

PRAYAS

JEE 2025

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Lecture - 68

Physics

Waves

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Topics *to be covered*

1 Sound wave, velocity of sound wave

2 Characteristics of sound

3 Loudness

4

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Sound wave

- Longitudinal wave
- Particle vibrates in the dirⁿ of wave motion (parallel/Antiparallel)
- As sound travel through air, the elements of air vibrates to produce change in density and pressure along the direction of motion of the wave.

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when no sound wave



$$\beta S_0 k = \Delta P_0$$

when sound wave travel



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Rarefaction

λ



Eqⁿ of sound wave

$$S = S_0 \sin(\omega t - kx + \phi)$$

Displacement of particle

Amplitude



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$$V_w = \frac{\omega}{k}$$

ω, k Sign opposite $\Rightarrow +x$ dirⁿ $\frac{d}{dt}$

ω, k " Same $\Rightarrow -x$ " "

$$\frac{\partial s}{\partial t} = v_p$$

$$\frac{\partial^2 s}{\partial t^2} = a_p$$

$$v_p = -v_w \frac{\partial s}{\partial x}$$

Variation of excess pressure in gas due to propagation of longitudinal wave.

Let s is displacement of particle at x

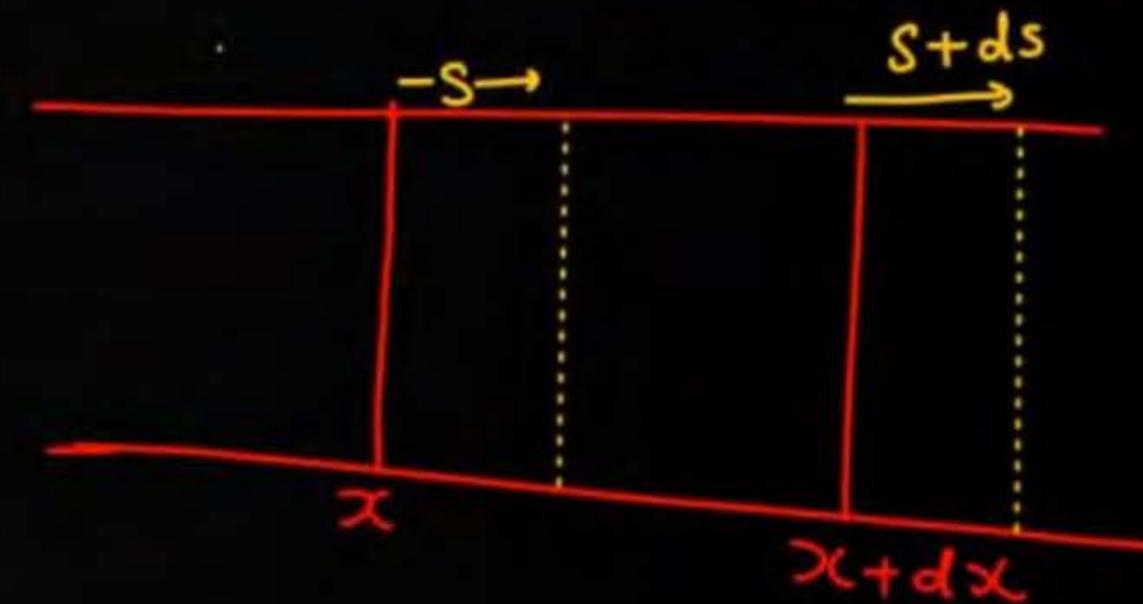
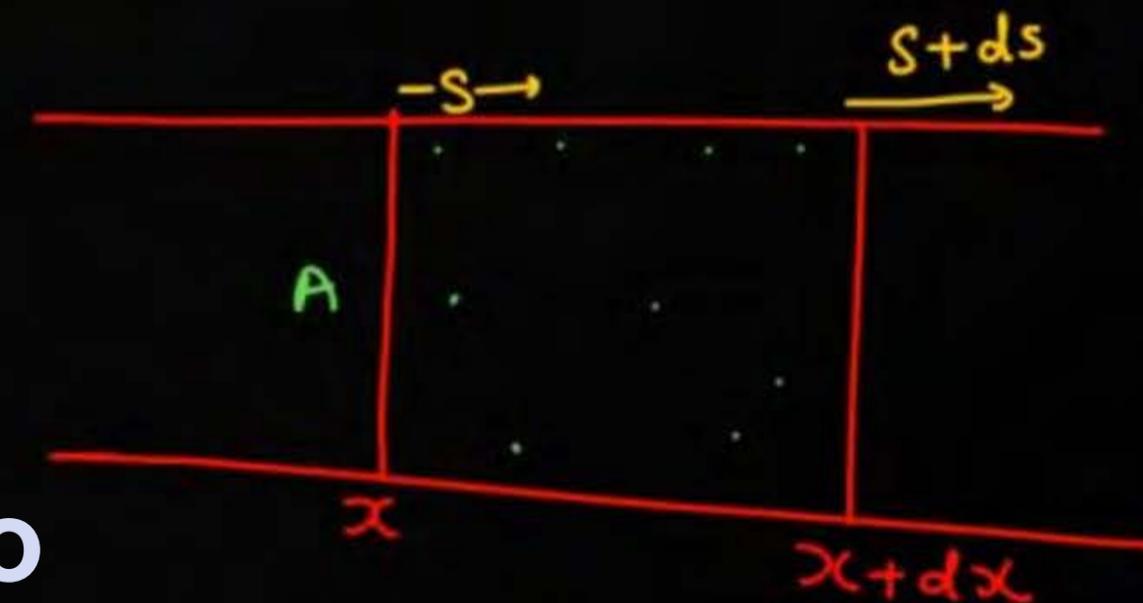
$s+ds$ is displacement of particle at $x+dx$

Change in volume of elements = ΔV

$V_i = \text{Initial vol}^n = A dx$

$\Delta V = \text{change in vol}^n = A ds$

$$\frac{\Delta V}{V_i} = \frac{A ds}{A dx} = \frac{\partial s}{\partial x}$$





$$B = - \frac{\Delta P}{\frac{\Delta V}{V}}$$

Excess pressure

$$\Delta P = -B \frac{\Delta V}{V} = -B \frac{\partial s}{\partial x}$$

$$\Delta P = -B \frac{\partial s}{\partial x}$$

Excess pressure

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$$s = s_0 \sin(\omega t - kx + \phi)$$

$$\Delta P = -B \frac{\partial s}{\partial x}$$

$$\Delta P = + \boxed{B s_0 k} \cos(\omega t - kx + \phi)$$

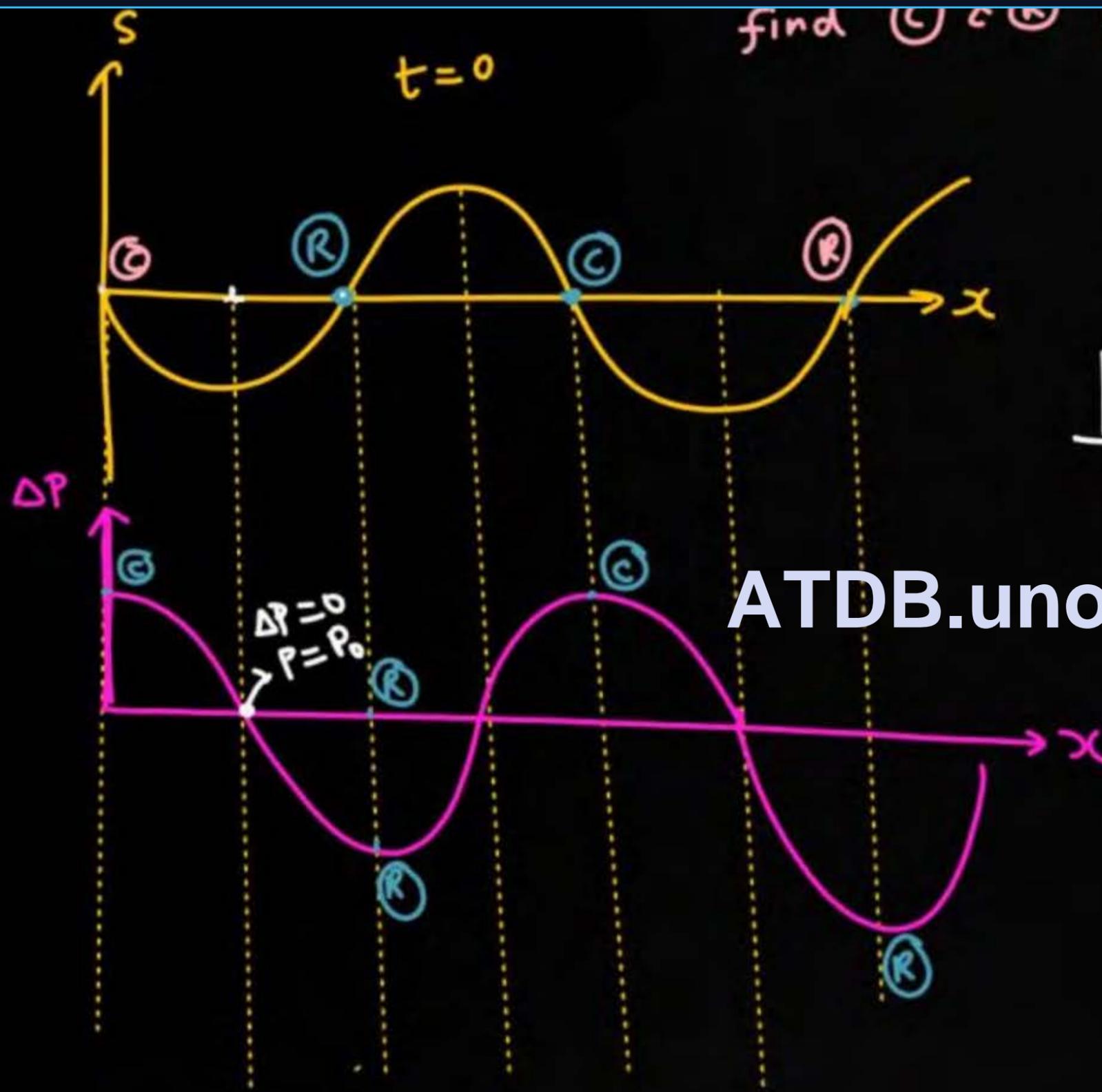
90°
phase diff

$$\Delta P = \Delta P_0 \cos(\omega t - kx + \phi)$$

Excess pressure
Amplitude

$$\Delta P_0 = B s_0 k$$

At most compressed volⁿ $\Rightarrow P = P_0 + \Delta P_0 = P_0 + B s_0 k$
 " rarified volⁿ $\Rightarrow P = P_0 - B s_0 k$



$$s = -s_0 \sin kx$$

$$\Delta P = -B \frac{ds}{dx}$$

$$\Delta P = +B s_0 k \cos kx$$

$$\Delta P = \Delta P_0 \cos kx$$

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Q A sound wave of wavelength 40 cm travel in air. If the difference b/w maximum & minimum pressure at a point is $4.0 \times 10^{-3} \text{ N/m}^2$, find the amplitude of vibration of particle of medium. - Bulk modulus = $1.4 \times 10^5 \frac{\text{N}}{\text{m}^2}$ of air

Solⁿ

$$\lambda = 40$$

$$4 \times 10^{-3} = (P_0 + BS_0K) - (P_0 - BS_0K)$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{40}$$

$$= 5\pi$$

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$$4 \times 10^{-3} = 2BS_0K$$

$$S_0 = \frac{4 \times 10^{-3}}{2 \cdot B \cdot K} = \frac{4 \times 10^{-3}}{2 \times 1.4 \times 10^5 \times 5 \times \frac{22}{7}}$$



Q Equation of sound wave in air is given by

$$P = 2 \sin(3000t - 9x) \text{ where all variables are in S.I. Unit.}$$

① find λ , frequency, speed of sound

$$\omega = 2000 = 2\pi f$$

$$k = 9 = \frac{2\pi}{\lambda}$$

$$V_w = \frac{\omega}{k} = \frac{3000}{9} = \frac{1000}{3}$$

$$f = \frac{2000}{2\pi}$$

$$\lambda = \frac{2\pi}{9}$$

② If the equilibrium pressure of air is $1.0 \times 10^5 \text{ N/m}^2$ what are max & min pressure at a point as wave passes through the points.

$$P_0 = 1.0 \times 10^5$$

$$P_{\text{max}} = P_0 + 2$$

$$P_{\text{min}} = P_0 - 2$$

$$\textcircled{3} B = 1.4 \times 10^5 \checkmark$$

$$S_0 = \frac{\Delta P_0}{BK}$$

Velocity of sound in Longitudinal wave

$$v_w = \sqrt{\frac{E}{\rho}}$$

$\xrightarrow{\text{modulus of rigidity}}$ E $\xrightarrow{\text{density}}$ ρ

① Solid $E = \gamma$ (young's modulus)

$$v_w = \sqrt{\frac{\gamma}{\rho}}$$

② Fluid / gas $E = B$

$$v_w = \sqrt{\frac{B}{\rho}}$$

Velocity of sound in air

Newton \rightarrow isothermal process

$$B = P$$

$$PM = \rho RT$$

$$v_w = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}} \approx 280 \text{ m/s}$$

15-17%

Laplace Correction

propagation of sound in air \rightarrow Adiabatic process

$$B = \gamma P$$

$$v_w = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

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Q If velocity of sound in oxygen is v_0 find velocity of sound in nitrogen gas at same temp.

$$v = \sqrt{\frac{\gamma RT}{m}}$$

$$\begin{array}{l} \gamma \rightarrow \text{same} \\ T \rightarrow \end{array}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\frac{v_{N_2}}{v_0} = \sqrt{\frac{32}{28}}$$



Q find velocity of sound in oxygen at 20°C is V_0 find
velocity of sound in He at 40°C

$$\text{He} \equiv \gamma_{\text{mono}} = \frac{5}{3} = \gamma_2$$

$$\text{O}_2 \equiv \gamma_{\text{dia}} = \frac{7}{5} = \gamma_1$$

$$\text{oxygen} \Rightarrow V_0 = V_1 = \sqrt{\frac{\gamma_1 R (273 + 20)}{m_1}}$$

He

$$V_2 = \sqrt{\frac{\gamma_2 R (273 + 40)}{m_2}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{\gamma_1}{\gamma_2} \frac{293}{313} \frac{m_2}{m_1}}$$

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Q If pressure of gas is increase to four times at const temp.
velocity of sound will become

$$V_w = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma R T}{M}} \longrightarrow \text{same}$$

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$$\beta = - \frac{dP}{\frac{dV}{V}}$$

isothermal

$$pV = \text{const}$$

$$P + V \frac{dP}{dV} = 0$$

$$P = - \frac{dP}{\left(\frac{dV}{V}\right)}$$

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Adeba

$$PV^\gamma = \text{const}$$

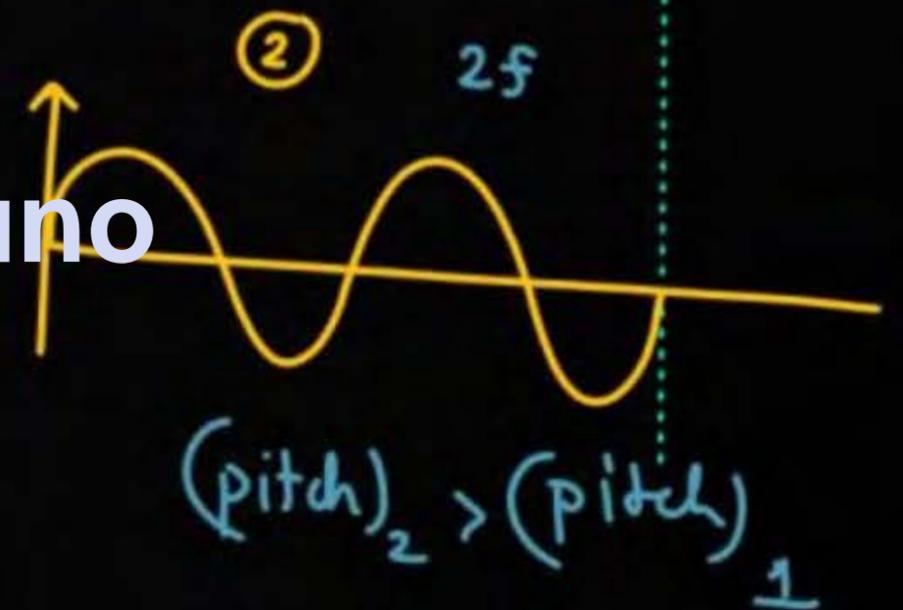
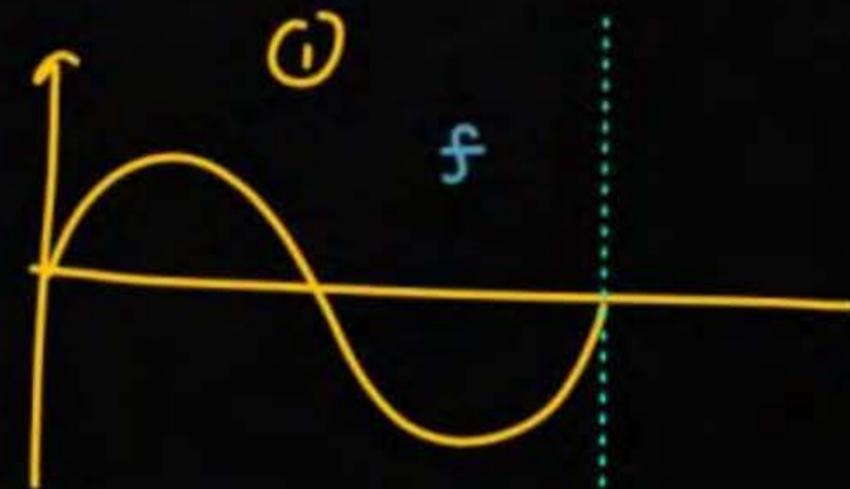
$$P \gamma V^{\gamma-1} + V^\gamma \frac{dP}{dV} = 0$$

$$\gamma \frac{P}{V} V^\gamma + \frac{dP}{dV} \cdot V^\gamma = 0$$

$$\frac{\gamma P}{V} = - \frac{dP}{\left(\frac{dV}{V}\right)} = \beta$$

Characteristics of sound

① Pitch \rightarrow depends on freq.
freq \uparrow , pitch \uparrow

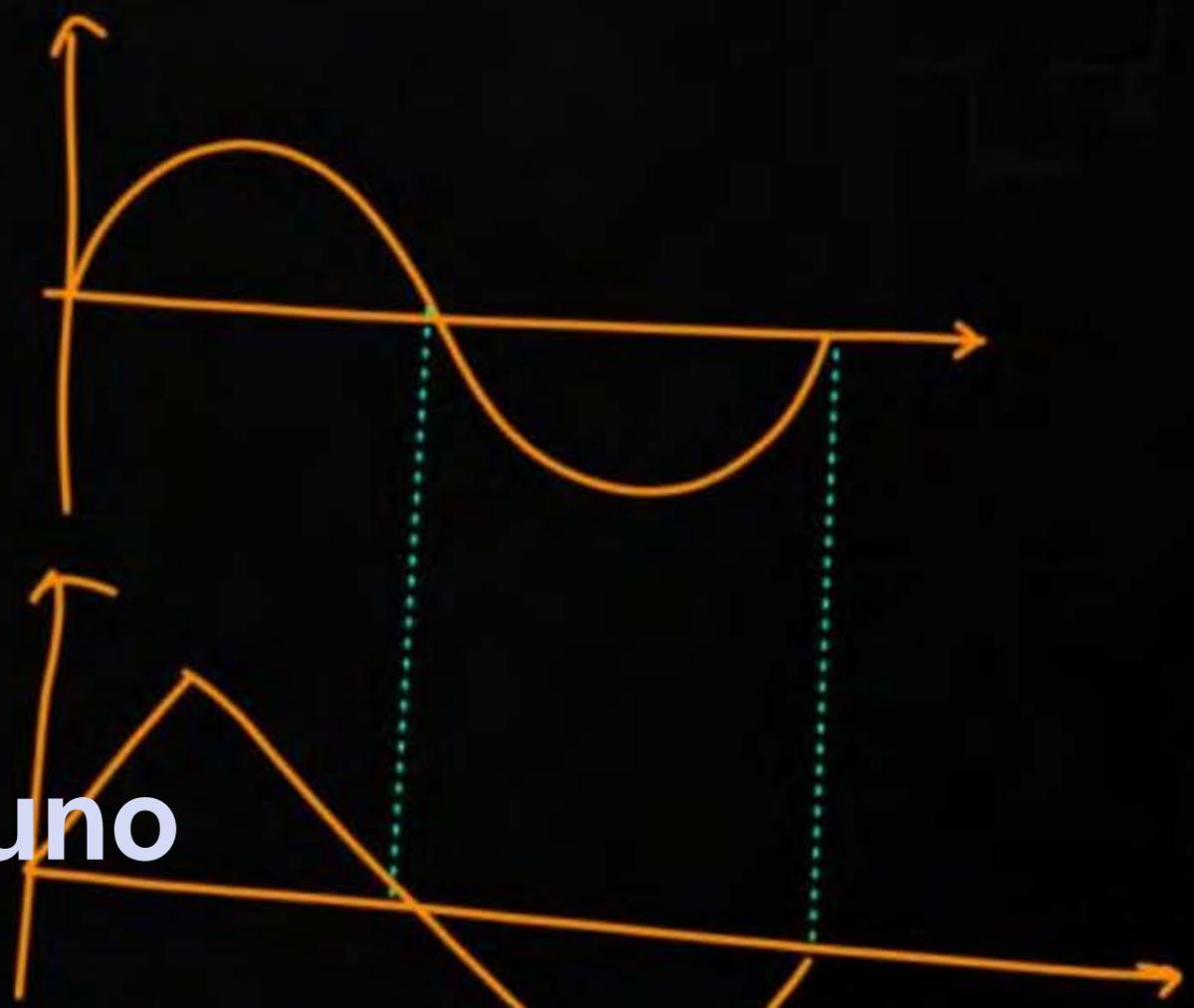


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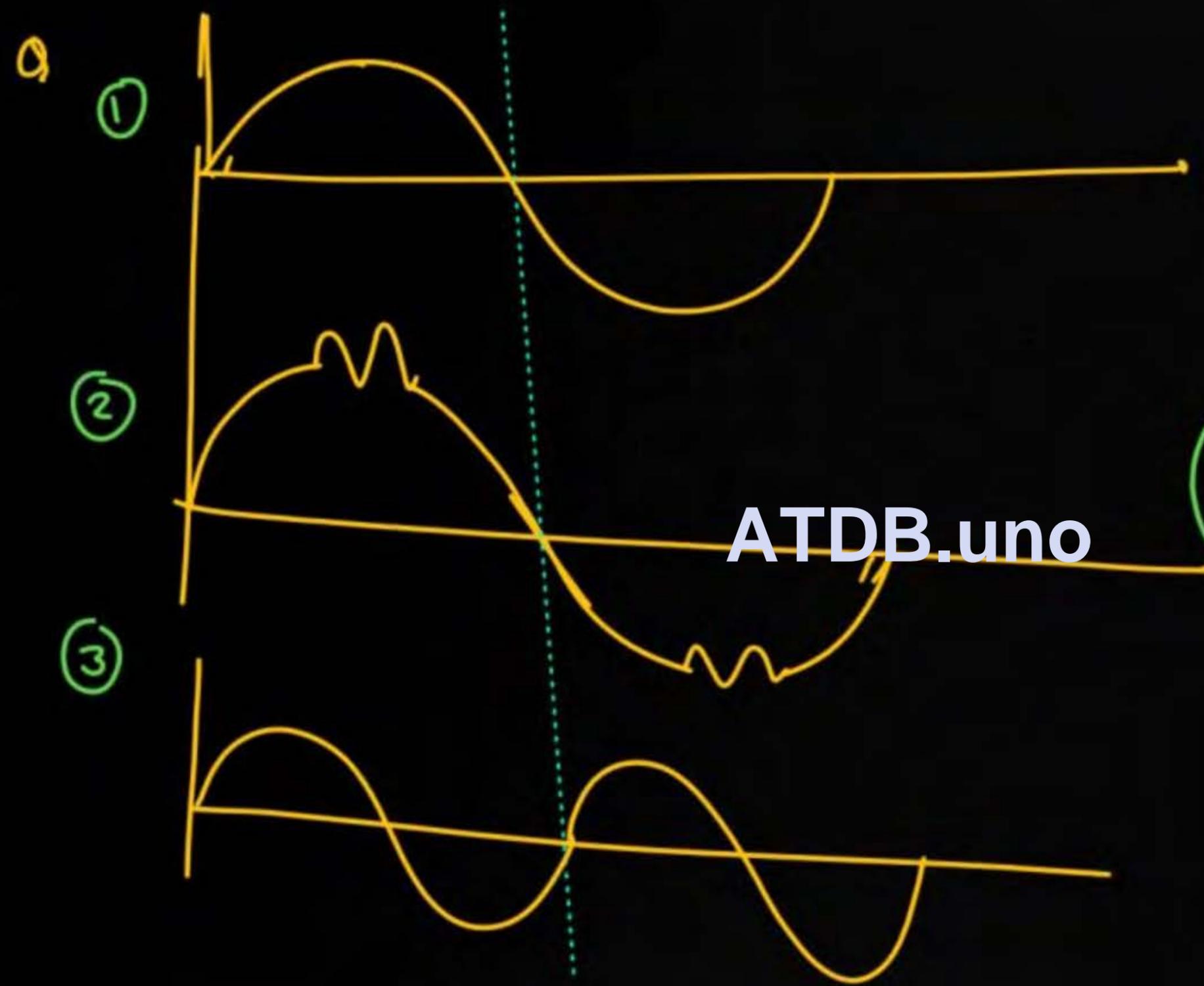
② Quality → depends on waveform

freq → Same
pitch → Same
quality → diff.



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sharpness ↑, Quality ↓



1, 3 → Same quality
 diff-freq.

(1, 2 ⇒ freq same
 Quality different)

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Loudness \longrightarrow depends on Intensity.

$I \longrightarrow$ Energy per Unit area per Unit time.

$t = 1 \text{ sec}, A = 1 \text{ m}^2$

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Loudness - It depends on intensity of sound
Loudness of sound is measured in decibel (dB)

$$L = 10 \log_{10} \left(\frac{I}{I_0} \right)$$

$I_0 \rightarrow$ min Intensity that can be heard by normal person.
 $= 10^{-12} \text{ w/m}^2$

Q

$$I = 10^{-10} \text{ watt/m}^2$$

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$$L = 10 \log_{10} \left(\frac{10^{-10}}{10^{-12}} \right) = 20 \text{ dB}$$



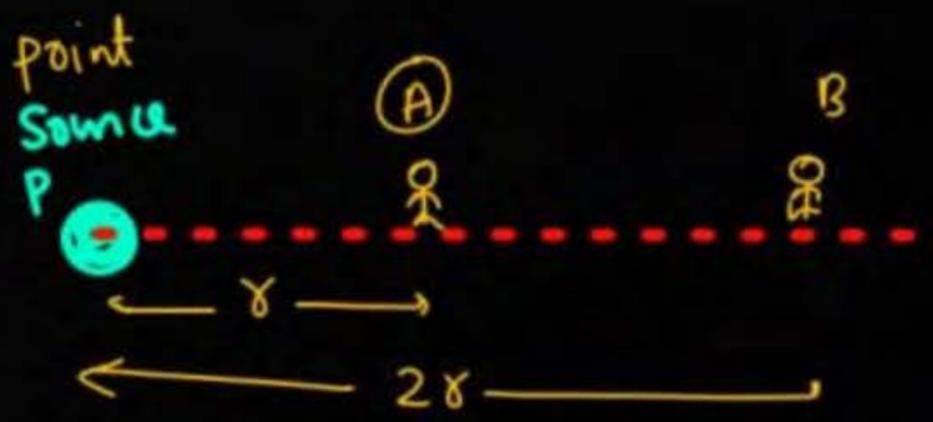
Intensity → Energy per Unit area · per Unit time
 power per Unit area

① point source
 Intensity at A



$$I = \frac{P}{4\pi r^2}$$

$I \propto \frac{1}{r^2}$ point source



$$I_A = \frac{P}{4\pi r^2}$$

$$I_B = \frac{P}{4\pi (2r)^2}$$

$$\frac{I_A}{I_B} = \frac{(2r)^2}{(r)^2}$$

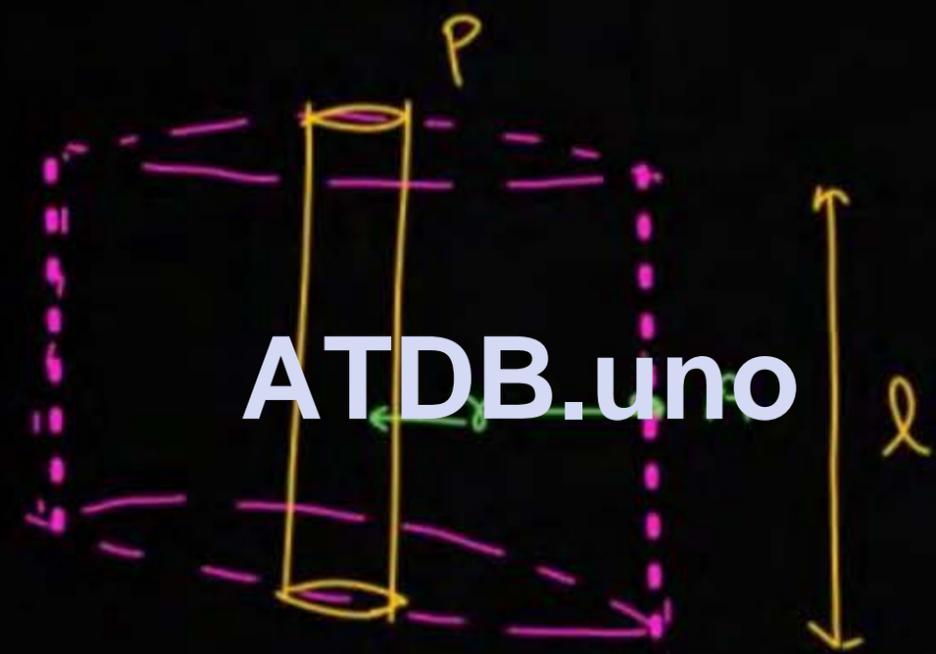
$$I \propto \frac{1}{r^2}$$



② Linear Source

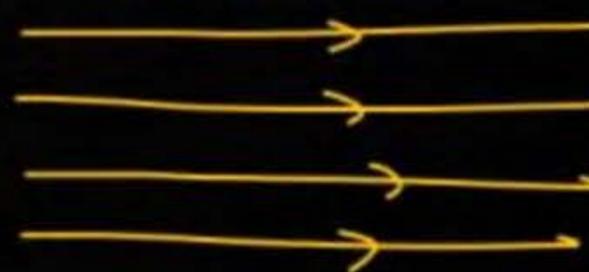
$$I_A = \frac{P}{2\pi r l}$$

$$I \propto \frac{1}{r}$$



$$\begin{array}{l} \text{point source } I \propto \frac{1}{r^2} \\ \text{linear " } I \propto \frac{1}{r} \end{array}$$

③ parallel beam



$$I \propto r^0$$

$$I \rightarrow \text{const}$$

Q Consider a point source emitting sound wave, find difference in decibel scale reading, (find difference in Loudness) between two points at a distance 5m and 10m from source respectively.

$$L_1 = 10 \log \frac{I_1}{I_0}$$

$$L_2 = 10 \log \frac{I_2}{I_0}$$

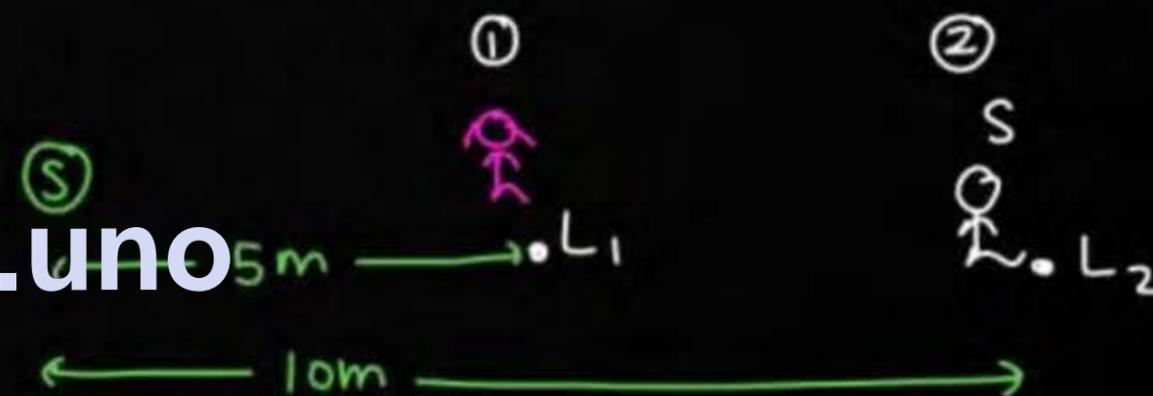
$$L_1 - L_2 = 10 \left(\log \frac{I_1}{I_0} - \log \frac{I_2}{I_0} \right)$$

$$= 10 \log \left(\frac{I_1}{I_2} \right) = 10 \log \left(\frac{r_2}{r_1} \right)^2 = 20 \log \left(\frac{10}{5} \right) = 20 \log 2$$

$$I \propto \frac{1}{r^2}$$

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1} \right)^2$$

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Q A certain sound level is increased by 20 dB by what multiple its intensity increased. (source is point source)

$$L_2 - L_1 = 20$$

$$10 \log \frac{I_f}{I_0} - 10 \log \frac{I_i}{I_0} = 20$$

$$\frac{I_f}{I_i} = 100$$

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$$I_f = 100 I_i$$

$$10 \log \frac{I_f}{I_i} = 20$$

$$\log_{10} \frac{I_f}{I_i} = 2$$



Home work

Prarambh \Rightarrow 22, 20, 23-27,

Prabal \Rightarrow 14, 15, 17, 18, 19, 20, 29, 30

HCV \rightarrow wave on string \Rightarrow ~~(4, 7, 2)~~ \rightarrow Adv level

Sound wave \Rightarrow 4, 5, 6, 8, 13, 12, 14, 16, (18-23)



THANK YOU

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