

PRAYAS

JEE 2025



Lecture No 04

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Physics

Modern Physics
(Bohr model)

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Topics *to be covered*

1 Bohr Model and Energy Level Diagram

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2

3

4



Bohr incorporated the following new ideas now regarded as postulates of Bohr's theory.

1. The centripetal force required for an encircling electron is provided by the electrostatic attraction between the nucleus and the electron

$$\text{i.e. } \frac{1}{4\pi\epsilon_0} \frac{(Ze)e}{r^2} = \frac{mv^2}{r} \dots (i)$$

ϵ_0 = Absolute permittivity of free space = $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

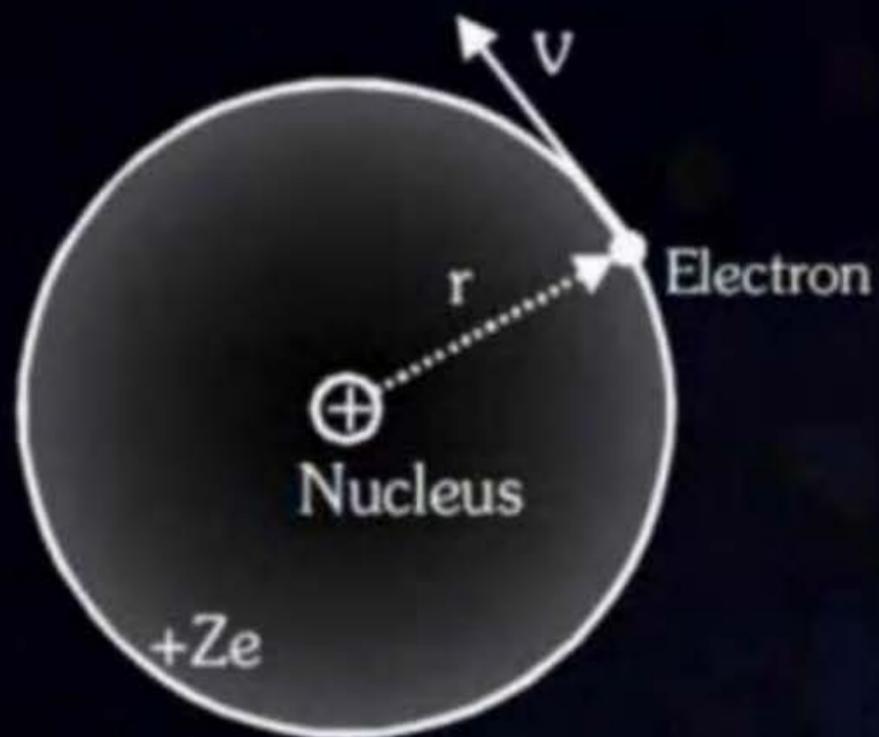
m = Mass of electron

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v = Velocity (linear) of electron

r = Radius of the orbit in which electron is revolving.

Z = Atomic number of hydrogen like atom.





Bohr incorporated the following new ideas now regarded as postulates of Bohr's theory.

2. Electrons can revolve only in those orbits in which angular momentum of electron about nucleus is an integral multiple of $\frac{h}{2\pi}$. i. e., $mvr = \frac{nh}{2\pi} \dots (ii)$

$$mvr = \frac{nh}{2\pi}$$

n = Principal quantum number of the orbit in which electron is revolving.

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3. In stable orbit (specific orbit), total energy of e^- remains const its energy changes when it makes a transition from one orbit to another.



Bohr incorporated the following new ideas now regarded as postulates of Bohr's theory.

3. Electrons in an atom can revolve only in discrete circular orbits called stationary energy levels (shells). An electron in a shell is characterised by a definite energy, angular momentum and orbit number. While in any of these orbits, an electron does not radiate energy although it is accelerated.

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$$\frac{kze^2}{r^2} = \frac{mv^2}{r}$$

$$mv r = \frac{nh}{2\pi} \Rightarrow$$

$$v = \frac{nh}{2\pi m r}$$

$$\frac{kze^2}{r^2} = \frac{m}{r} \frac{n^2 h^2}{4\pi^2 m^2 r^2}$$

$$r = \frac{1}{m} \frac{n^2 h^2}{4\pi^2 z e^2 k}$$

$$r = \frac{h^2}{m 4\pi^2 e^2 k} \left(\frac{n^2}{z} \right) = 0.529 \frac{n^2}{z} \text{ \AA}$$

$r = ?$

$$r \propto \frac{n^2}{z}$$

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$n \rightarrow$ no. of bohr orbit

$$v = \frac{nh}{2\pi m r} = \frac{nh}{2\pi m} \frac{m 4\pi^2 e^2 k z}{h^2 n^2}$$

$$v = \frac{h}{2\pi} \frac{4\pi^2 e^2 k}{h^2} \left(\frac{z}{n} \right)$$

$$v = 2.18 \times 10^6 \frac{z}{n} \text{ m/s}$$



$$\textcircled{1} \quad r \propto \frac{n^2}{Z}$$

$$\textcircled{2} \quad v \propto \frac{Z}{n}$$

$$\textcircled{3} \quad \text{Time period } T = \frac{2\pi r}{v}$$

$$T \propto \frac{r}{v} \propto \frac{n^2/Z}{Z/n} \propto \frac{n^3}{Z^2}$$

$$T \propto \frac{n^3}{Z^2}$$

$$\textcircled{6} \quad \text{Centripetal } = \frac{v^2}{r} \propto \frac{(Z/n)^2}{n^2/Z}$$

$$\textcircled{4} \quad \text{frequency } f = \frac{1}{T} \Rightarrow f \propto \frac{Z^2}{n^3}$$

$$a_c \propto \frac{Z^3}{n^4}$$

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$$\textcircled{5} \quad \text{Angular velocity } \omega = \frac{v}{r} = \frac{2\pi}{T}$$

$$\omega \propto \frac{Z/n}{n^2/Z} \propto \frac{Z^2}{n^3}$$

$$\textcircled{7} \quad \text{momentum } = mv \propto \frac{Z}{n}$$

$$\textcircled{8} \quad KE = \frac{1}{2}mv^2 \propto \frac{Z^2}{n^2}$$

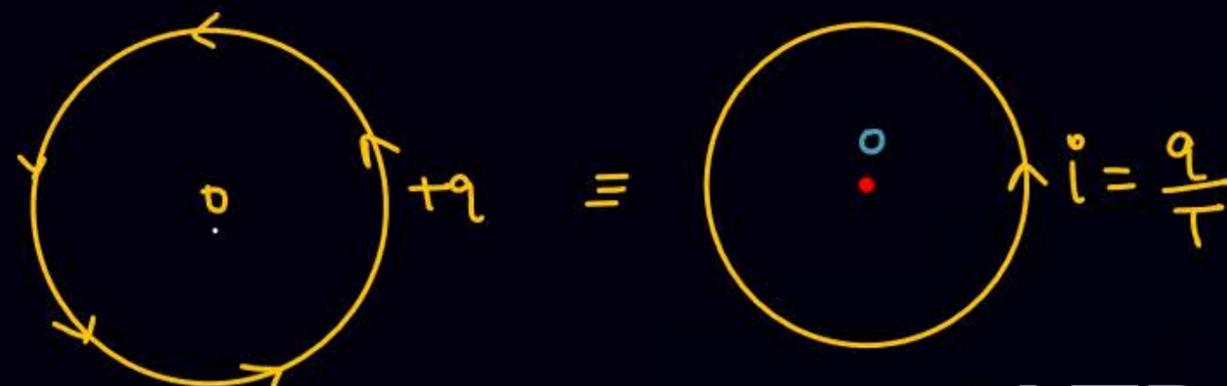
$$\textcircled{9} \quad PE = -\frac{k \cdot Z e \cdot e}{r}$$

$$PE \propto \frac{Z}{r} \propto \frac{Z}{n^2/Z} \propto \frac{Z^2}{n^2}$$



⑩

$$\text{orbital current} = \frac{q}{T} \propto \frac{1}{T} \propto \frac{Z^2}{n^3}$$



⑫ magnetic moment

$$\vec{m} = i \vec{A}$$

$$m \propto i \pi r^2 \propto \frac{Z^2}{n^3} \left(\frac{n^2}{Z} \right)^2$$

$$\boxed{m \propto n}$$

⑪

Magnetic field at center due to revolving \$e^-\$

$$B_{\text{center}} = \frac{\mu_0 i}{2r}$$

$$B \propto \frac{i}{r} \propto \frac{Z^2/n^3}{n^2/Z}$$

$$\boxed{B \propto \frac{Z^3}{n^5}}$$

⑬ Angular momentum = \$mvr \propto n\$

$$L \propto vr \propto \frac{Z}{n} \cdot \frac{n^2}{Z}$$

$$\boxed{L \propto n}$$

$$\boxed{L = mvr = \frac{nh}{2\pi}}$$



| Physical quantity | Formula | Ratio Formulae of hydrogen atom |
|--|--|--|
| Radius of Bohr orbit (r_n) | $r_n = \frac{n^2 h^2}{4\pi^2 m k Z e^2}$; $r_n = 0.53 \frac{n^2}{Z} \text{ \AA}$ | $r_1 : r_2 : r_3 \dots r_n = 1 : 4 : 9 \dots n^2$ |
| Velocity of electron in n^{th} Bohr orbit (v_n) | $v_n = \frac{2\pi k Z e^2}{n h}$; $v_n = 2.2 \times 10^6 \frac{Z}{n}$ | $v_1 : v_2 : v_3 \dots v_n = 1 : \frac{1}{2} : \frac{1}{3} \dots \frac{1}{n}$ |
| Momentum of electron (p_n) | $p_n = \frac{2\pi m k e^2 z}{n h}$; $p_n \propto \frac{Z}{n}$ | $p_1 : p_2 : p_3 \dots p_n = 1 : \frac{1}{2} : \frac{1}{3} \dots \frac{1}{n}$ |
| Angular velocity of electron (ω_n) | $\omega_n = \frac{8\pi^3 k^2 Z^2 m c^4}{n^3 h^3}$; $\omega_n \propto \frac{Z^2}{n^3}$ | $\omega_1 : \omega_2 : \omega_3 \dots \omega_n = 1 : \frac{1}{8} : \frac{1}{27} \dots \frac{1}{n^3}$ |

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| Physical quantity | Formula | Ratio Formulae of hydrogen atom |
|-----------------------------------|--|--|
| Time Period of electron (T_n) | $T_n = \frac{n^3 h^3}{4\pi^2 k^2 Z^2 m e^4} ; T_n \propto \frac{n^3}{Z^2}$ | $T_1 : T_2 : T_3 \dots T_n = 1 : 8 : 27 : \dots : n^3$ |
| Frequency (f_n) | $f_n = \frac{4\pi^2 k^2 Z^2 e^4 m}{n^3 h^3} ; f_n \propto \frac{Z^2}{n^3}$ | $f_1 : f_2 : f_3 \dots f_n = 1 : \frac{1}{8} : \frac{1}{27} \dots \frac{1}{n^3}$ |
| Orbital current (I_n) | $I_n = \frac{4\pi^2 k^2 Z^2 m e^5}{n^3 h^3} ; I_n \propto \frac{Z^2}{n^3}$ | $I_1 : I_2 : I_3 \dots I_n = 1 : \frac{1}{8} : \frac{1}{27} \dots \frac{1}{n^3}$ |
| Angular momentum (J_n) | $J_n = \frac{nh}{2\pi} ; J_n \propto n$ | $J_1 : J_2 : J_3 \dots J_n = 1 : 2 : 3 \dots n$ |

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| Physical quantity | Formula | Ratio Formulae of hydrogen atom |
|------------------------------------|--|---|
| Centripetal acceleration (a_n) | $a_n = \frac{16\pi^4 k^3 Z^3 m e^6}{n^4 h^4}; a_n \propto \frac{Z^3}{n^4}$ | $a_1 : a_2 : a_3 \dots a_n = 1 : \frac{1}{16} : \frac{1}{81} \dots \frac{1}{n^4}$ |
| Kinetic energy (E_{K_n}) | $E_{K_n} = \frac{RchZ^2}{n^2}; E_{K_n} \propto \frac{Z^2}{n^2}$ | $E_{K_1} : E_{K_2} \dots E_{K_n} = 1 : \frac{1}{4} : \frac{1}{9} \dots \frac{1}{n^2}$ |
| Potential energy (U_n) | $U_n = \frac{-2RchZ^2}{n^2}; U_n \propto \frac{Z^2}{n^2}$ | $U_1 : U_2 : U_3 \dots U_n = 1 : \frac{1}{4} : \frac{1}{9} \dots \frac{1}{n^2}$ |
| Total energy (E_n) | $E_n = \frac{-RchZ^2}{n^2}; E_n \propto \frac{Z^2}{n^2}$ | $E_1 : E_2 : E_3 \dots E_n = 1 : \frac{1}{4} : \frac{1}{9} \dots \frac{1}{n^2}$ |

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Q

Plot the graph between
 $\ln A$ Vs $\ln n$ for H atom

$A \rightarrow$ Area $n \rightarrow$ no. of orbit

$$A = \pi r^2$$

$$r = k \frac{n^2}{Z}$$

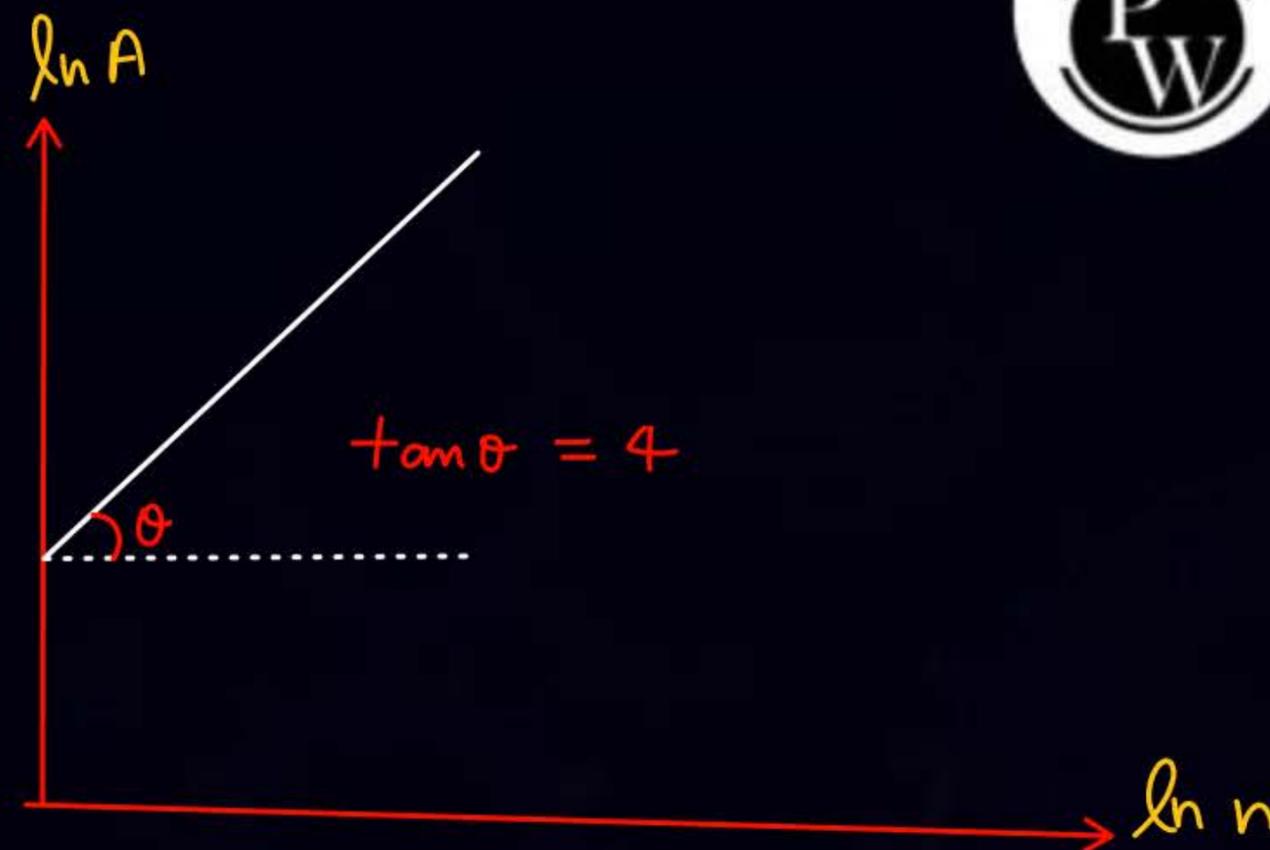
$$A = \pi \left(\frac{kn^2}{Z} \right)^2$$

$$A = k' n^4$$

$$\ln A = \ln k' + 4 \ln n$$

$$y = mx + c$$

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Energy

$$\rightarrow \frac{kze^2}{r^2} = \frac{mv^2}{r}$$

$$K.E. = \frac{1}{2}mv^2 = \frac{kze^2}{2r}$$

$$P.E = \frac{kze(-e)}{r} = -\frac{kze^2}{r}$$

$$\text{Total Energy} = K.E + P.E = -\frac{kze^2}{2r}$$

$$K.E = |T.E| = \frac{|P.E|}{2}$$



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$$T.E = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{2r}$$

$$r = 529 \frac{n^2}{Z} \text{Å}^0$$

$$T.E = -13.6 \frac{z^2}{n^2} \cdot \text{eV}$$

* $K.E = |T.E| = \frac{|P.E|}{2}$

for $Z=1$

(Energy level diagram)

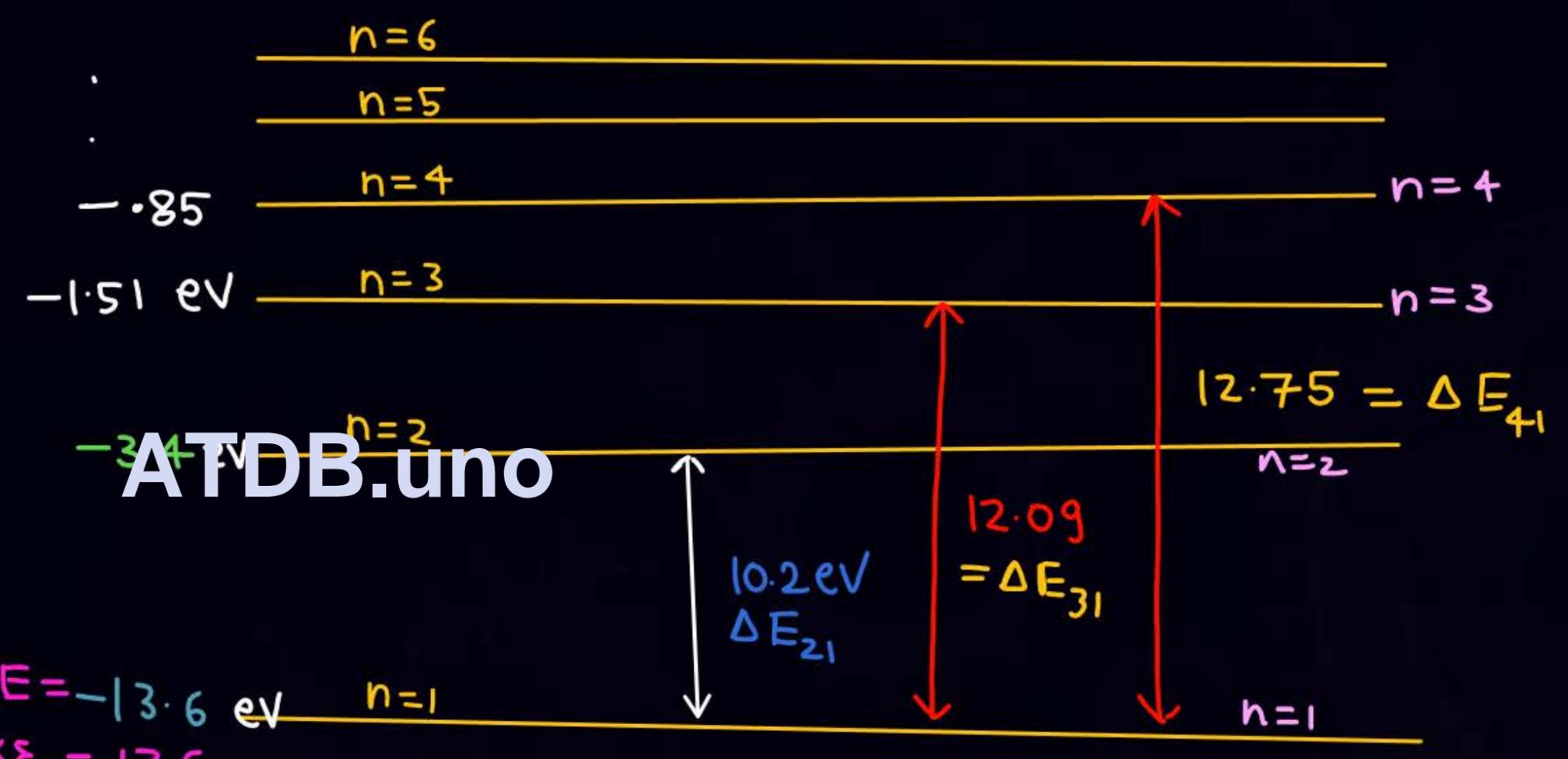


$$T.E = -13.6 \frac{Z^2}{n^2}$$

* $n=2$, $T.E = \frac{-13.6 \times 1^2}{2^2} = -3.4$

* $n=3$, $T.E = \frac{-13.6 \times 1^2}{3^2} = -1.51$

$T.E = -13.6 \text{ eV}$
 $K.E = 13.6$
 $P.E = -27.2$



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- ☀️ Electron in lower energy state absorb a photon of energy Equal to energy difference between two state to get excited to higher energy state.
- ☀️* when an electron make transition from a higher energy level to lower energy level it emit a photon with energy equal to the energy difference between initial & final levels.

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- In ground state e^- can absorb only those photon which have energies equal to the difference in energies of the stable energy level with ground state.

Energy required to ionise an atom is called ionisation energy of atom for that particular energy level

$$\begin{aligned}\Delta E_{n \rightarrow \infty} &= E_{\infty} - E_n = 0 - \left(-\frac{13.6Z^2}{n^2} \right) \\ &= \frac{13.6Z^2}{n^2}\end{aligned}$$



Energy of photon required to excite e^- from $n_i = n_1$ to $n_f = n_2$

will be equal to

$$\Delta E = 13.6 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = \frac{hc}{\lambda} = \frac{1240}{\lambda \text{ (nm)}}$$



If an electron de-excites from n_2 to n_1 (higher \rightarrow lower)

then energy of photon emitted = $\Delta E = 13.6 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = \frac{1240}{\lambda}$

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Q  Find energy of photon required to excite electron of $z=1$ atom from $n=1$ to $n=5$

$$\Delta E = 13.6 \times 1^2 \left(\frac{1}{1^2} - \frac{1}{5^2} \right) = \frac{1240}{\lambda} \checkmark$$

Q  Find energy of photon required to excite electron of $z=1$ atom from first excited state to 3rd excited state.

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Solⁿ

$$n=2 \longrightarrow n=4$$

$$\Delta E = 13.6 \times 1^2 \left(\frac{1}{2^2} - \frac{1}{4^2} \right)$$



Step

* When external radiation is given to the hydrogen atom, electron in ground state jump to the higher energy state & atom is now called in excited state. Any excited state is unstable state & max lifetime of an excited state is of order of 10^{-8} seconds. After the lifetime of the excited state e^- jump to the ground state again directly or indirectly by emitting one or more electromagnetic Radiation. It may have many path to come to ground state.

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* Excitation of atom \equiv excitation of e^-



Deexcitation

☀️ when electron deexcite from $n = n$ to ground state

$$\text{Total no. of way} = 2^{n-2}$$

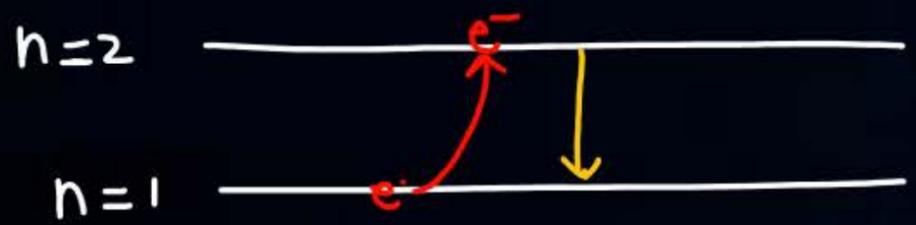
Total no. of different

$$\lambda \text{ of photon possible} \Rightarrow n C_2$$

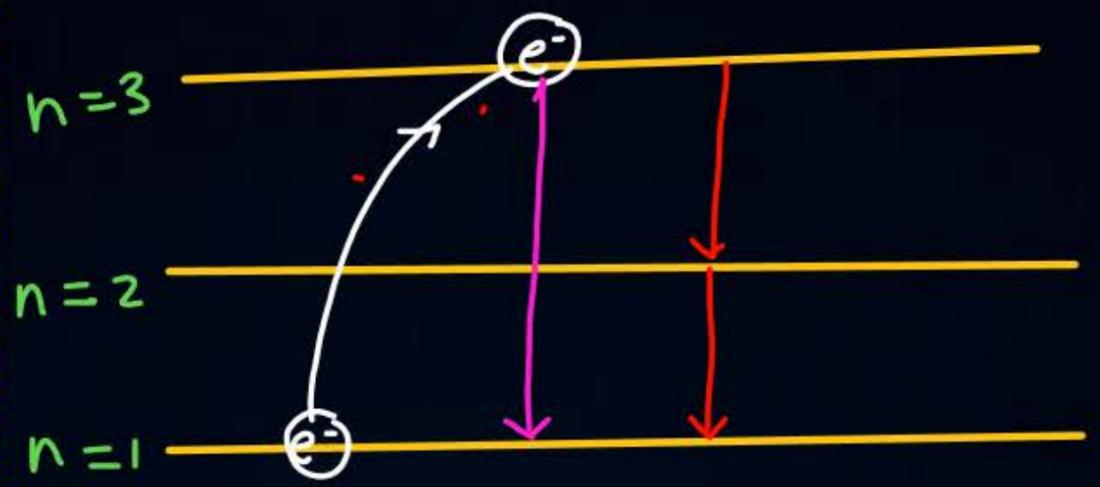
☀️ An e^- in higher energy state emit a photon of energy exactly equal to the energy difference b/w two corresponding state to return to a lower energy state.

$$\Delta E = 13.6 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

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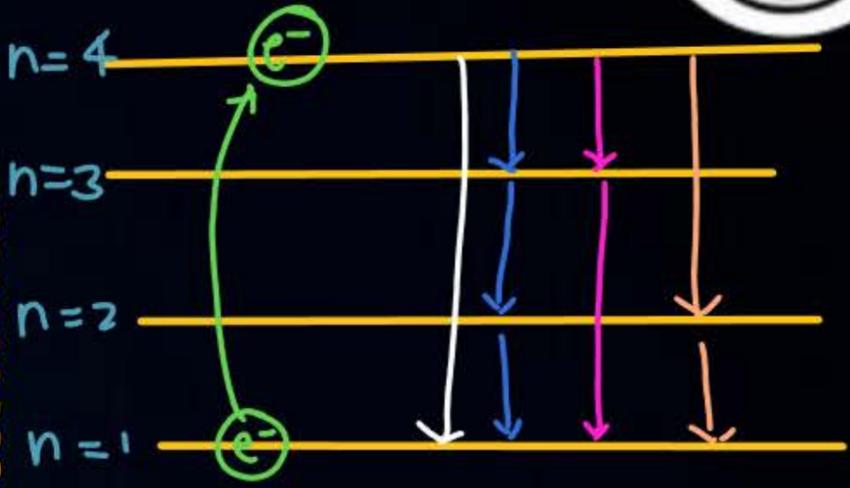


$nC_2 \equiv \text{Different } \lambda$
 $2^{n-2} \equiv \text{Diff. ways}$



$3 \rightarrow 1$
 $3 \rightarrow 2 \rightarrow 1$ } 2 No. of ways
 * 3 diff λ of photon

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* $4 \rightarrow 1$
 $4 \rightarrow 3 \rightarrow 2 \rightarrow 1$
 $4 \rightarrow 3 \rightarrow 1$
 $4 \rightarrow 2 \rightarrow 1$ } 4 ways

* $\Delta E_{41}, \Delta E_{43}, \Delta E_{42}, \Delta E_{31}$
 ΔE_{32}

6 तरंग λ





Q  If an e^- deexcite from $n=3$ to $n=1$ of $Z=1$ atom directly.

find

① Energy of photon release in eV

$$\Delta E = 13.6 \times 1^2 \times \left(\frac{1}{1^2} - \frac{1}{3^2} \right) \text{ eV}$$

Joule में

② wavelength of photon released

$$13.6 \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = \frac{1240}{\lambda}$$

* ③ Frequency of photon released.

$$c = f \lambda$$

$$E = h \nu$$

(Joule)



☀ 1st line of Lyman series



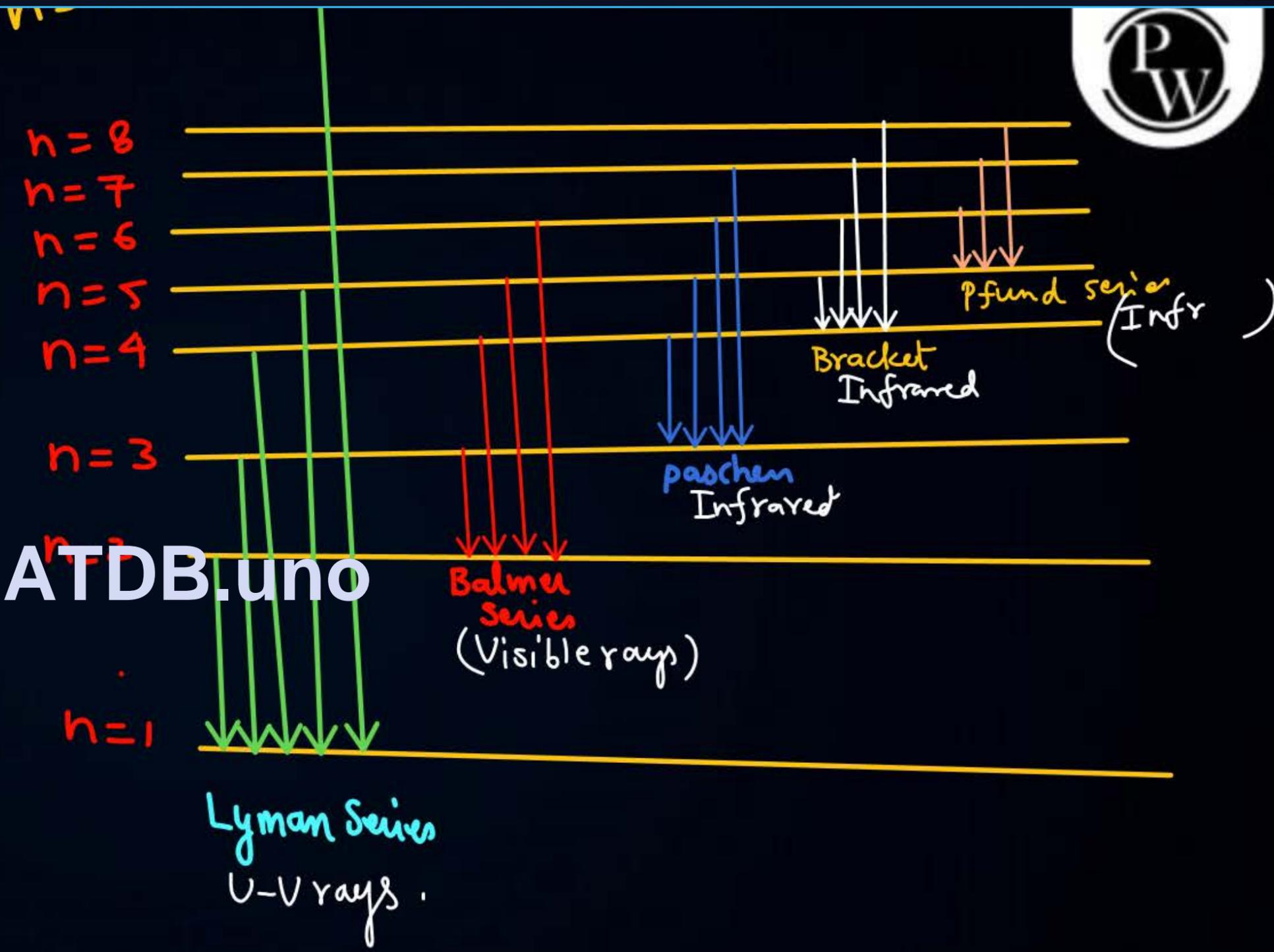
☀ 2nd Line of Lyman Series



☀ 5th line of balmer series



☀ 2nd line of paschen Series



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Q. Find the wavelength of photon emitted in 2nd line of Balmer series for $Z=1$

Sol



$$\Delta E = 13.6 \left(\frac{1}{2^2} - \frac{1}{4} \right) = \frac{1240}{\lambda}$$

λ (nm)



Q  find the wavelength of photon emitted in 4th line of paschen series for $Z=1$

solⁿ

7 \longrightarrow 3

$$\Delta E = 13.6 \left(\frac{1}{3^2} - \frac{1}{7^2} \right) = \frac{1240}{\lambda} \quad \text{ATDB.uno} \quad \text{---} \rightarrow (n_2)$$



$$\# \text{☀} \quad \Delta E = \frac{hc}{\lambda} = 13.6 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = \frac{1240}{\lambda_{nm}}$$

$$\frac{1}{\lambda} = \frac{13.6}{hc} Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\frac{1}{\lambda} = R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad R = \frac{13.6}{hc} = 1.097 \times 10^7 \text{ m}^{-1}$$

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 $1.097 \times 10^7 \text{ m}^{-1}$

Rydberg const.

$$\frac{1}{\lambda} = \text{wavenumber}$$

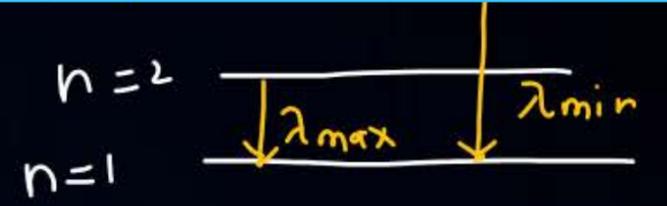


*  Lyman Series \Rightarrow
(Z=1)

λ_{min} when $\infty \longrightarrow 1$

λ_{max} = Longest λ possible $\Rightarrow (\Delta E)_{min} \Rightarrow n=2 \longrightarrow n=1$

$\lambda \equiv (91.6 \text{ nm to } 121.6 \text{ nm})$
for Z=1

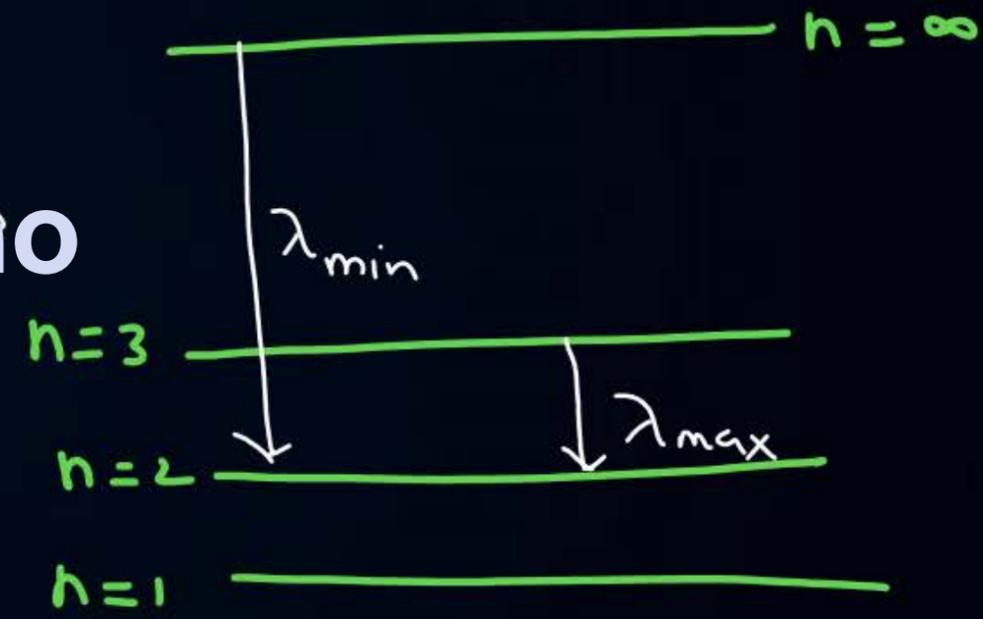


*  Balmer Series

$\lambda_{min} \Rightarrow \infty \longrightarrow 2$

$\lambda_{max} \Rightarrow 3 \longrightarrow 2$

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$$\frac{13.6Z^2}{n^2}$$

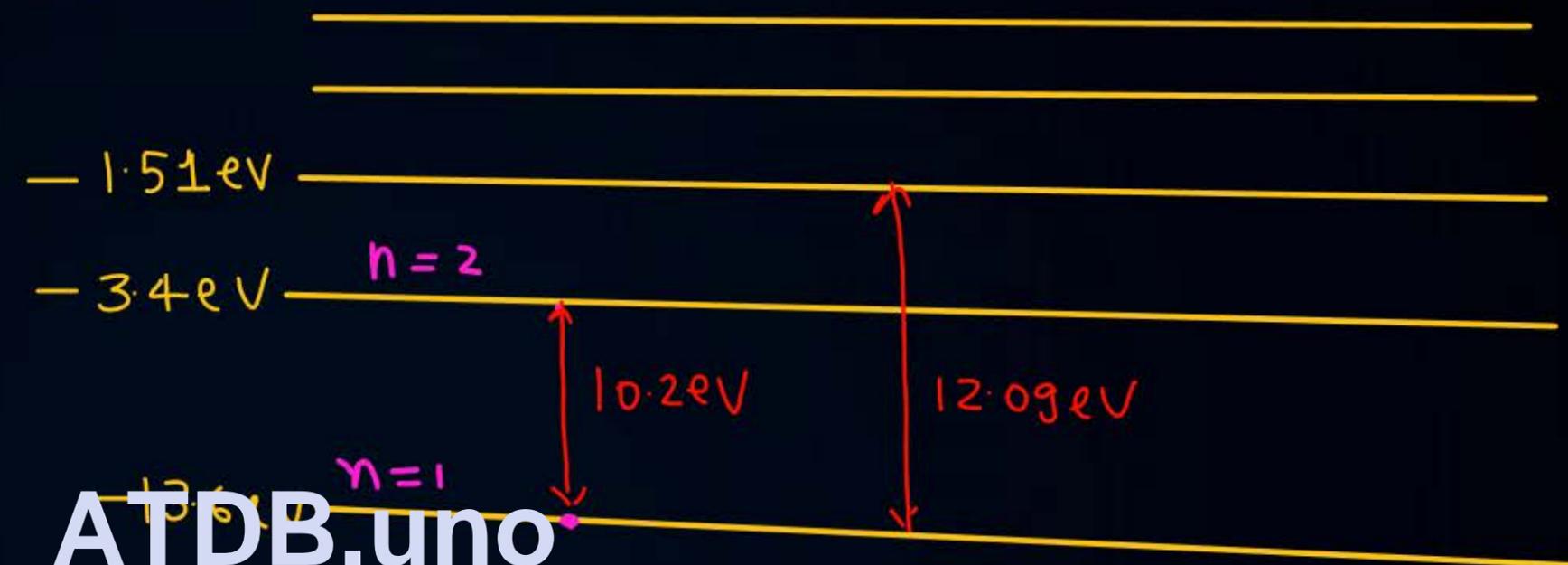
$$Z=1$$

* Ionisation energy
for $n=1 \Rightarrow 13.6 \text{ eV}$

* Ionisation potential for
 $n=1 \Rightarrow 13.6 \text{ Volt}$

* Excitation energy for
 $n=1$ to $n=2 \Rightarrow 10.2 \text{ eV}$

$$\text{or } 13.6Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$



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just
take
reading

atom corresponds to infinite separation between electron and nucleus. Total positive energy implies that the atom is ionized and electron is in unbound (isolated) state moving with certain kinetic energy. Minimum energy required to move an electron from ground state to $n = \infty$ is called ionization energy of the atom or ion.

$$E_{\text{ionization}} = E_{\infty} - E_n = -E_n = \frac{13.6Z^2}{n^2} \text{ eV}$$

Ionization energy of H atom = 13.6 eV

Ionization energy of He⁺ ion = 54.4 eV

Ionization energy of Li⁺⁺ ion = 122.4 eV

Ionization potential (IP): Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionization energy of the atom is called ionization potential of the atom.

$$V_{\text{ionization}} = \frac{E_n}{e} = \frac{13.6Z^2}{n^2} \text{ V}$$

IP of H atom = 13.6 V

IP of He⁺ ion = 54.4 V





THANK YOU

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