto each string, where it travels to the end, reflects off of the mass or the pulley as the case may be, and returns. Which of the following is true?



Solve again if $u < < m$ String is light

(a) The pulse on string A returns first

(b) The pulse on string B returns first

(c) The pulse on string A returns at the same time as the pulse on string B

(d) Cannot be predicted

QUESTION

A wire of density 8×10^3 kg m^{-3} is stretched between two clamps 0.5 m apart. The extension developed in the wire is 3.2×10^{-4} m. If $Y = 8 \times 10^{10}$ Nm^{-2}, the fundamental frequency of vibration in the wire will be _____ Hz.

[11 April 2023 - Shift 2]

Ans : (80)

QUESTION

A transverse harmonic wave on a string is given by $y(x, t) = 5\sin(6t + 0.003x)$ where x and y are in cm and t in sec. The wave velocity is _____ ms^{-1}

[10 April 2023 - Shift 1]

Ans : (20)

QUESTION

A steel wire with mass per unit length 7.0×10^{-3} kg m^{-1} is under tension of 70 N. The speed of transverse waves in the wire will be: **[01 February 2023 - Shift 1]**

1. $200\ \pi$ m/s
2. 100 m/s
3. 10 m/s
4. 50 m/s

Ans : (2)

QUESTION

A transverse wave is represented by $y = 2\sin(\omega t - kx)$ cm. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity, will be:

[JEE Mains 2022]

1. 4π
2. 2π
3. π
4. 2

Ans : (1)

QUESTION

The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form $y = -0.21 \sin(x + 30t)$ is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of x is.
(Round-off to the nearest integer).

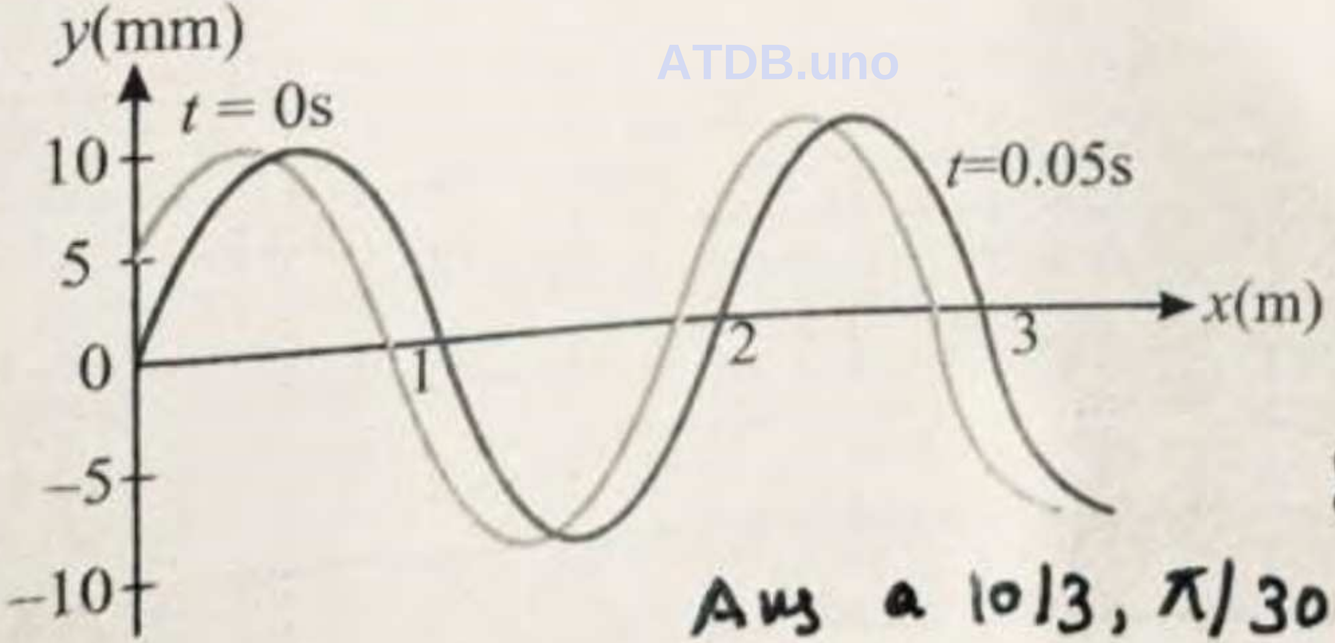
[JEE Mains 2021]

Ans : 1215

A string with linear mass density $\lambda = 5.00 \times 10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

Ans 512

Figure represents two snaps of a travelling wave on a string of mass per unit length $\mu = 0.25$ kg/m. The first snap is taken at $t = 0$ and the second is taken at $t = 0.05$ s. Determine **(a)** the speed of the wave, **(b)** the wavelength and frequency of the wave, **(c)** the maximum speed of the particle, **(d)** the tension in the string **(e)** the equation of the wave.



$y = 10 \sin(\pi x)$

Ans a $10/3, \pi/30, \frac{25}{3}, \left(-\frac{\pi}{3}10t + \frac{\pi}{6}\right)$

A circular loop of string rotates about its axis on a frictionless horizontal plane at a uniform rate so that the tangential speed of any particle of the string is v. If a small transverse disturbance is produced at a point of the loop, with what speed (relative to the string) will this disturbance travel on the string?

4. A travelling wave pulse is given by $y = \dfrac{10}{5+(x+2t)^2}$

In which direction and with what velocity is the pulse propagating? What is the amplitude of the pulse? (-2, 2)

5. A travelling wave pulse is given by

$$y = \frac{0.8}{(3x^2 + 12xt + 12t^2 + 4)}$$

where x and y are in m and t is in seconds. Find the velocity and amplitude of the wave. (2, .2)

6. If the displacement relation for a particle in a wave is given by $y = 5\sin\left(\dfrac{t}{0.04} - \dfrac{x}{4}\right)$, determine the maximum speed of the particle in SI units. (125)

A wave pulse is travelling along $+x$ direction on a string at 2 m/s. Displacement y (in cm) of the particle at $x = 0$ at any time t is given by $2/(t^2 + 1)$. Find

(a) Expression of the function $y = (x, t)$, i.e., displacement of a particle at position x and time t.

Homework

- DPP
- Module → Praranbh (1–13), ⇒ Prabal ⇒ (2–9)
- HCV yesterday H.W