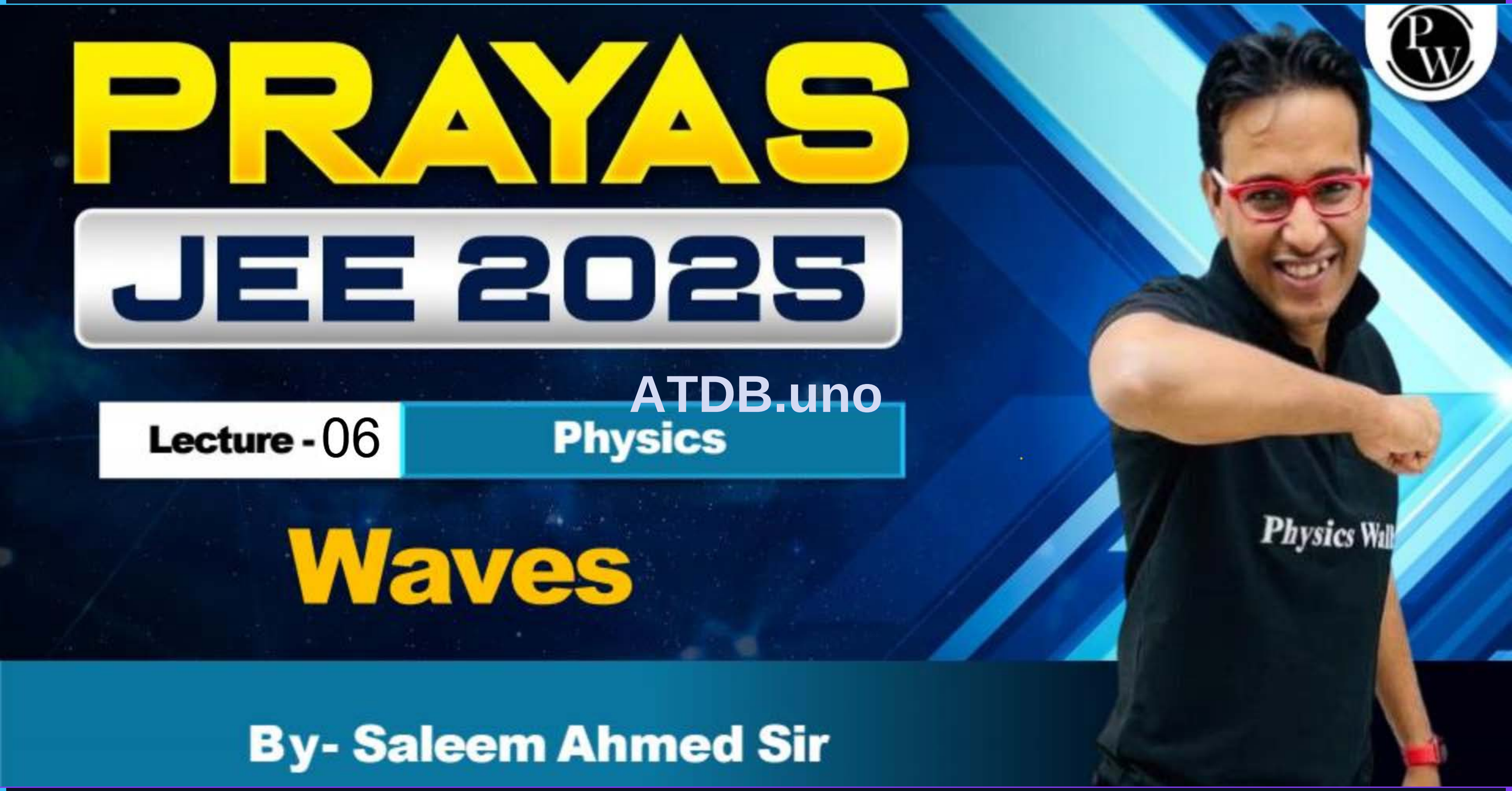
PRAYAS

JEE 2025

Lecture - 06

Physics

Waves



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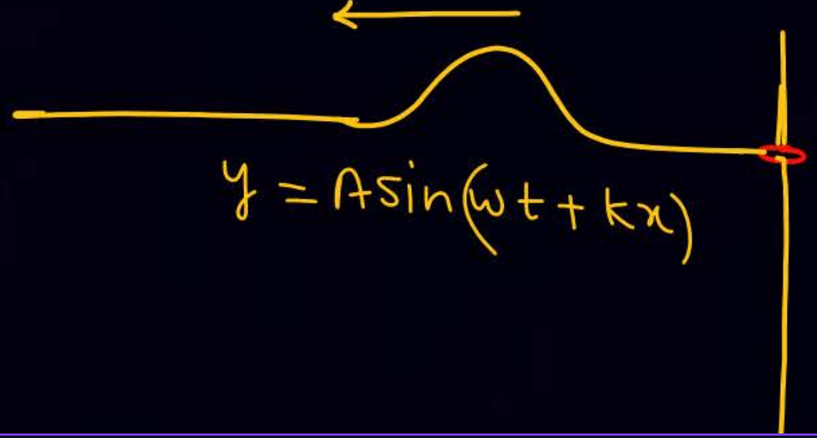
Topics to be covered

1. Standing wave
2.
3.
4.

Reflection from fixed end

Reflection from free end



$y = A\sin(\omega t - kx)$

$y_r = A\sin(\omega t + kx + \pi)$ fixed

$y_f = A\sin(\omega t + kx)$ free

Consider the following wave functions:

(a) $y = A\sin(\omega t - kx)$, (b) $y = A\sin(kx - \omega t)$

(c) $y = A\cos(\omega t - kx)$, (d) $y = A\cos(kx - \omega t)$

(e) $y = A\sin(\omega t + kx)$, (f) $y = A\cos(\omega t + kx)$

Write the equations of reflected wave after reflection from a free and a fixed boundary. Also find the resulting stationary waves formed by the superposition of its reflected wave.

Sol.

	Incident wave $\omega = \frac{2\pi}{T}, k = \frac{2\pi}{\lambda}$	Reflected wave from free boundary, $\phi = 0$	Reflected wave from fixed boundary, $\phi = \pi$
(a)	$y = A\sin(\omega t - kx)$	$y = A\sin(\omega t + kx)$	$y = A\sin(\omega t + kx + \pi)$ $= -A\sin(\omega t + kx)$
(b)	$y = A\sin(kx - \omega t)$	$y = A\sin(-kx - \omega t)$ $= -A\sin(kx + \omega t)$	$y = A\sin(-kx - \omega t + \pi)$ $= A\sin(kx + \omega t)$
(c)	$y = A\cos(\omega t - kx)$	$y = A\cos(\omega t + kx)$	$y = A\cos(\omega t + kx + \pi)$ $= -A\cos(\omega t + kx)$
(d)	$y = A\cos(kx - \omega t)$	$y = A\cos(-kx - \omega t)$ $= A\cos(kx + \omega t)$	$y = A\cos(-kx - \omega t + \pi)$ $= -A\cos(kx + \omega t)$
(e)	$y = A\sin(\omega t + kx)$	$y = A\sin(\omega t - kx)$	$y = A\sin(\omega t - kx + \pi)$ $= -A\sin(\omega t - kx)$
(f)	$y = A\cos(\omega t + kx)$	$y = A\cos(\omega t - kx)$	$y = A\cos(\omega t - kx + \pi)$ $= -A\cos(\omega t - kx)$

Standing wave.

If two sinosoidal wave of same amplitude same frequency & in same wavelength travel in opposite dirn in a streach string, then interference produce standing wave.

If $\begin{cases} y_1 = A\sin(kx - \omega t) \\ y_2 = A\sin(kx + \omega t) \end{cases}$

$y_{net} = \underbrace{2A\sin kx}_{\text{Amplitude}} \cos\omega t \equiv$ Eq^n of standing wave

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$

$$y_{net} = y_1 + y_2$$

$$y_{net} = A\sin kx\cos\omega t - A\cos kx\sin\omega t + A\sin kx\cos\omega t + A\cos kx\sin\omega t$$

$$y_{net} = \underbrace{2A\sin kx}_{\text{Amplitude}}\cos\omega t$$

Amplitude

$$\text{Amplitude} = 2A\sin kx$$

Amplitude = 0 (Node)

$$2A\sin kx = 0$$

$$\sin kx = 0$$

$$kx = 0, \pi, 2\pi, 3\pi \ldots$$

$$\frac{2\pi}{\lambda}x = 0, \pi, 2\pi, \ldots$$

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots$$

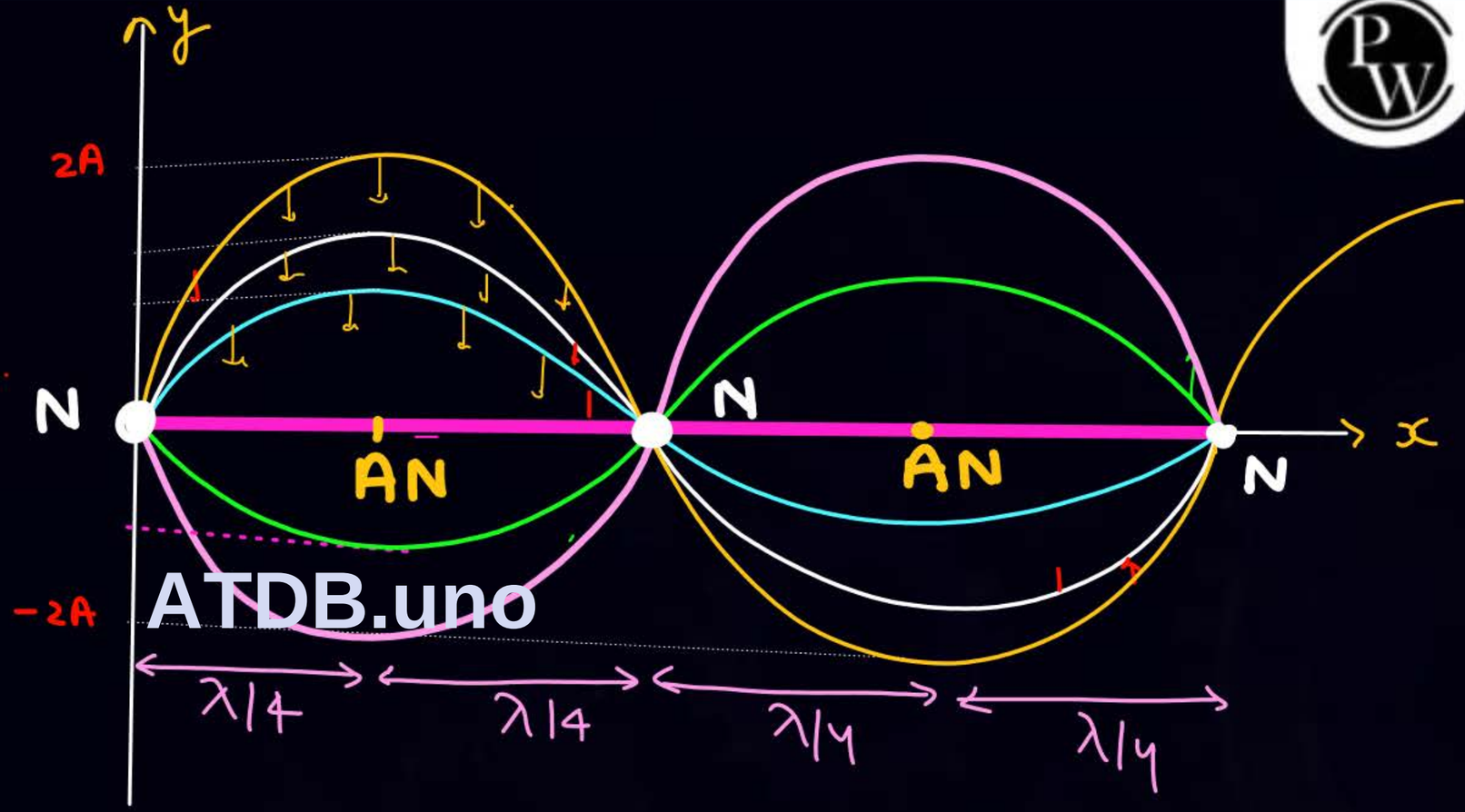
Amplitude ≡ max ⇉ Antinode

$$2A\sin kx = \pm 2A$$

$$\sin kx = \pm 1$$

$$\frac{2\pi}{\lambda}x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$$

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$



Hence we can conclude that

- Stationary wave are formed due to superposition of two identical simple harmonic travelling in opposite dir^n with same speed.
- particle having zero amplitude called Node they are at rest. and
- " " max amplitude " antinode ⇒ having min. strain. having max strain.
- Node & antinode are formed alternately.
- Distance b/w two consecutive [illegible] is λ/2
- " " " AN " "
- " " " Node & AN is λ/4
- All the particles in one particular segments vibrates in same phase but particle b/w two adjacent segments differ in phase by 180°

काम का सवाल (SKC)

—

(*) → $y = (2A \sin Kx) \cos \omega t$

$2A \sin Kx$ → Amplitude.

* In a loop ≡ all the particle are in same phase &
having 180° phase difference with Baju वाला loop

$\lambda = \frac{}{\lambda}, \lambda = \frac{}{2}$

Q $y = 20 \sin 4\pi x \cos 2\pi t$

find ① Location of node

$A = 20 \sin 4\pi x = 0$

$4\pi x = 0, \pi, 2\pi, 3\pi$

$x = 0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}$

② location of antinode

$\sin 4\pi x = \pm 1$

$4\pi x = \pi/2, 3\pi/2, 5\pi/2$ - -

$x = \frac{1}{8}, \frac{3}{8}, \frac{5}{8}$ - -

③ location of the point whose amplitude is half of the max amplitude

$A = 20 \sin 4\pi x$

$\pm 10 = 20 \sin 4\pi x$

$\sin 4\pi x = \pm \frac{1}{2}$

$4\pi x = \pi/6, 5\pi/6, 7\pi/6$

$x = \frac{1}{24}, \frac{5}{24}, \frac{7}{24}$

H.W

⑧ If string is fixed b/w two end and lenght of string is 2m find no. of node & antinode on string.

$$y = 2A \sin kx \cos \omega t$$

$$y = 2A \sin kx \sin \omega t$$

$$y = 2A \cos kx \sin \omega t$$

$$y = 2A \cos kx \cos \omega t$$

$$y = 2A \sin(kx + \pi) \cos kx$$

Home work

— All the best for AITS

- Probable → (2–9),

Programme → (1 – 8), 14, 15,