PRAYAS

JEE 2025

Lecture - 07 | Physics

Modern Physics

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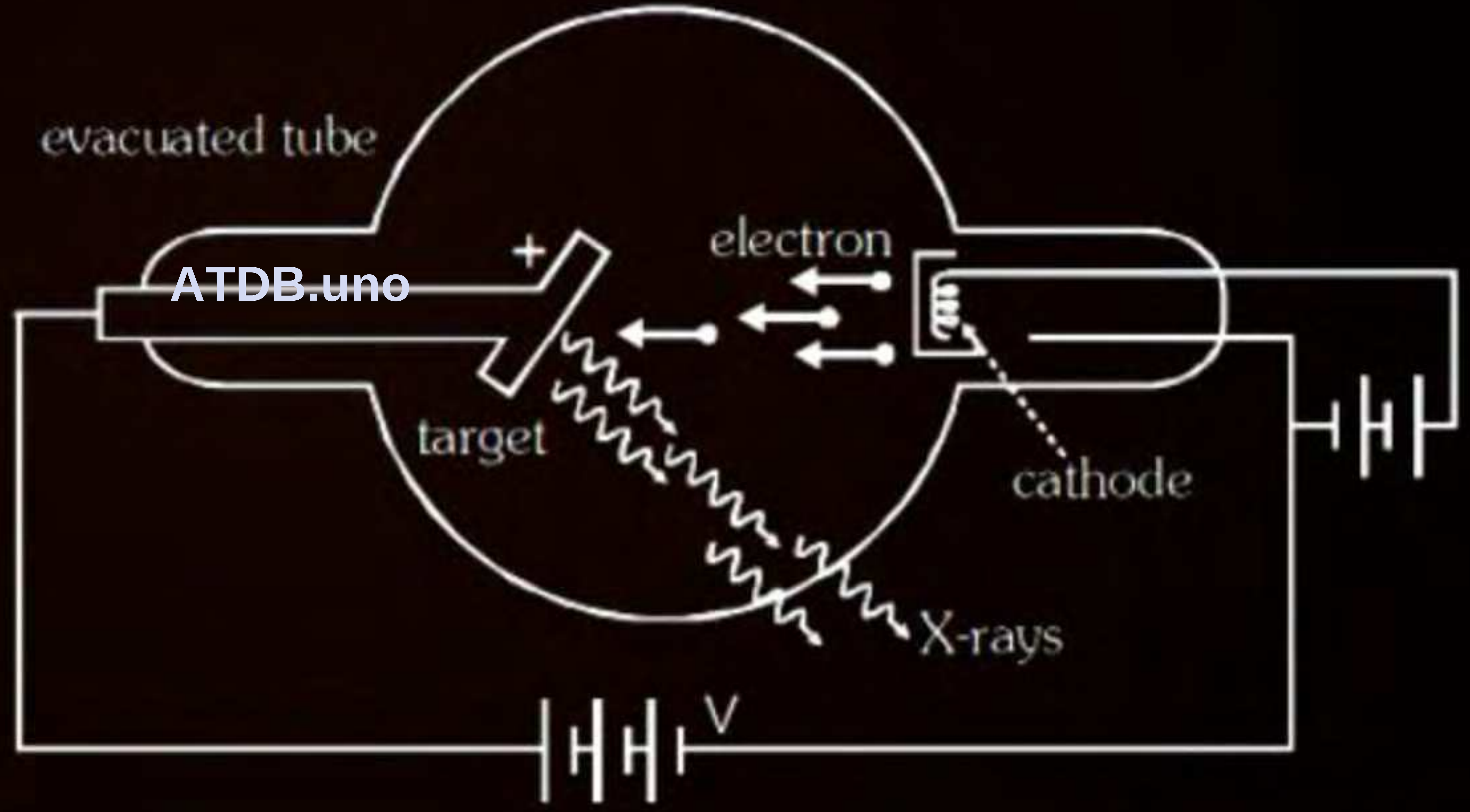
Topics *to be covered*

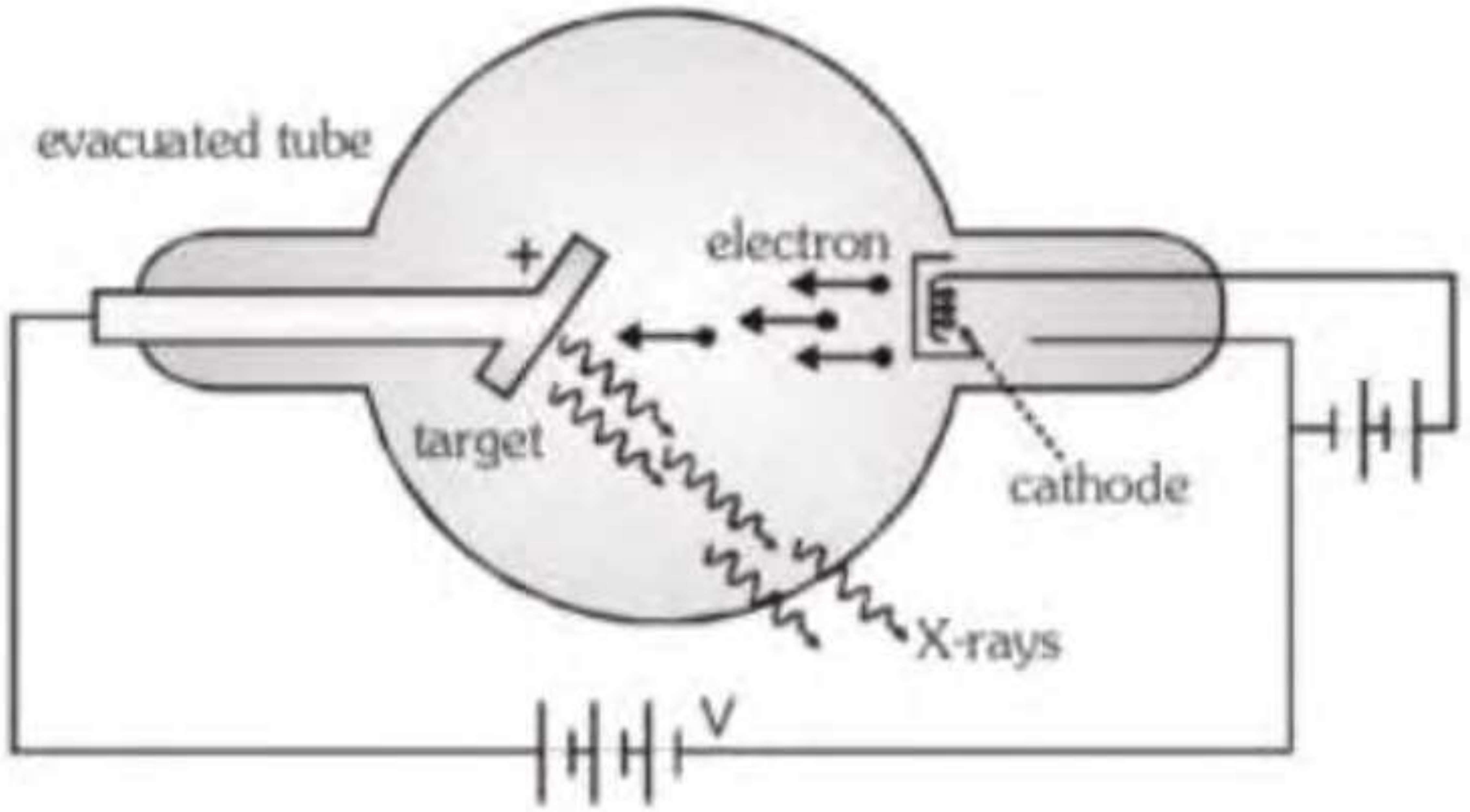
1. X-ray
2.
3.
4.

Production of X-rays:



ROENTGEN EXPERIMENT

Roentgen discovered X–ray. While performing experiment on electric discharge tube Roentgen observed that when pressure inside the tube is 10^{-3}mm of Hg and applied potential is kept 25kV then some unknown radiation are emitted by anode. These are known as X–ray. X–rays are produced by bombarding high speed electrons on a target of high atomic weight and high melting point.



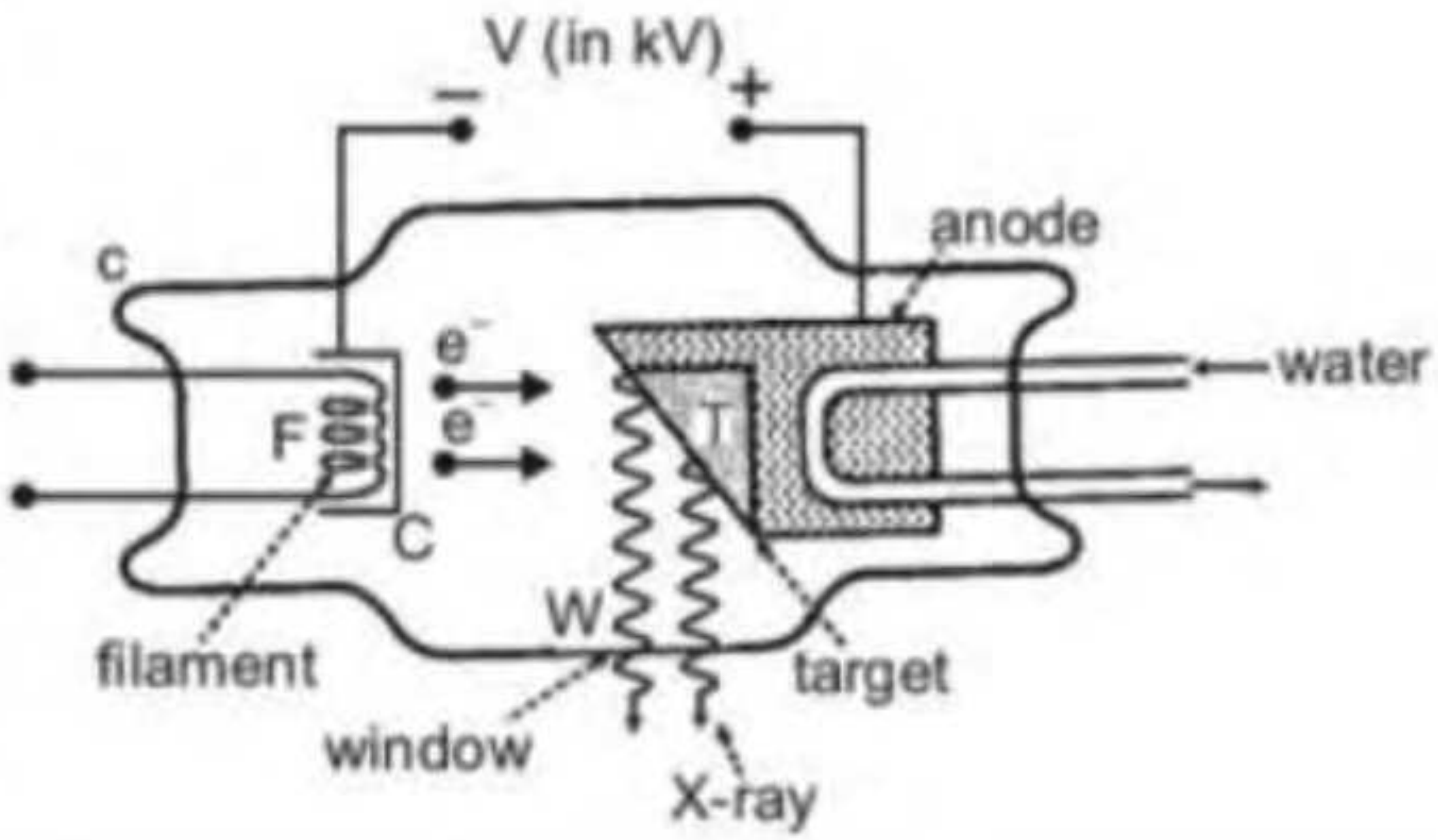
To Produce X–ray Three Things are Required

(i) Source of electron (ii) Means of accelerating these electron to high speed

(iii) Target on which these high speed electron strike

COOLIDGE METHOD

Coolidge developed thermoionic vacuum X–ray tube in which electron are produced by thermoionic emission method. Due to high potential difference electrons (emitted due to thermoionic method) move towards the target and strike from the atom of target due to which X–ray are produced. Experimentally it is observed that only 1% or 2% kinetic energy of electron beam is used to produce X–ray. Rest of energy is wasted in form of heat.



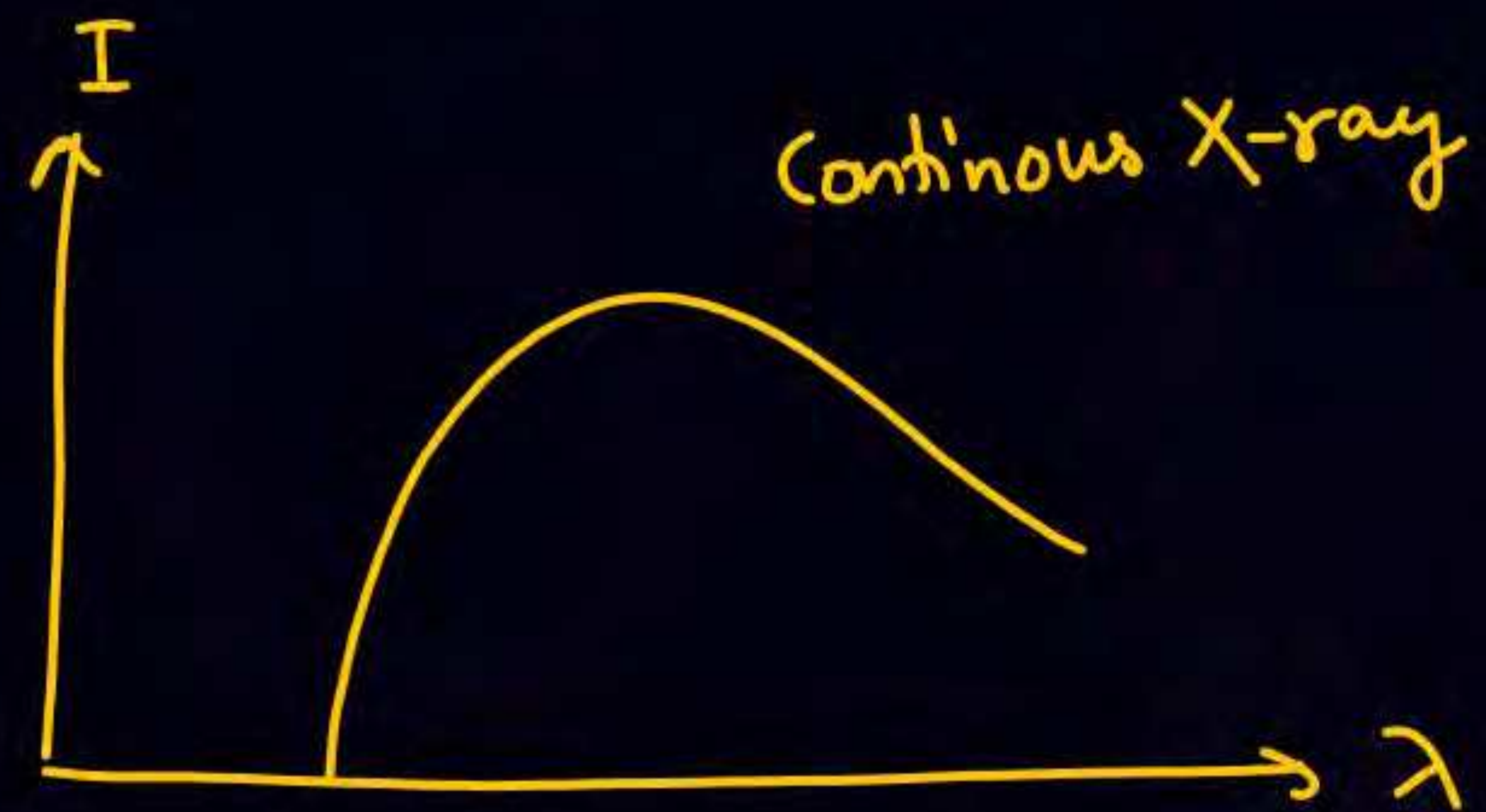
$(K.E)_{final}$ of e^- just before striking the target material $= eV_0$ → Accelerating Voltage

$$(KE)_e = eV_0 = \frac{hc}{\lambda_{min}} = \frac{1240}{\lambda_{min}} eV$$

(λ_{min} → (nm))

$$\lambda_{threshold} = \lambda_{cutt\ off} = \lambda_{min} = \frac{1240}{V_0} \text{ (nm)}$$

$$\lambda_{min} = \frac{12400}{V_0} (A^0)$$



* X-ray ($\cdot 1 - 100\mathring{A}$)
  - Soft x-ray ⇒ Energy ↓ , λ ↑ ⇒ ($\lambda = 10\mathring{A} - 100\mathring{A}$), Less penetrating power
  - Hard x-ray ⇒ Energy ↑, λ ↓ ⇒ $\lambda = (\cdot 1\mathring{A} - 10\mathring{A})$, high penetrating power.

* Intensity of x-ray depends on no. of electron striking the target & no. of e^- depends on temp of filament which can be control by filament current.
* If accelerating Voltage V_0 increases λ_{min}, λ_{cutoff} decreases. (Graph shift left)
* X-ray always travel with speed of light, b'coz X-ray are E.M.W.
* There is no charge on x-ray as they are not deflected by E.F & M.F.

Characteristics of target

$Z \uparrow$, M.P $\uparrow$, K $\uparrow$ → thermal cond.

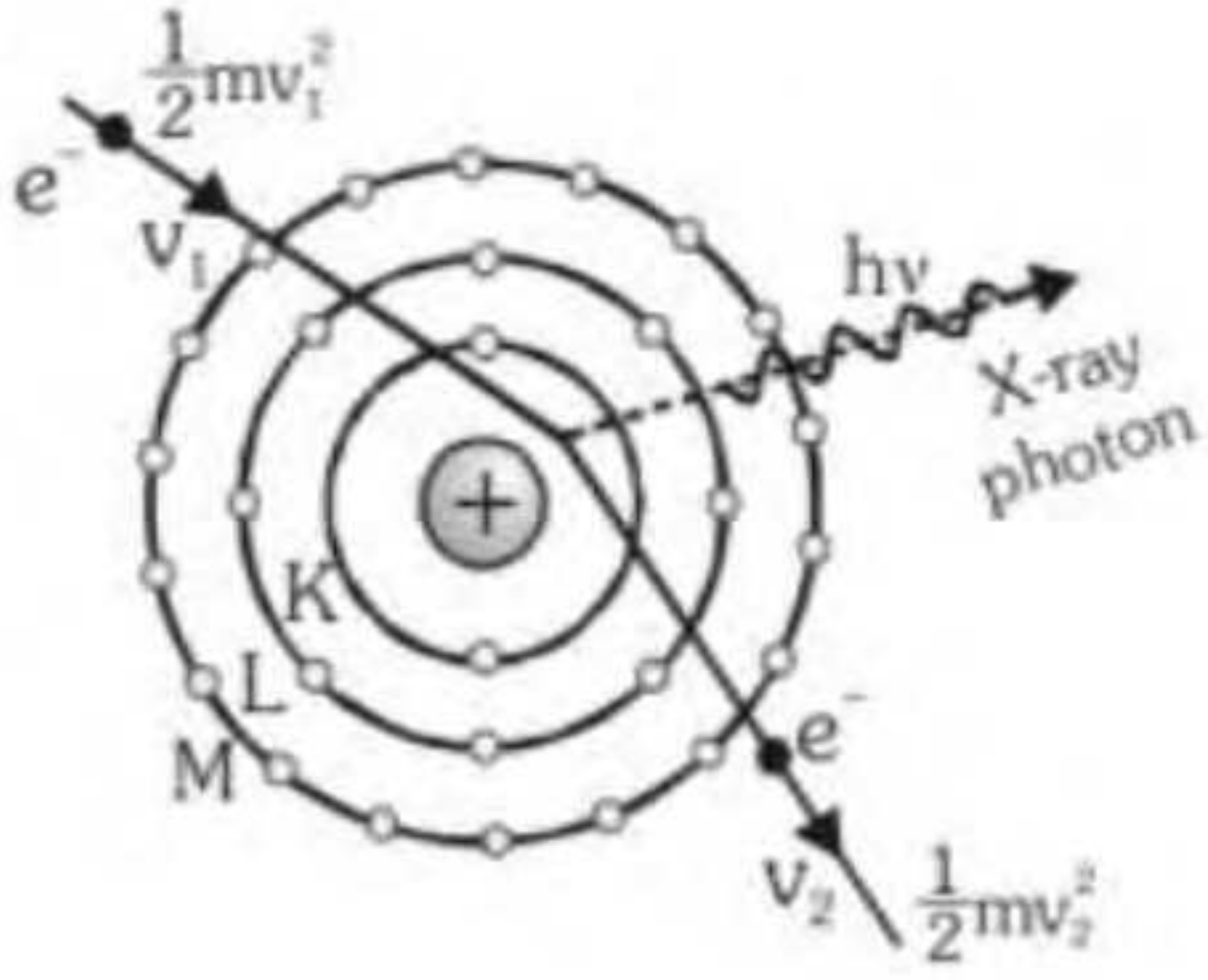
(a) Must have high atomic number to produce hard X–rays.

(b) High melting point to withstand high temperature produced.

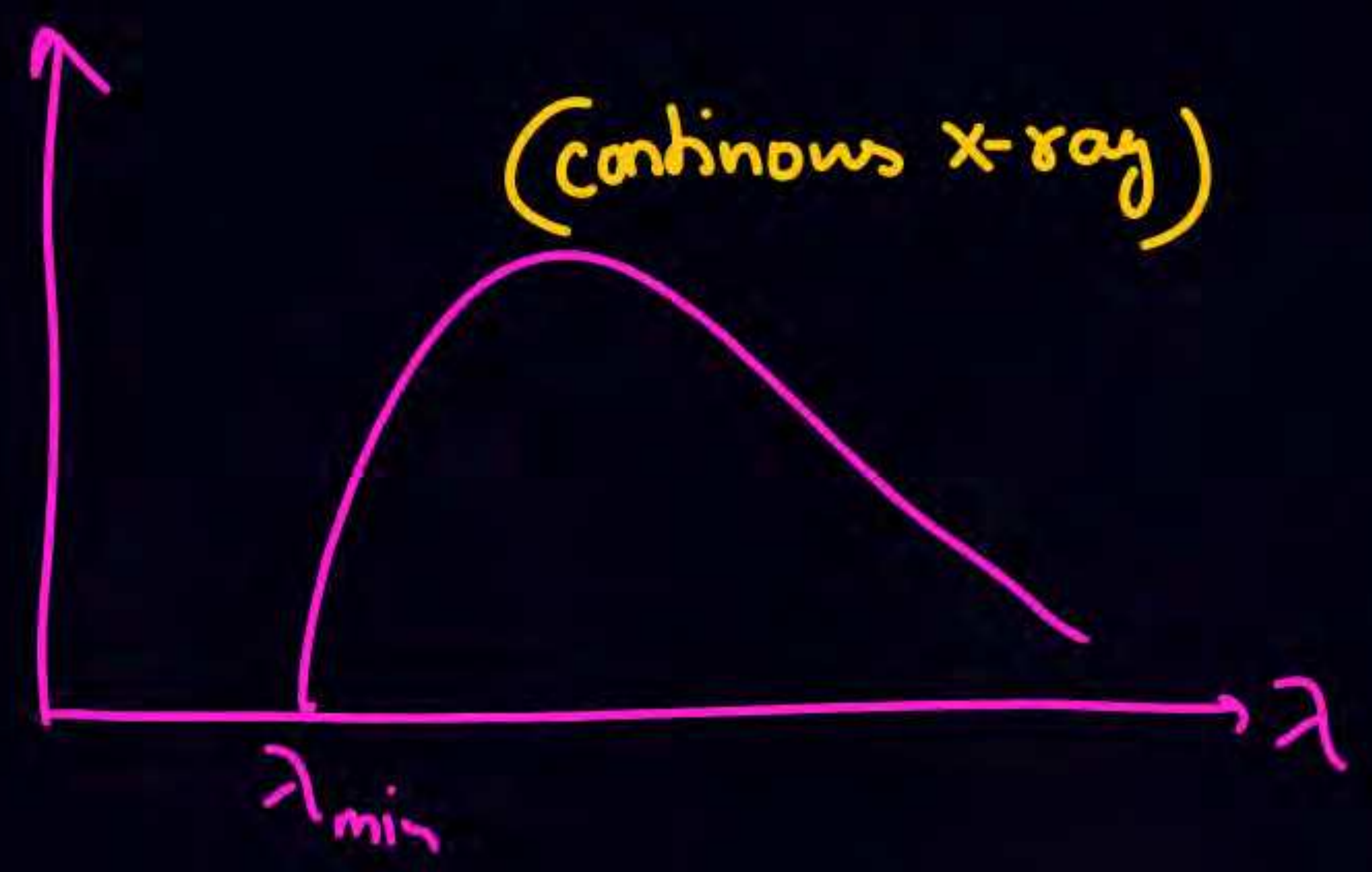
(c) High thermal conductivity to remove the heat produced

(d) Tantalum, platinum, molybdenum and tungsten serve as target materials

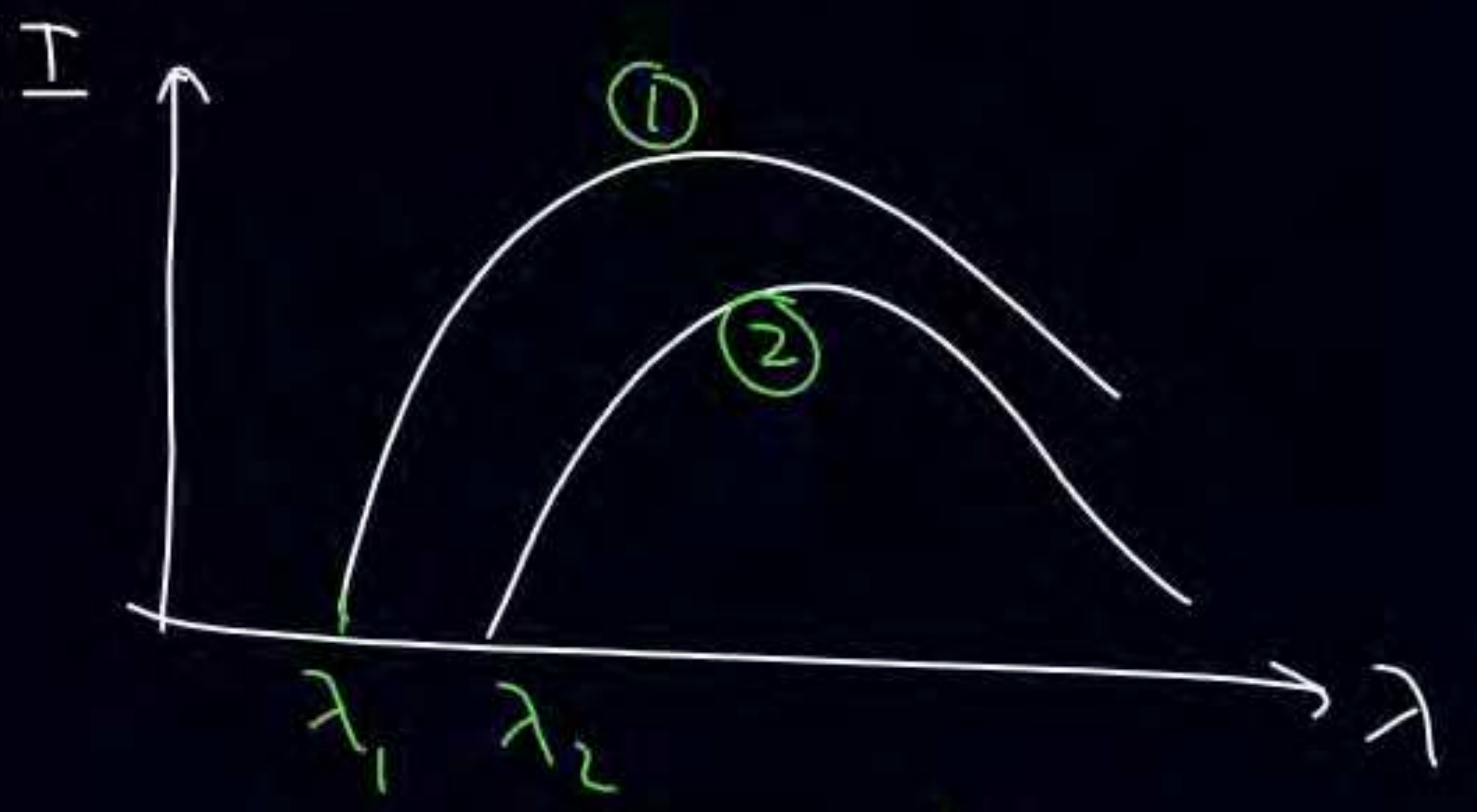
- **Continuous spectrum of X–ray** : When high speed electron collides from the atom of target and passes close to the nucleus. There is coulomb attractive force due to this electron is deaccelerated i.e. energy is decreased. The loss of energy during deacceleration is emitted in the form of X–rays. X–ray produced in this way are called Braking or Bremstralung radiation and form continuous spectrum. In continuous spectrum of X–ray all the wavelength of X–ray are present but below a minimum value of wavelength there is no X–ray. It is called cut off or threshold or minimum wavelength of X–ray. The minimum wavelength depends on applied potential.



$$\lambda_{cutt\ off} = \lambda_{min} = \frac{1240}{V_0}(nm) = \frac{12400}{V_0}\ A^\circ$$



If $V_0 \uparrow \Rightarrow \lambda_{min} \downarrow$



$$(V_0)_1 > (V_0)_2$$

अगर q charge को हम pot-diff 'V' से Rest से acc करें तो

final KE = qV होती है

$$\lambda_{debrog} = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2m(KE)}} = \frac{h}{\sqrt{2mqV}}$$

Q A x-ray tube is operated at 30KV

① max frequency of x-ray emitted

$$eV_0 = h\nu$$

$$1.6\times10^{-19}\times 30000 = 6.6\times10^{-34}\,\nu$$

$$\nu = (\checkmark)\,Hz$$

② λ_{min} or λ_{cutt} of x-ray emitted.

$$\lambda_{min} = \frac{1240}{30000}(nm).$$

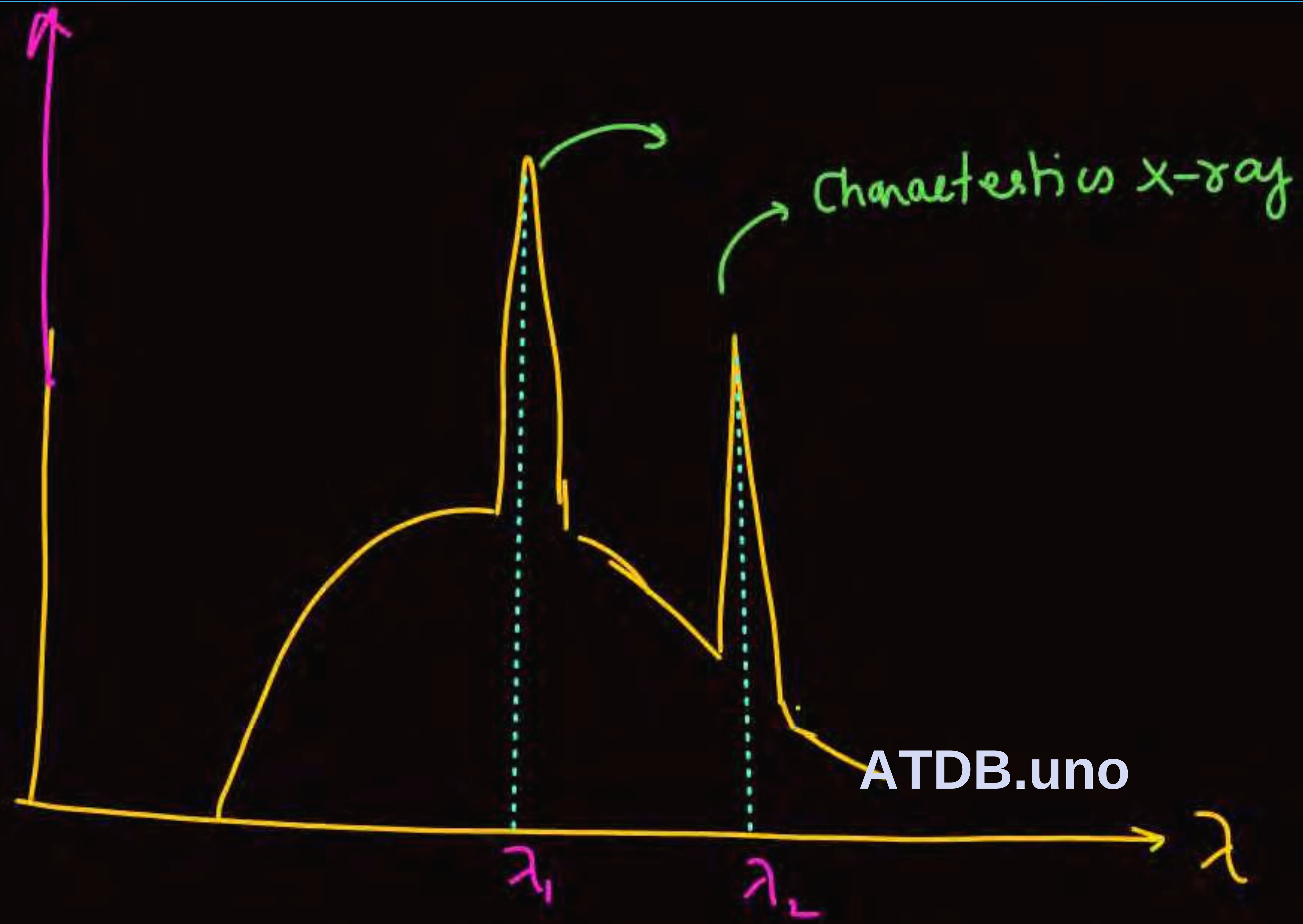
③ If a particular electron looses 5% of its kinetic energy to emit an x-ray photon at first collision find wavelength corresponding to photon

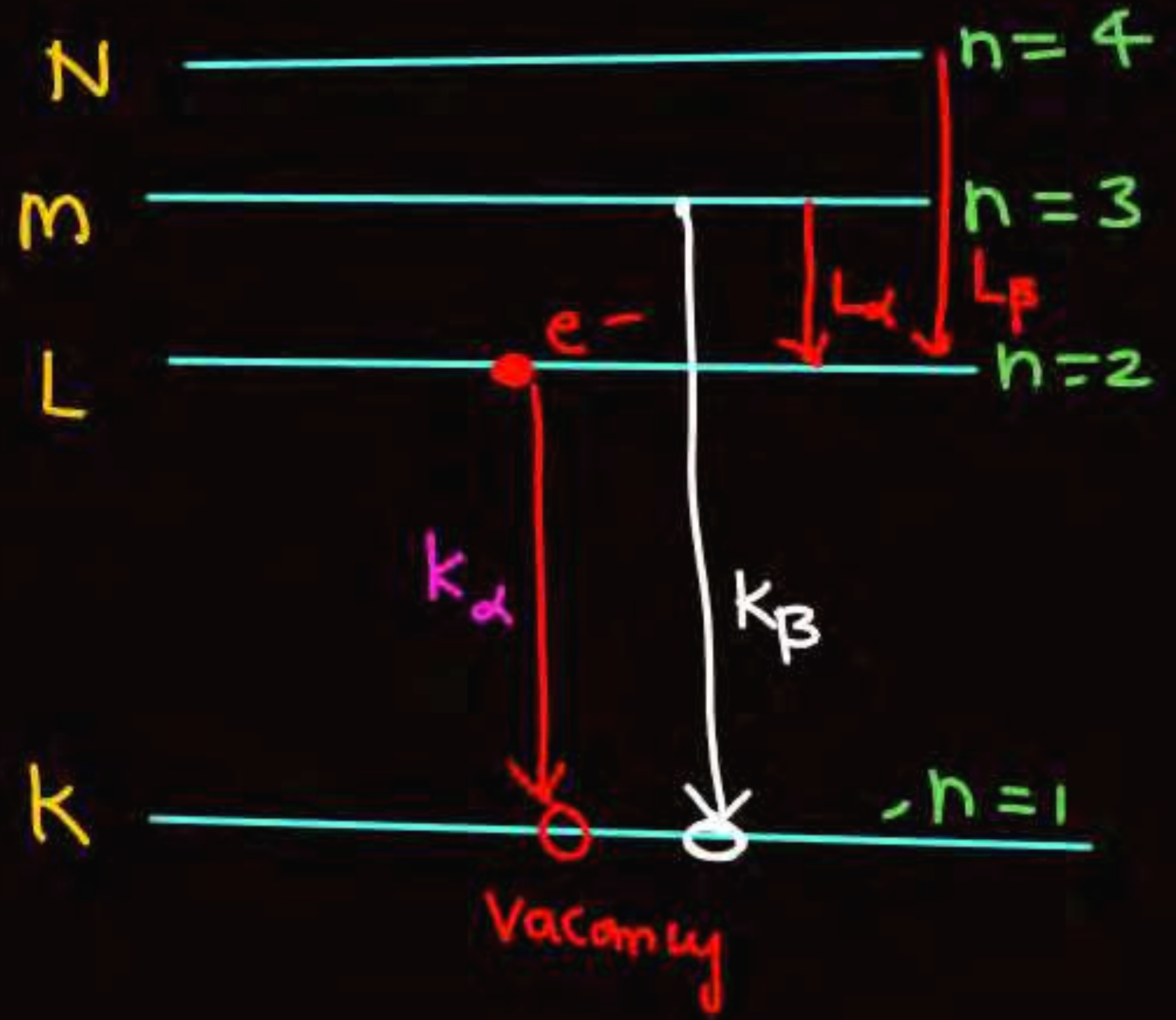
Sol^n

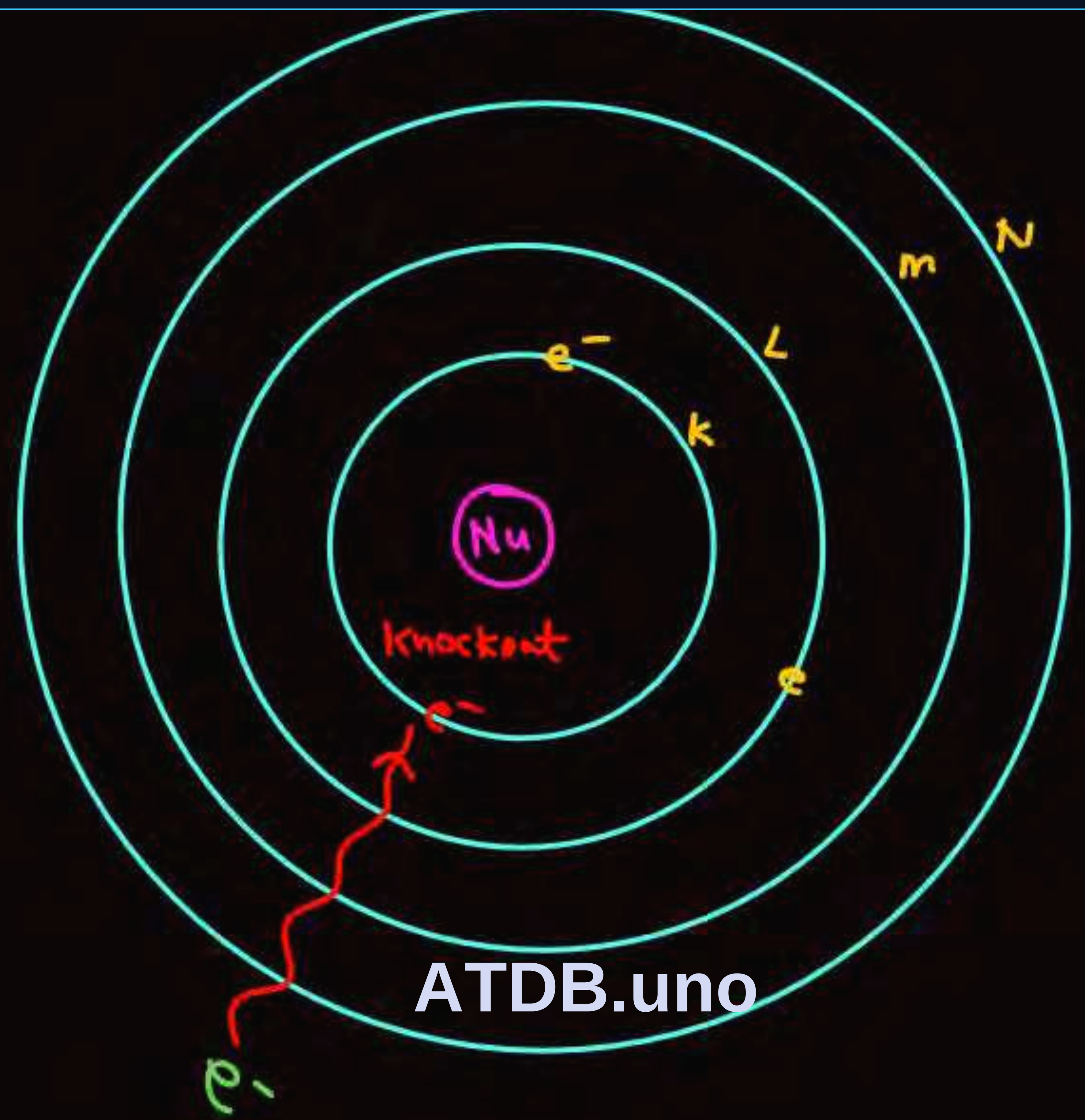
$$(eV_0)\frac{5}{100} = \frac{hc}{\lambda}$$

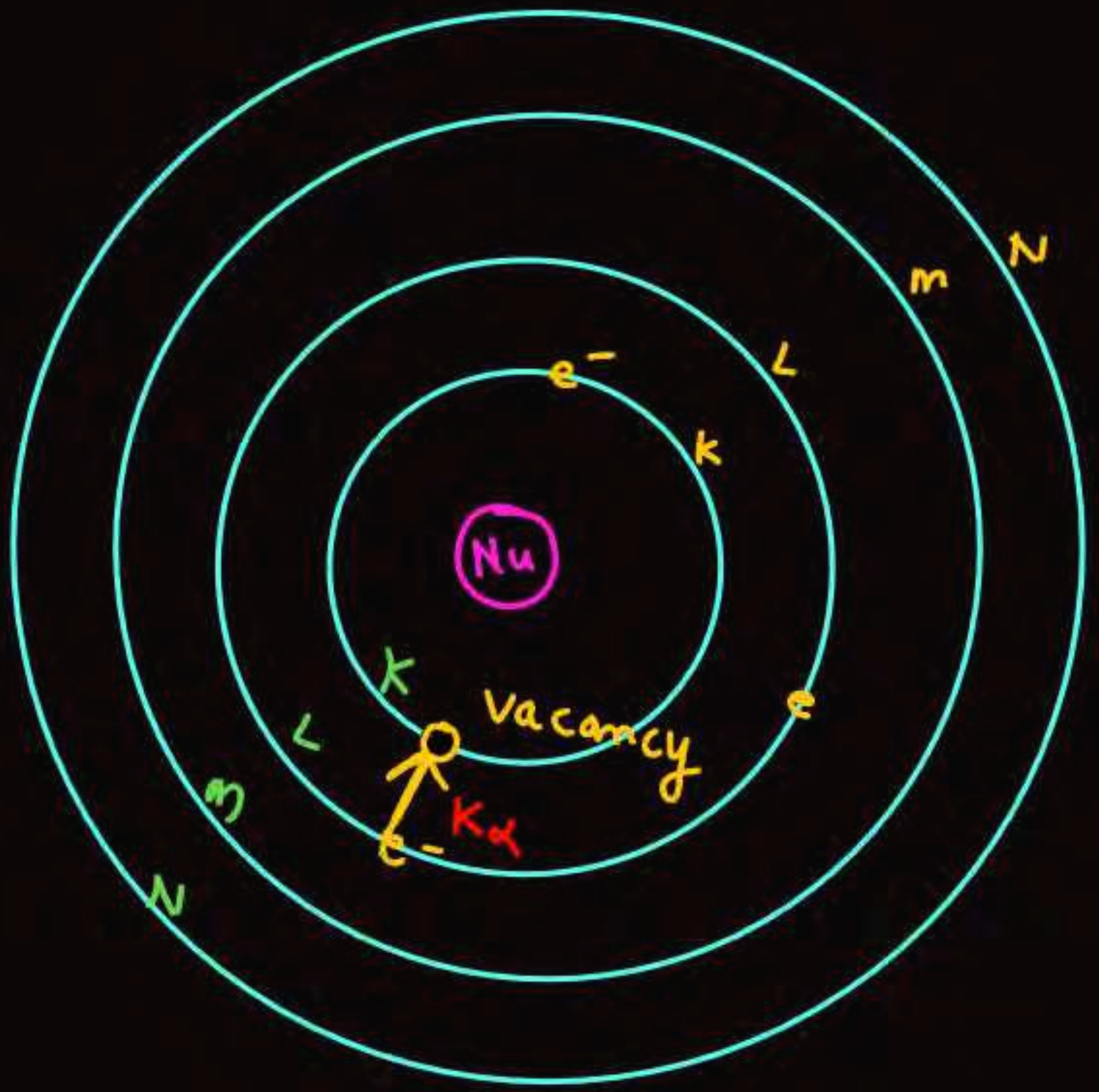
$$e\times 30000(volt)\times\frac{5}{100} = \frac{1240}{\lambda_{(nm)}}\,eVolt$$

$$\lambda = \frac{1240\times100}{30000\times5} = \checkmark$$


Characteristics x-ray
λ
λ1
λ2







$$|E_L - E_K| = \frac{hc}{\lambda}$$

K_α series $\Rightarrow$ $n = 2 \longrightarrow n = 1$

K_β series $\Rightarrow$ $n = 3 \longrightarrow n = 1$

Energy corresponding in Characteristics X-ray

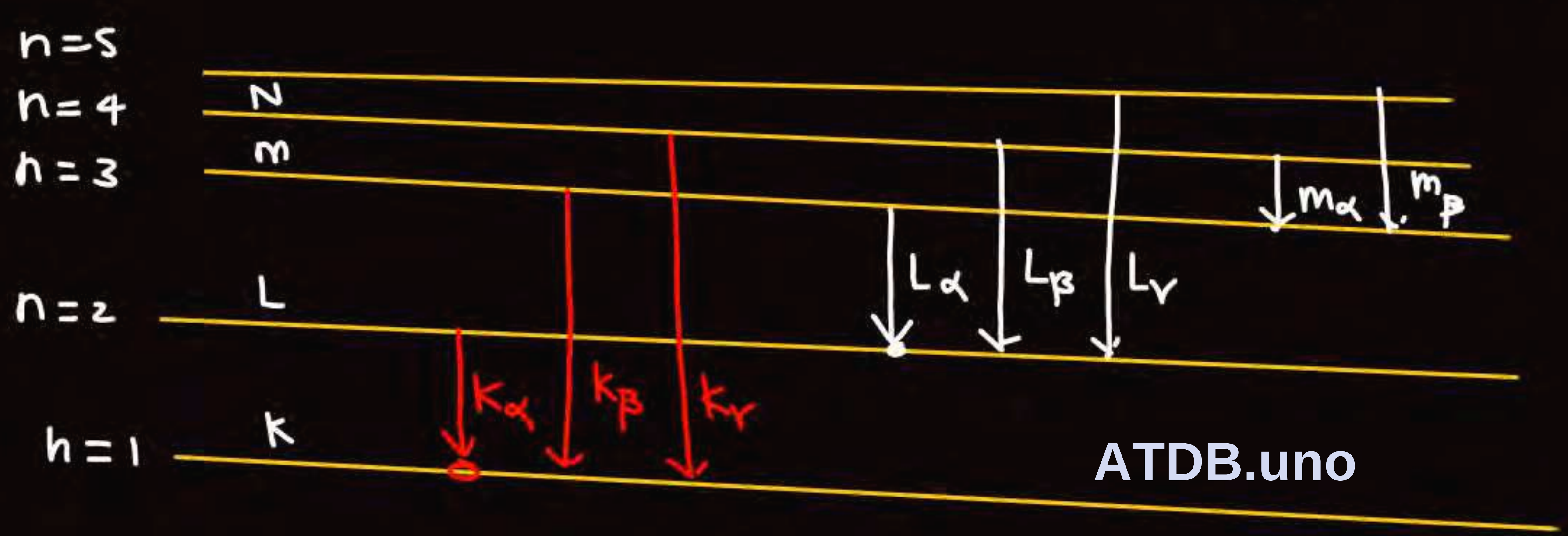
$$\Delta E = 13.6\,(Z-b)^2\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right) = \frac{hc}{\lambda} = \frac{1240}{\lambda}$$

λ in nm

$b \longrightarrow$ screening const

K series $b = 1$

L series $b = 7.4$



- $K_\alpha \Rightarrow 2 \longrightarrow 1$
- $K_\beta \Rightarrow 3 \longrightarrow 1$
- $L_\alpha \Rightarrow 3 \longrightarrow 2$
- $L_\beta \Rightarrow 4 \longrightarrow 2$

Q Find energy of K_α series for Al (Z = 13)

Sol^n $n = 2 \longrightarrow n = 1$

$$\Delta E = 13.6 (Z-b)^2 \left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$\Delta E = 13.6 \times (13-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

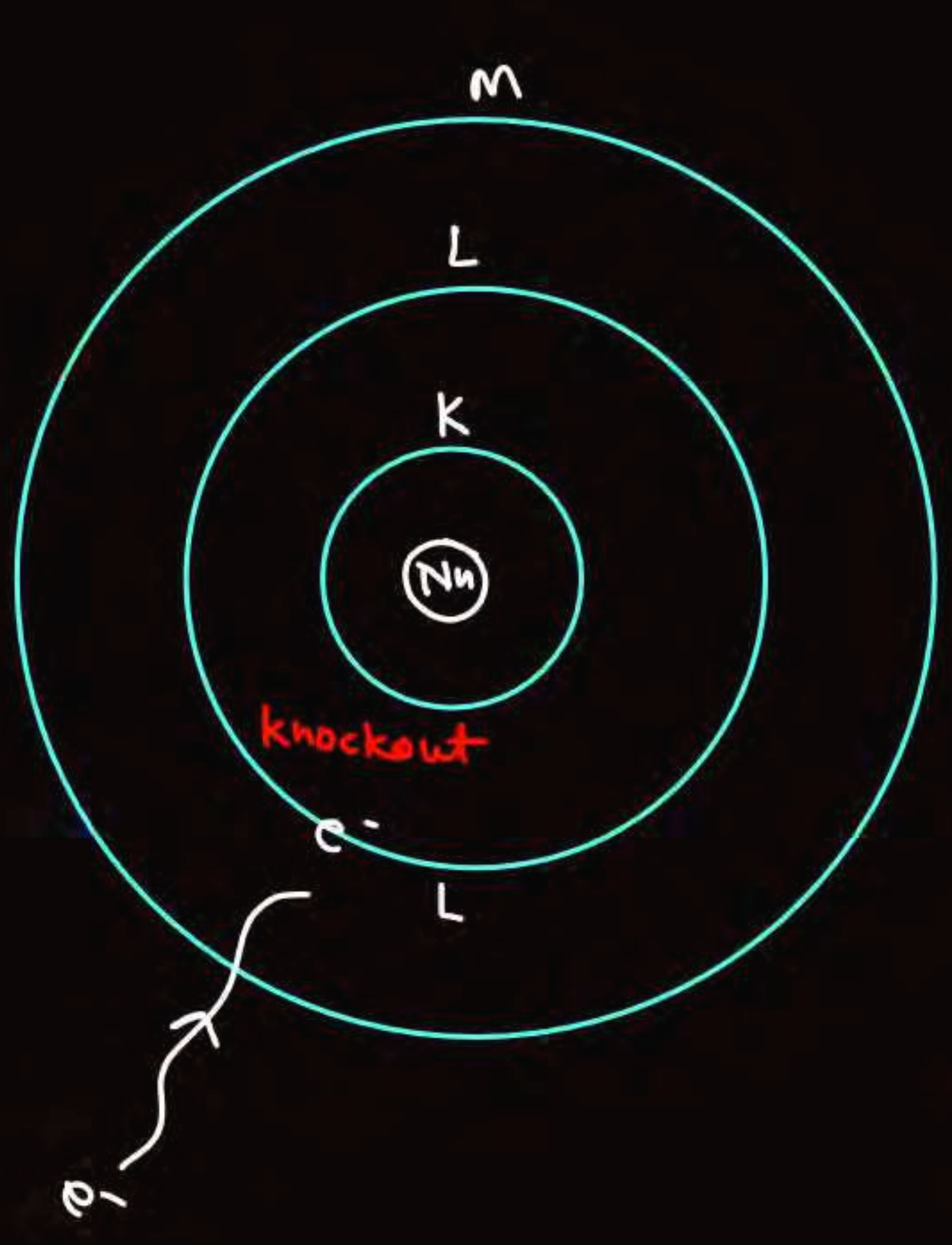
Q find λ for K_β for Z = 41

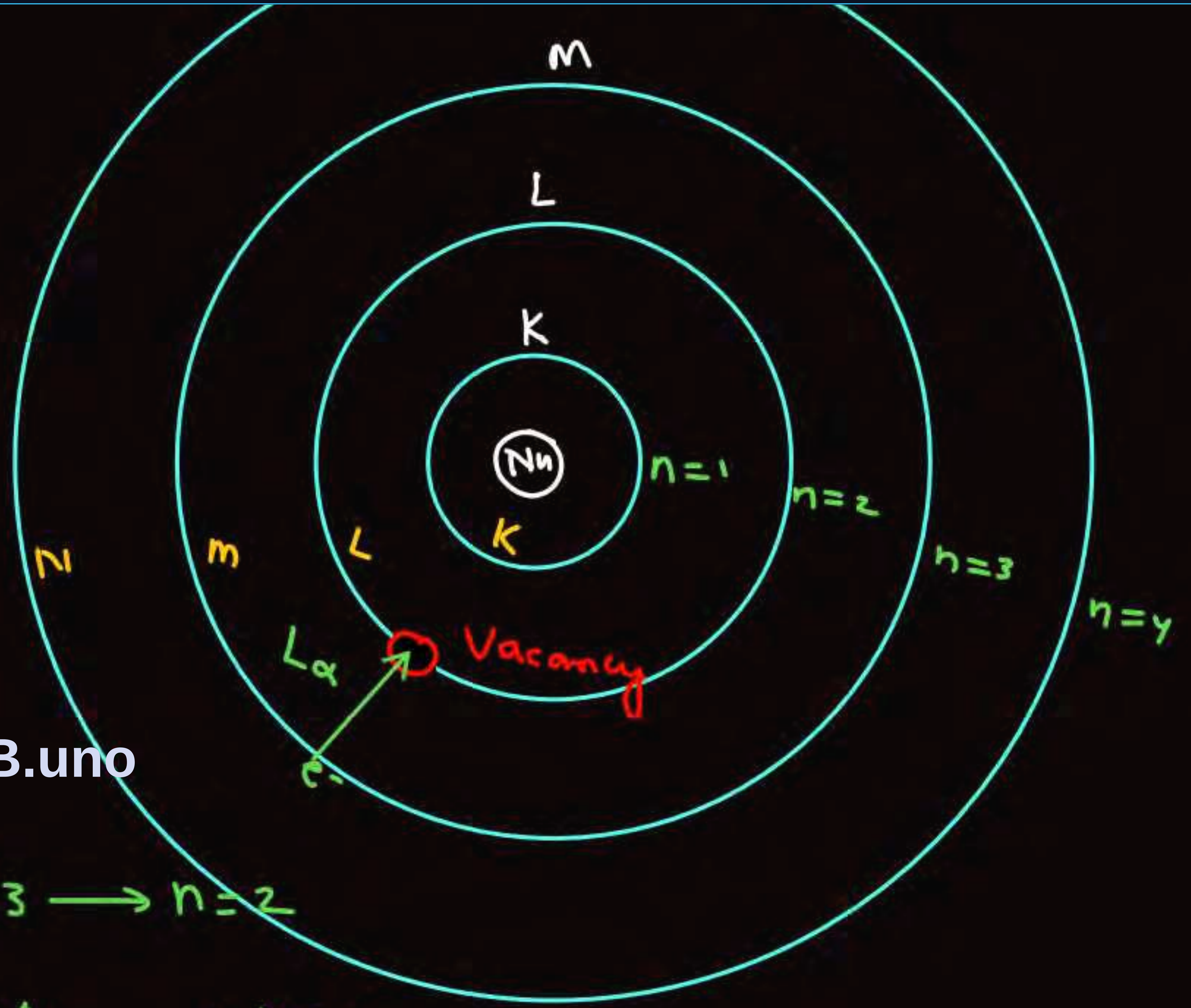
Sol^n

$$\Delta E = 13.6 (41-1)^2 \left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$= \frac{1240}{\lambda}$$

λ → (nm)

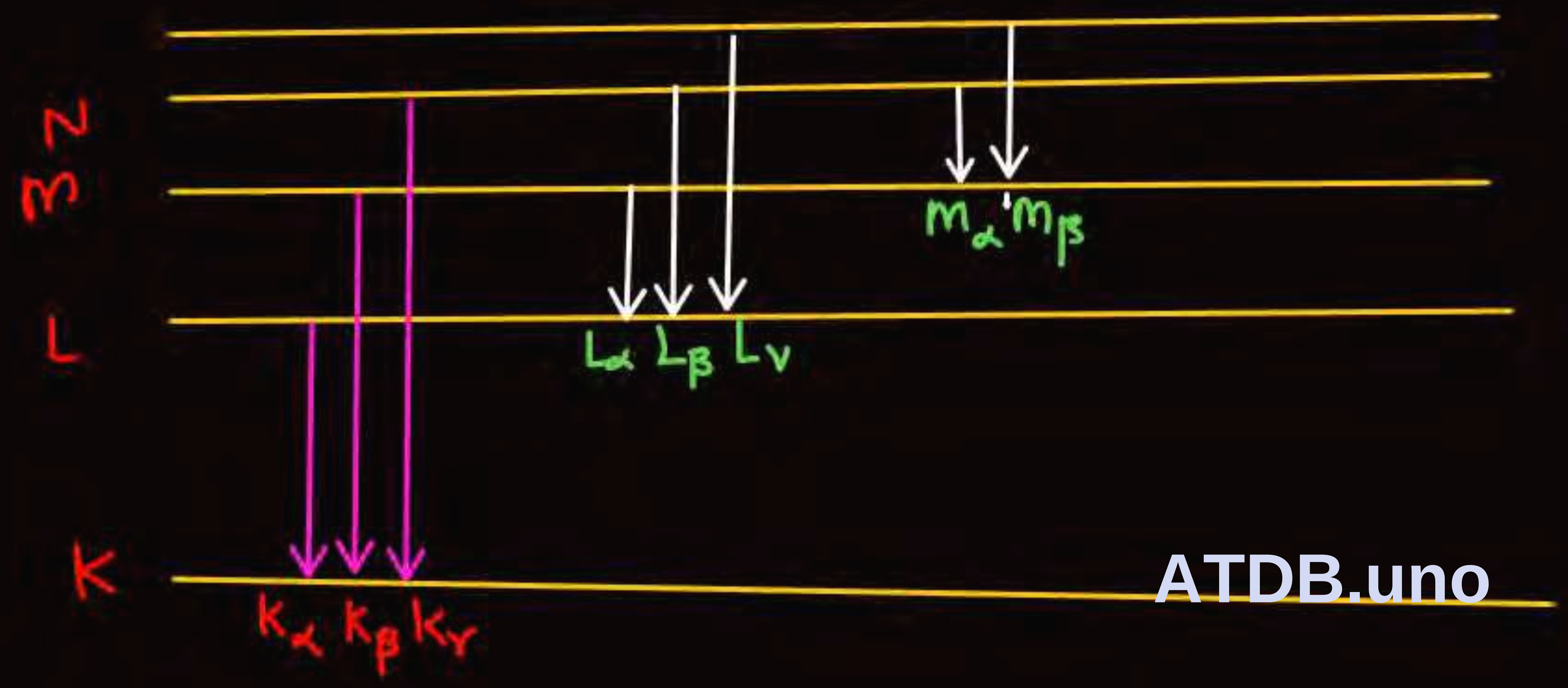




$L_\alpha \Longrightarrow n=3 \longrightarrow n=2$

$L_\beta \Rightarrow n=4 \longrightarrow n=2$

#

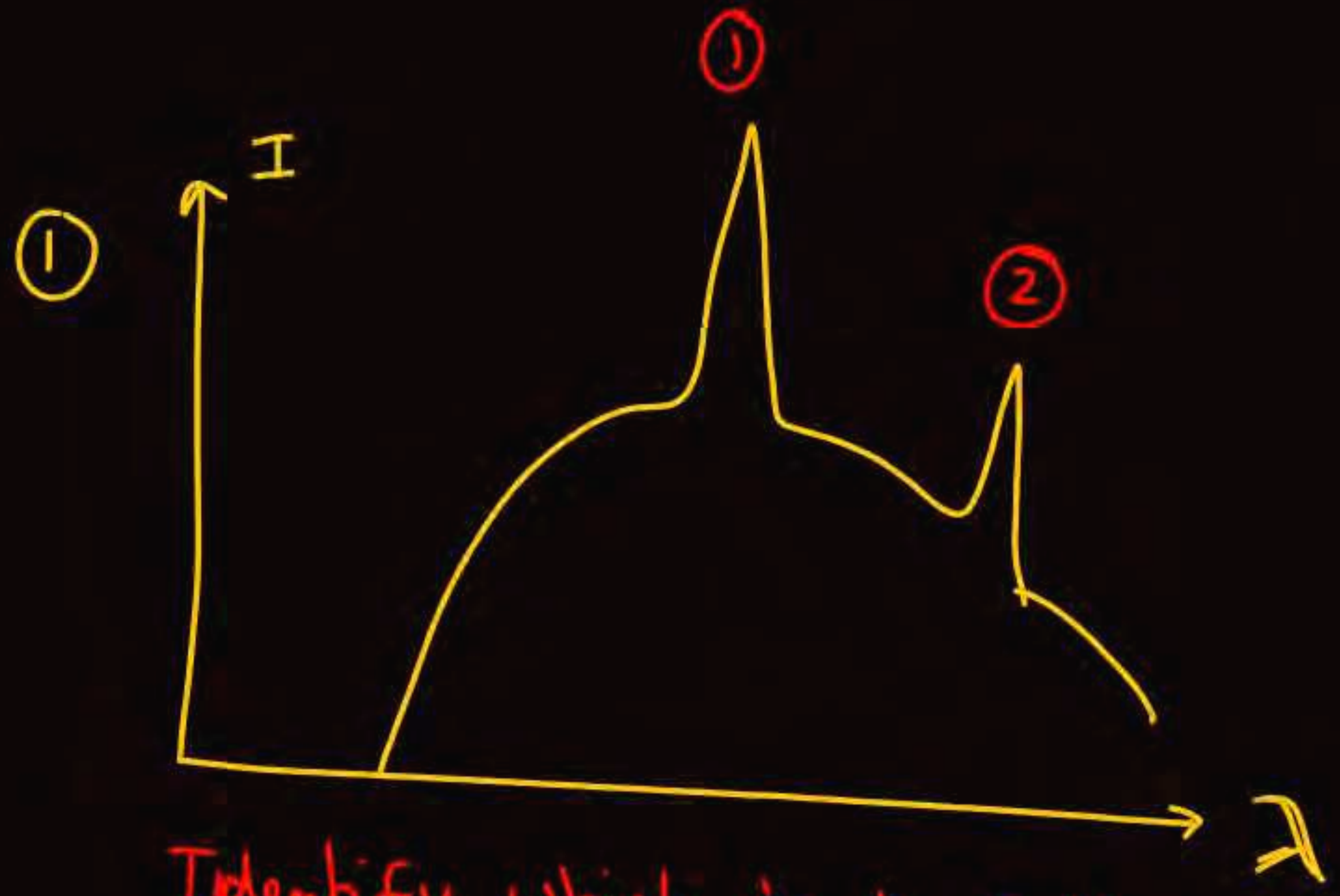


K_α K_β L_α L_β में

Energy $\Rightarrow$ $K_\beta > K_\alpha > L_\beta > L_\alpha$

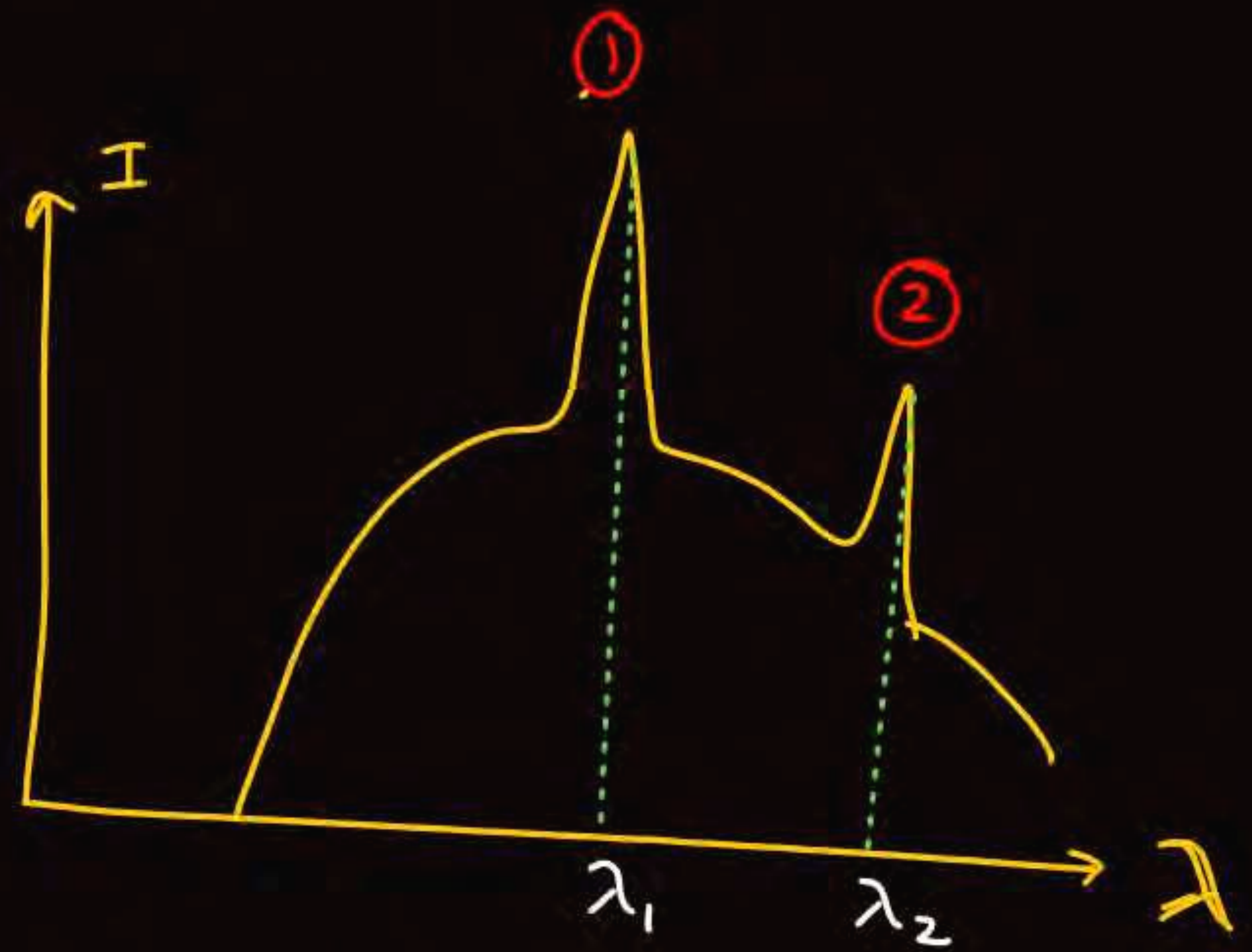
$\lambda \Rightarrow \lambda_{K_\beta} < \lambda_{K_\alpha} < \lambda_{L_\beta} < \lambda_{L_\alpha}$

Energy $\Rightarrow$ $K_\gamma > K_\beta > K_\alpha > L_\gamma > L_\beta > L_\alpha > M_\beta > M_\alpha$



Identify which is K_α & K_β

Sol^n



$\lambda_2 > \lambda_1$

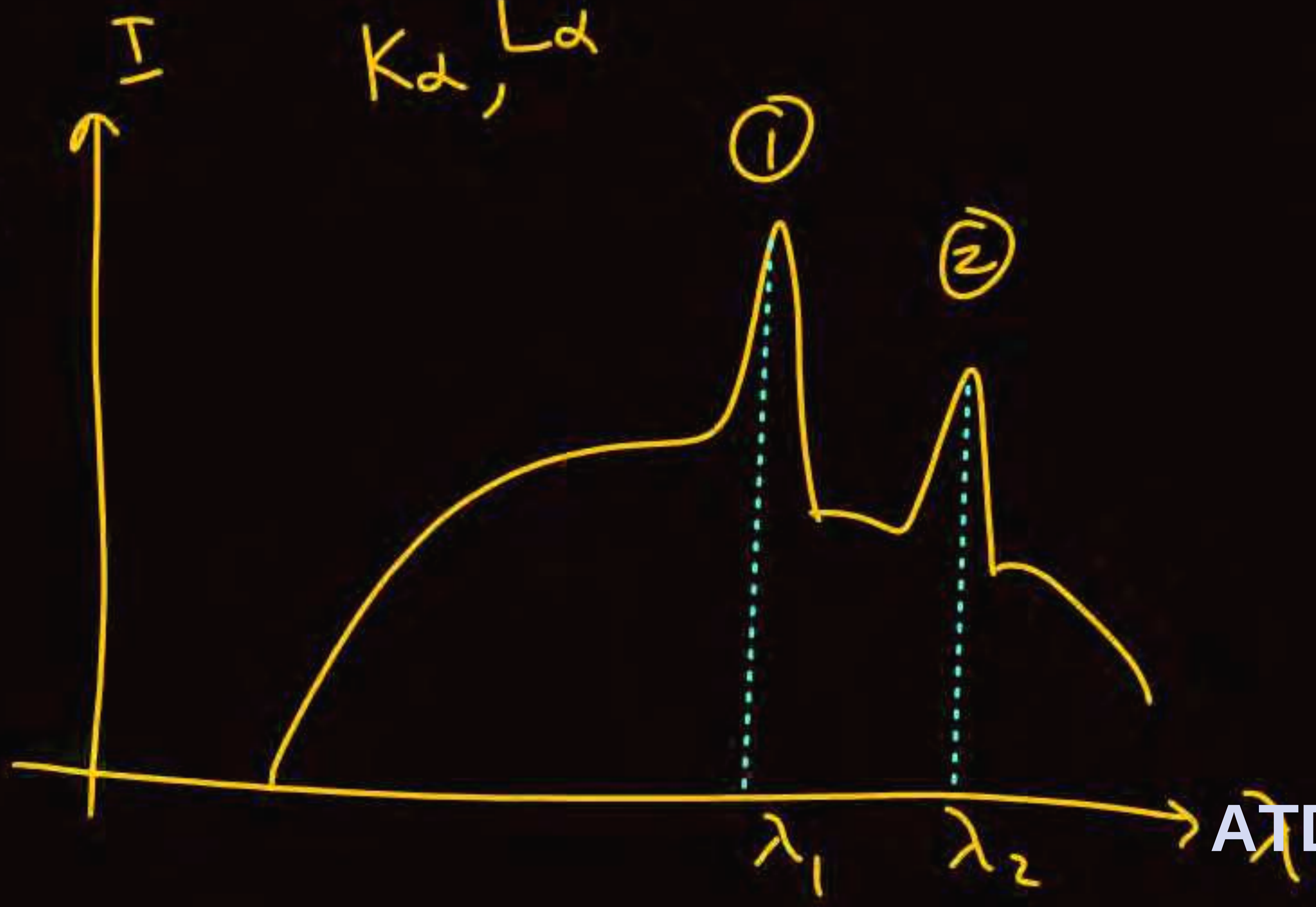
(E कम)

$K_\alpha < K_\beta$ (Energy)

② $\equiv K_\alpha$

① $\equiv K_\beta$

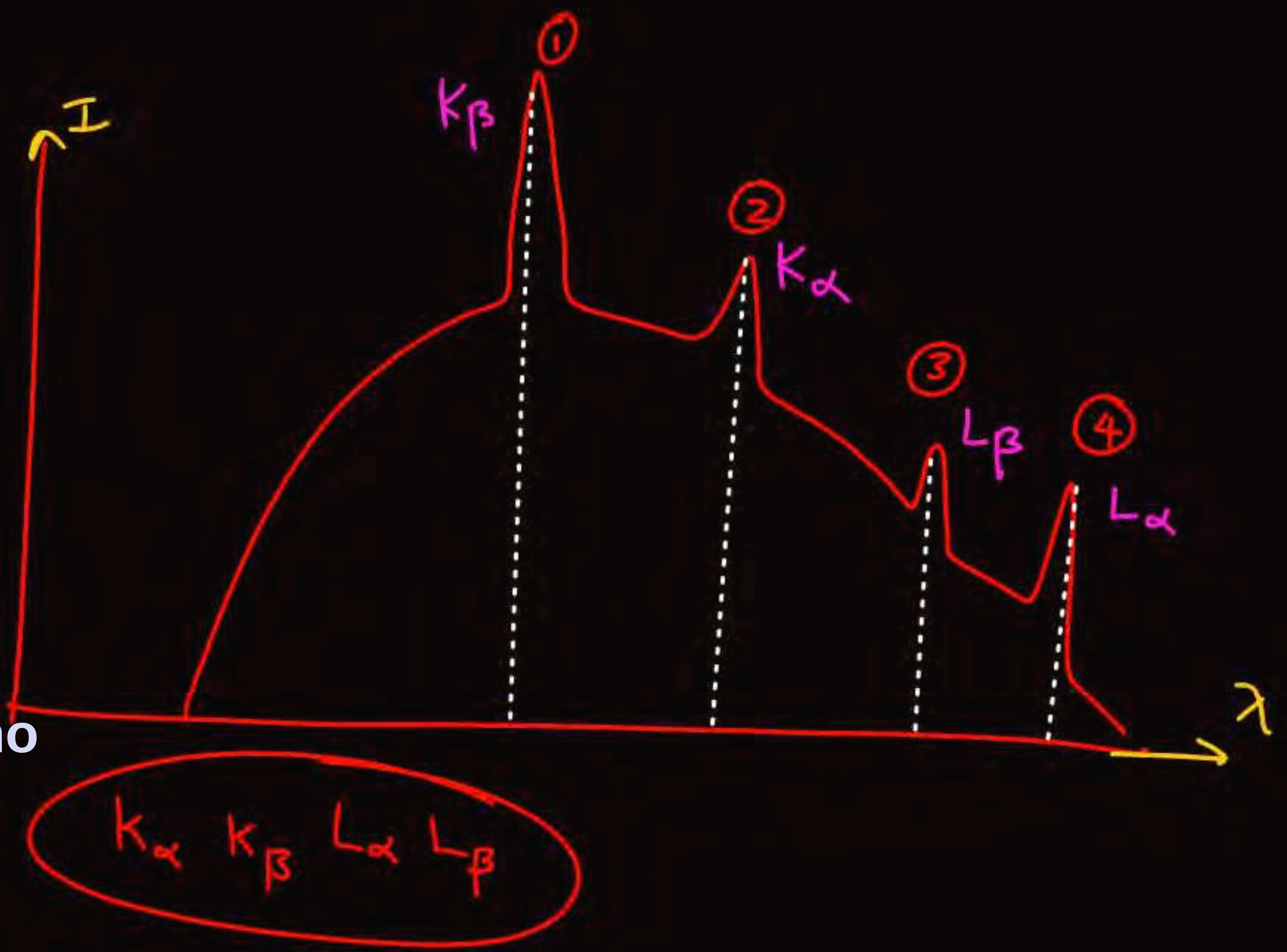
(2)



① ⇒ K_α

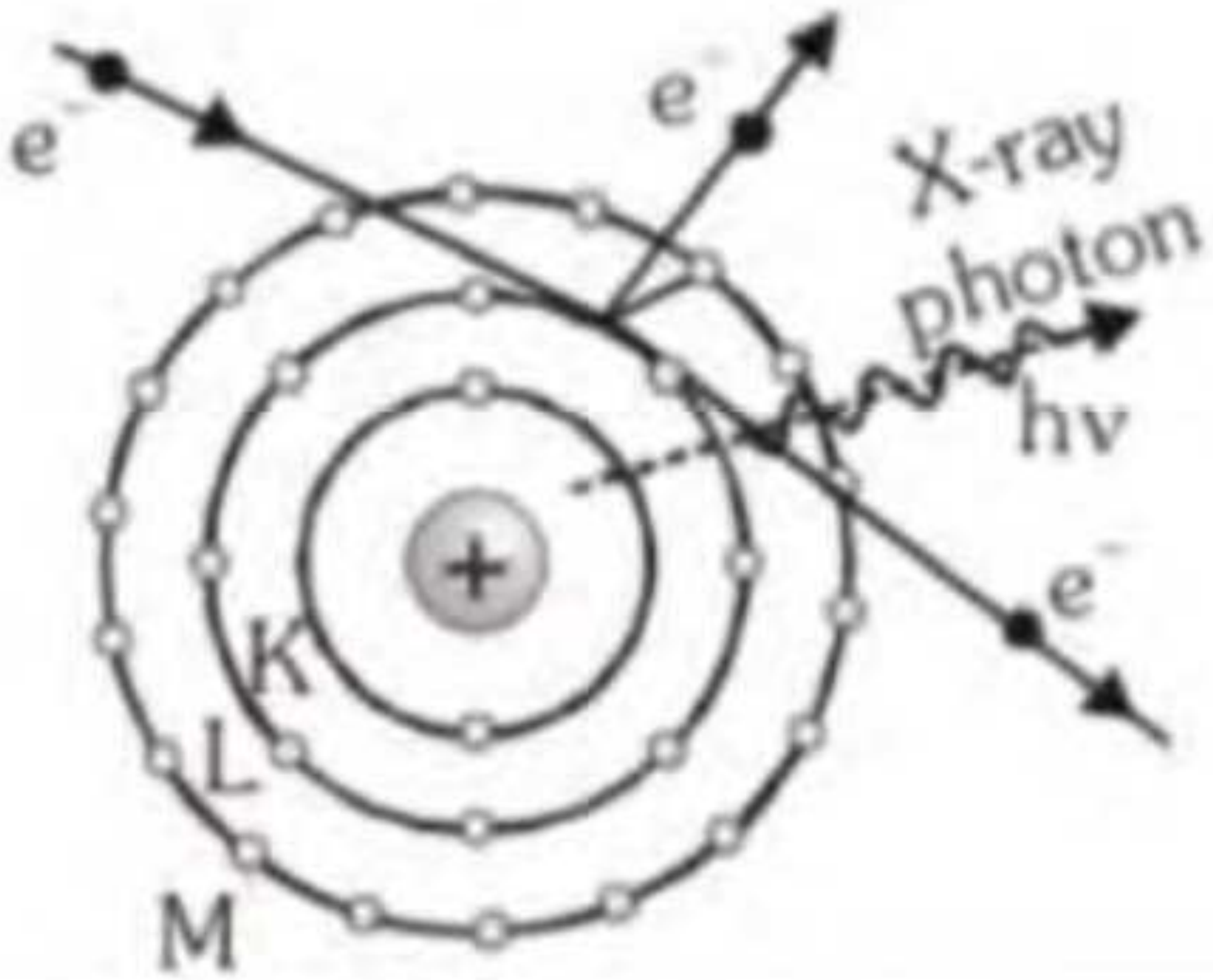
② ⇒ L_α

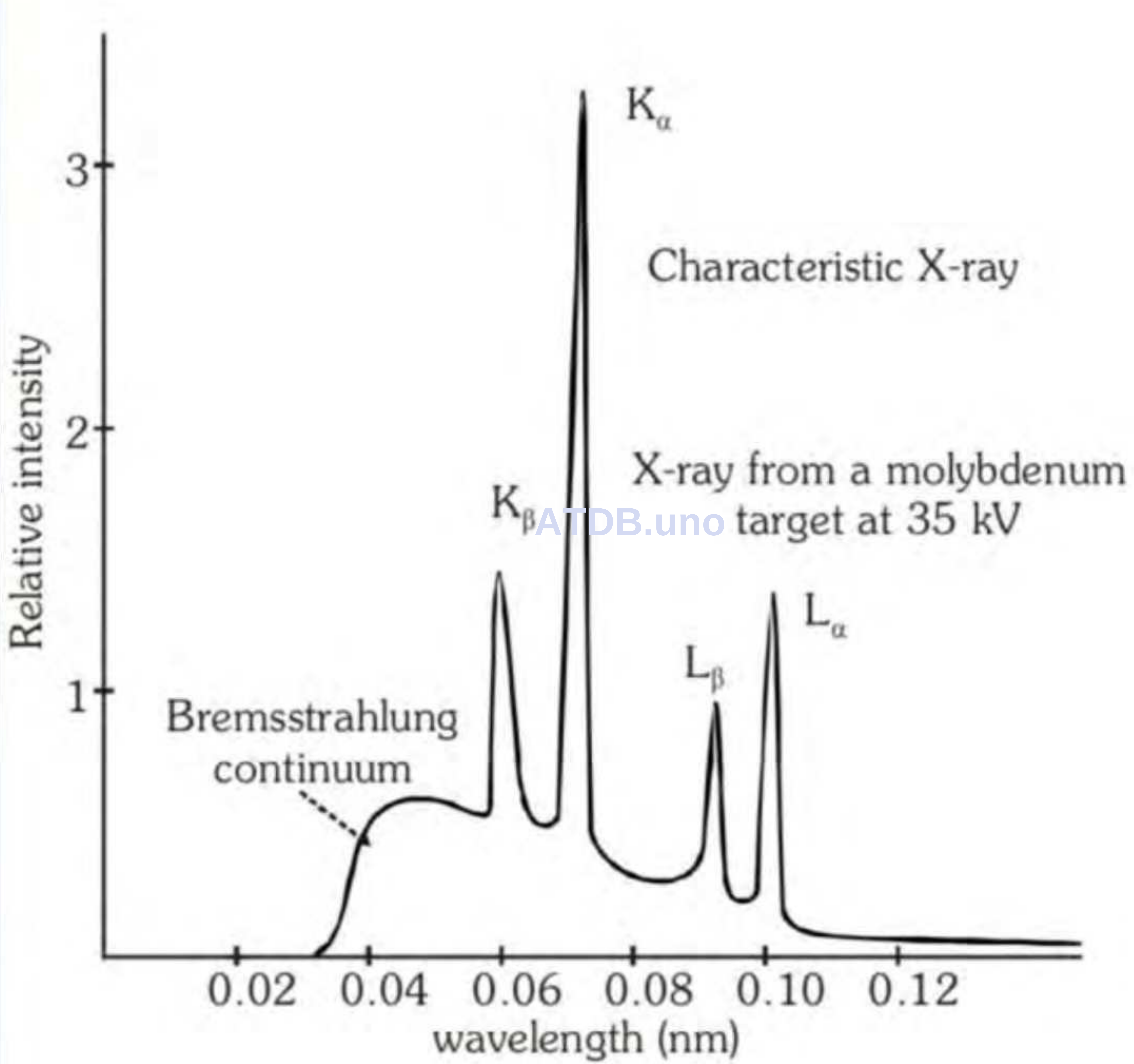
Q



- **Characteristic Spectrum of X–ray**

When the target of X–ray tube is collide by energetic electron it emits two type of X–ray radiation. One of them has a continuous spectrum whose wavelength depend on applied potential while other consists of spectral lines whose wavelength depend on nature of target. The radiation forming the line spectrum is called characteristic X–rays. When highly accelerated electron strikes with the atom of target then it knockout the electron of orbit, due to this a vacancy is created. To fill this vacancy electron jumps from higher energy level and electromagnetic radiation are emitted which form characteristic spectrum of X–ray. Whose wavelength depends on nature of target and not on applied potential.





Relative intensity
3
2
1
K_α
Characteristic X-ray
X-ray from a molybdenum target at 35 kV
K_β
L_α
L_β
Bremsstrahlung continuum
0.02
0.04
0.06
0.08
0.10
0.12
wavelength (nm)

From Bohr Model

$n_1 = 1,$ $n_2 = 2, 3, 4.......$ K series

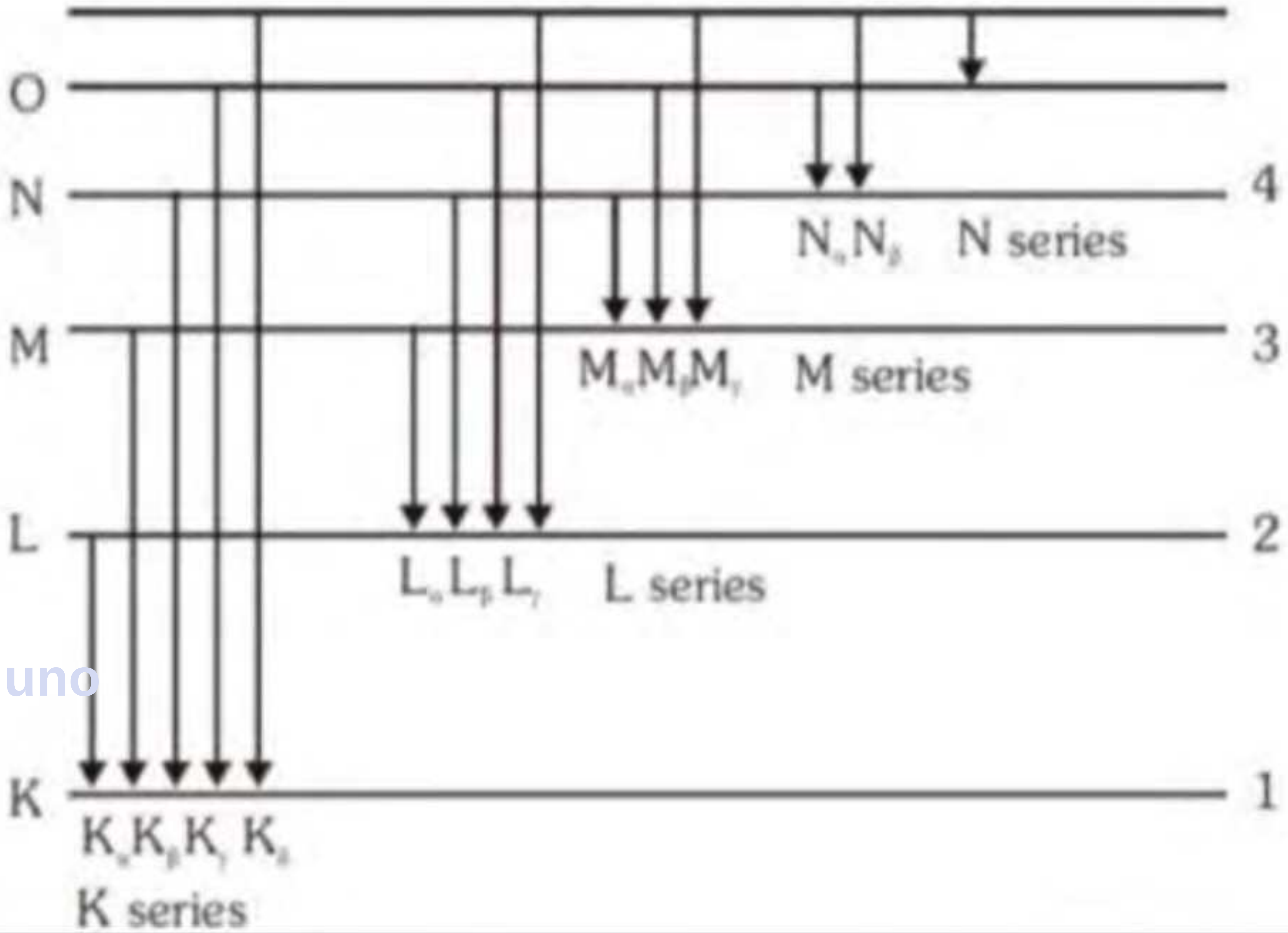
$n_1 = 2,$ $n_2 = 3, 4, 5.......$ L series

$n_1 = 3,$ $n_2 = 4, 5, 6.......$ M series

First line of series = α

Second line of series = β

Third line of series = γ



Transition	Wave–length	Energy	Energy difference	Wavelength
$L \rightarrow K$	$\lambda_{K\alpha}$	$h\nu_{K\alpha}$	$-(E_K - E_L)$	$\lambda_{K\alpha} = \frac{hc}{(E_K - E_L)}$
$(2 \rightarrow 1)$			$= h\nu_{K\alpha}$	$= \frac{12400}{(E_K - E_L)}$ eVÅ
$M \rightarrow K$	$\lambda_{K\beta}$	$h\nu_{K\beta}$	$-(E_K - E_M)$	$\lambda_{K\beta} = \frac{hc}{(E_K - E_M)}$
$(3 \rightarrow 1)$			$= h\nu_{K\beta}$	$= \frac{12400}{(E_K - E_M)}$ eVÅ
$M \rightarrow L$	$\lambda_{L\alpha}$	$h\nu_{L\alpha}$	$-(E_L - E_M)$	$\lambda_{L\alpha} = \frac{hc}{(E_L - E_M)}$
$(3 \rightarrow 2)$			$= h\nu_{L\alpha}$	$= \frac{12400}{[illegible]}$ eVÅ

Thus proportionality constant 'a' does not depend on the nature of target but depend on tr

Bohr model		K series b = 1 Moseley's correction	
1.	For single electron species	1.	For many electron species
2.	$\Delta E = 13.6Z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]eV$	2.	$\Delta E = 13.6\,(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]eV$
3.	$\nu = RcZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$	3.	$\nu = Rc(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$
4.	$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$	4.	$\frac{1}{\lambda} = R\,(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

$$\Delta E = 13.6\,(Z-b)^2\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right) = h\nu$$

$$\boxed{\frac{13.6}{h}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}(Z-b)^2 = \nu$$

$$A\,(Z-b)^2 = \nu$$

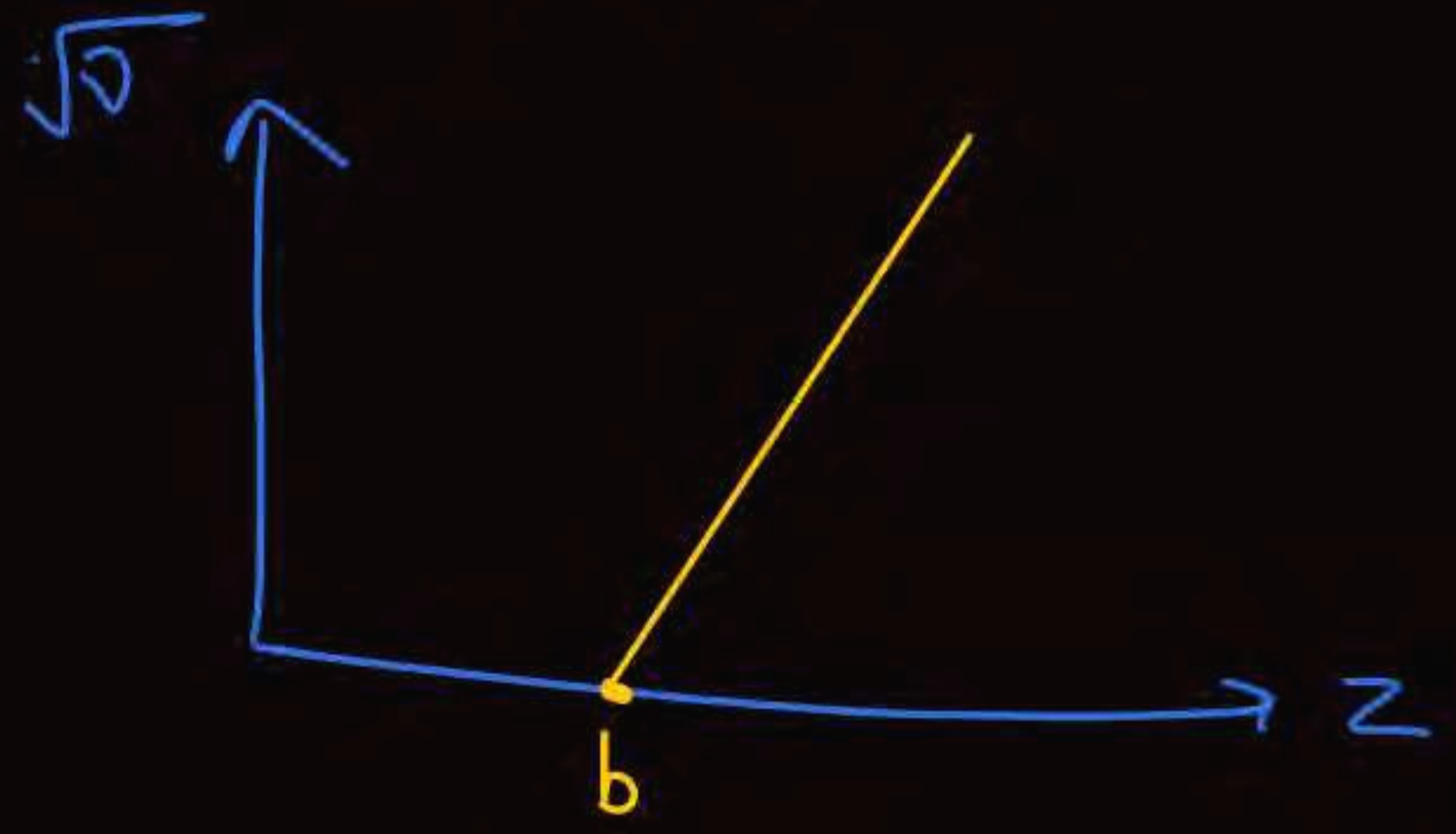
$$\sqrt{A}\,(Z-b) = \sqrt{\nu}$$

$$\boxed{a\,(Z-b) = \sqrt{\nu}}$$

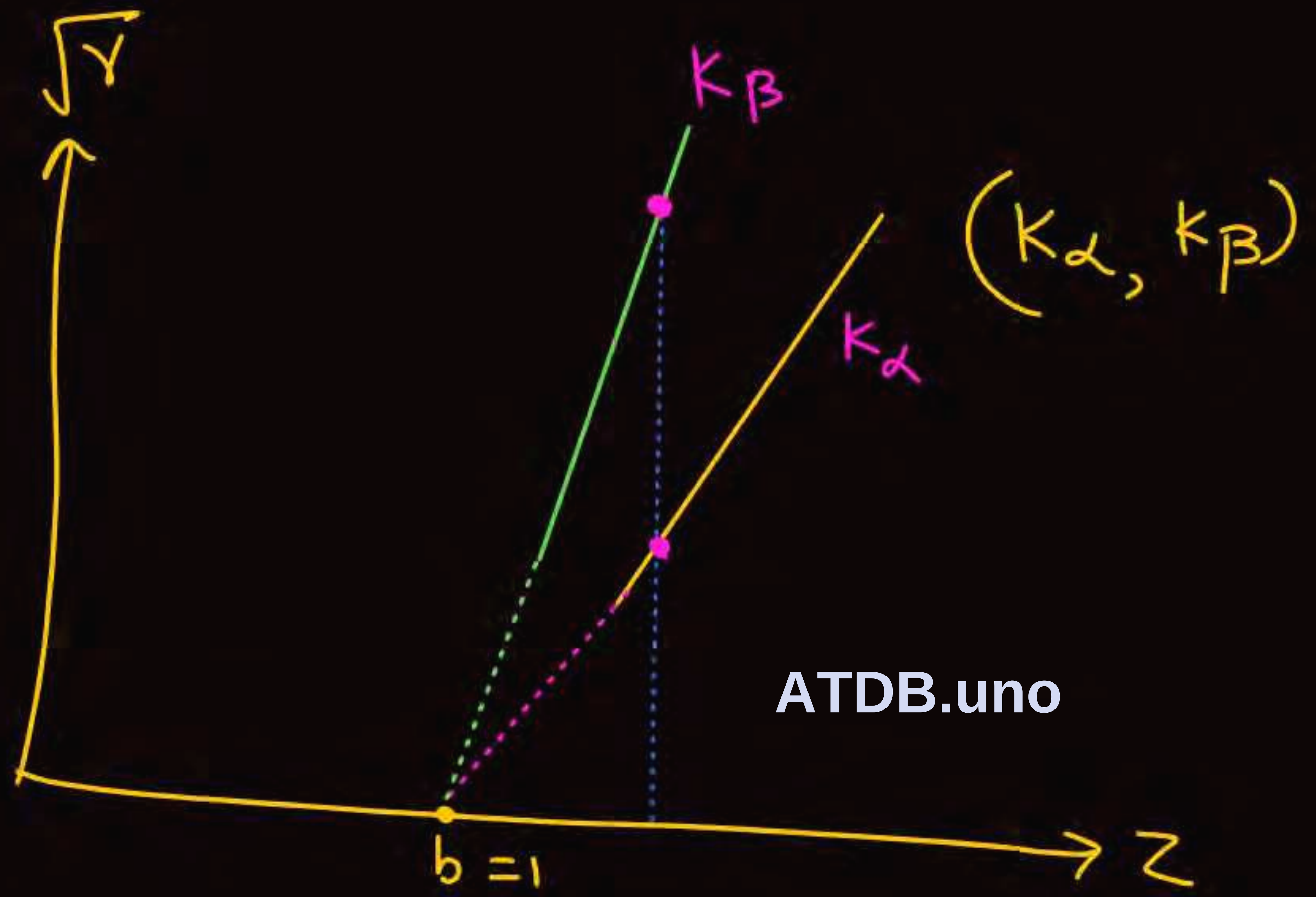
Moseley law

$$\sqrt{\nu} = a\,(Z-b)$$

b → screening const



Q



PROPERTIES OF X–RAY

- X–ray always travel with the velocity of light in straight line because X–rays are em waves
- X–ray is electromagnetic radiation it show particle and wave both nature
- In reflection, diffraction, interference, refraction X–ray shows wave nature while in photoelectric effect it shows particle nature.
- There is no charge on X–ray thus these are not deflected by electric field and magnetic field.
- X–ray are invisible.
- X–ray affects the photographic plate
- When X–ray incidents on the surface of substance it exerts force and pressure and transfer energy and momentum
- Characteristic X–ray can not obtained from hydrogen because the difference of energy level in hydrogen is very small.

ABSORPTION OF X–RAY

Not imp (25 year में 1 ques) in 2004 JEE mains*

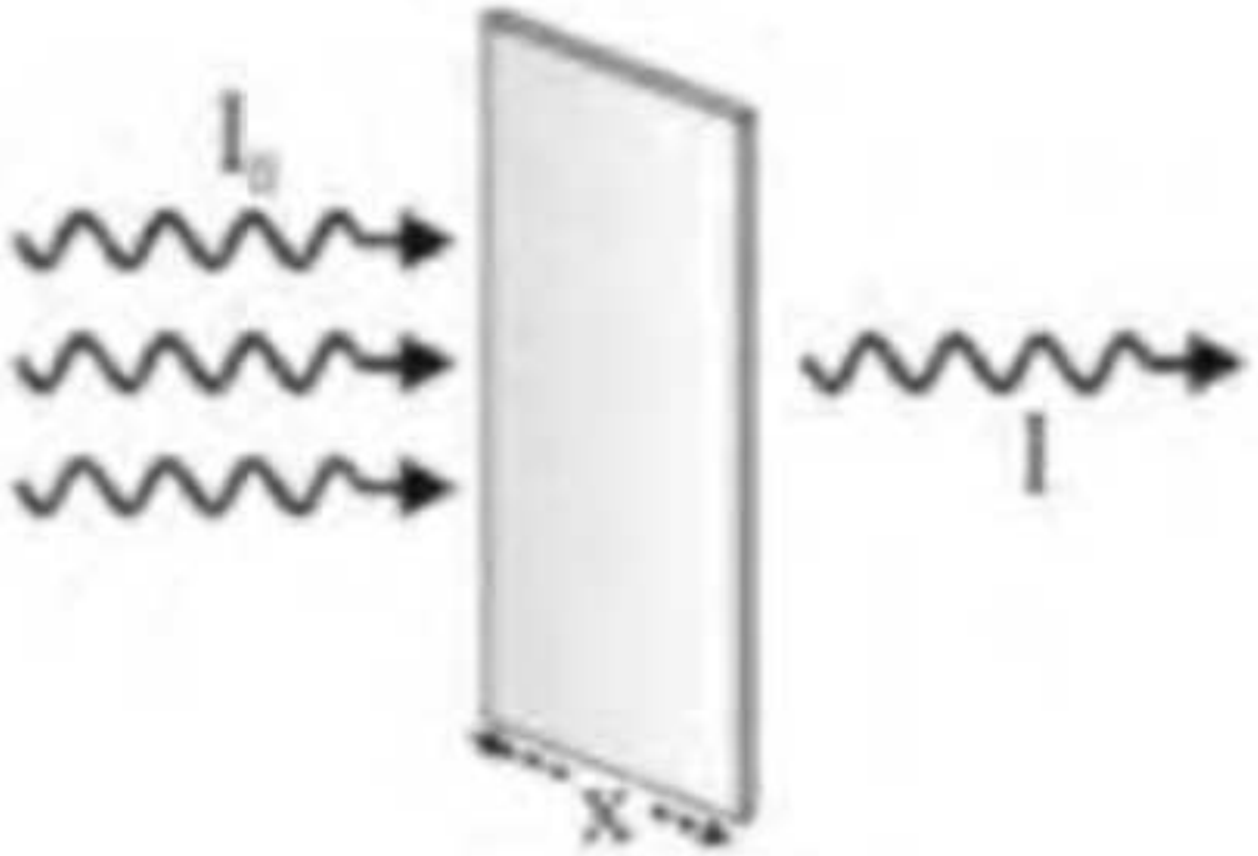
When X–ray passes through x thickness then its intensity $I = I_0 e^{-\mu x}$

I_0 = Intensity of incident X–ray

I = Intensity of X–ray after passing through x distance

μ = absorption coefficient of material

$$I = I_0 e^{-\mu x}$$



- Intensity of X–ray decrease exponentially.
- Maximum absorption of X–ray → Lead
- Minimum absorption of X–ray → Air

32. In an X-ray tube, electrons emitted from a filament (cathode) carrying current I hit a target (anode) at a distance d from the cathode. The target is kept at a potential V higher than the cathode resulting in emission of continuous and characteristic X-rays. If the filament current I is decreased to $\frac{I}{2}$, the potential difference V is increased to 2V, and the separation distance d is reduced to $\frac{d}{2}$, then

[JEE Advanced-2020]

(A) the cut-off wavelength will reduce to half, and the wavelengths of the characteristic X-rays will remain the same

(B) the cut-off wavelength as well as the wavelengths of the characteristic X-rays will remain the same

(C) the cut-off wavelength will reduce to half, and the intensities of all the X-rays will decrease

(D) the cut-off wavelength will become two times larger, and the intensity of all the X-rays will decrease

एक X-किरण नली में, धारा I ग्रहण किए हुए एक तन्तु (कैथोड)से उत्सर्जित इलेक्ट्रॉन, कैथोड से d दूरी पर एक लक्ष्य (एनोड) से टकराते हैं। लक्ष्य को कैथोड से उच्च विभव V पर रखा जाता है जिसके परिणामस्वरूप सतत् तथा अभिलाक्षणिक X-किरणों का उत्सर्जन होता है। यदि तन्तु की धारा I को I/2 तक घटाया जाता है, तब विभवान्तर V, 2V तक बढ़ जाता है तथा पृथक्करण दूरी d, d/2 तक घट जाती है, तब **[JEE Advanced-2020]**

(A) अंतक तरंगदैर्ध्य आधे तक घट जाएगी तथा अभिलाक्षणिक X-किरणों की तरंगदैर्ध्य समान बनी रहेगी।

(B) अंतक तरंगदैर्ध्य तथा अभिलाक्षणिक X-किरणों की तरंगदैर्ध्य समान बनी रहेगी।

(C) अंतक तरंगदैर्ध्य आधे तक घट जाएगी तथा सभी X-किरणों की तीव्रताऐं घटेगी।

(D) अंतक तरंगदैर्ध्य दो गुना अधिक हो जाएगी तथा सभी X-किरणों की तीव्रताऐं घटेगी।

Ans. (A,C)

$$\frac{\sqrt{\gamma_1}}{\sqrt{\gamma_2}} = \frac{a(Z_1-b)}{a(Z_2-b)} = \frac{\sqrt{\lambda_2}}{\sqrt{\lambda_1}}$$

Q.3 A metal target with atomic number $Z = 46$ is bombarded with a high energy electron beam. The emission of X-rays from the target is analyzed. The ratio r of the wavelengths of the K_α-line and the cut-off is found to be $r = 2$. If the same electron beam bombards another metal target with $Z = 41$, the value of r will be

(A) 2.53 (B) 1.27 (C) 2.24 (D) 1.58

(Adv – 2024) 3 marks

32) 81 (2.53
64
170
160
100

$$\frac{hc}{\lambda_{K_\alpha}} = (Z-1)^2 \times 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

(1)

$$r = \frac{\lambda_{K\alpha}}{\lambda_{cutoff}} = 2$$

Same

Z → change

λ_{cutt} → same $= \frac{1240}{V_0}$

$$\frac{(\lambda_{K\alpha})_1}{(\lambda_{K\alpha})_2} = \frac{2}{r} = \frac{(Z-1)_f^2}{(Z-1)_i^2} = \frac{(41-1)^2}{(46-1)^2}$$

$$\frac{2}{r} = \frac{40 \times 40}{45 \times 45}$$

$$r = \frac{45 \times 45 \times 2}{40 \times 40} = \frac{81}{32}$$

1. Which one of the following statements is **WRONG** in the context of X-rays generated from a X-ray tube ? **[JEE 2008]**

(A) Wavelength of characteristic X-rays decreases when the atomic number of the target increases

(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target

(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-rays tube

(D) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

X-किरण नली से निकली X-किरणों के संदर्भ में निम्न में से कौनसा वक्तव्य असत्य है?

(A) यदि लक्ष्य परमाणु का परमाणु क्रमांक बढ़ाया जाये तो अभिलक्षणिक X-किरणों की तरंगदैर्ध्य घटेगी।

(B) सतत् X-किरणों की अन्तक (cut–off) तरंगदैर्ध्य लक्ष्य परमाणु के परमाणु क्रमांक पर निर्भर करती है।

(C) अभिलक्षणिक X-किरणों की तीव्रता X-किरण नली को दी गई विद्युत शक्ति पर निर्भर करती है।

(D) सतत् X-किरणों की अन्तक तरंगदैर्ध्य X-किरण नली में इलेक्ट्रॉन की ऊर्जा पर निर्भर करती है।

Ans. (B)

20. The wavelength of characteristic K_α-line emitted by a hydrogen like element is 2.5 Å. Find the wavelength of the K_γ-line emitted by the same element (in Å). [Assume the shielding effect to be same as of K_α]

एक हाइड्रोजन सदृश तत्व द्वारा उत्सर्जित अभिलाक्षणिक K_α-रेखा की तरंगदैर्ध्य 2.5 Å है। इसी तत्व द्वारा उत्सर्जित K_γ-रेखा की तरंगदैर्ध्य (Å में) ज्ञात कीजिए। (माना कि परिरक्षण प्रभाव K_α के समान ही है)

Ans. 2

15. In an X-ray experiment target is made up of copper (Z=29) having some impurity. The K_α line of copper have wavelength λ_0. It was observed that another K_α line due to impurity have wavelength $\frac{784}{625}\lambda_0$. The atomic number of the impurity element is

$$\frac{hc}{\lambda_0} = 13.6\,(29-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

X-किरण प्रयोग में लक्ष्य तांबे (Z=29) से बना हुआ है, जिसमें कुछ अशुद्धि है। तांबे की K_α रेखा की तरंगदैर्ध्य λ_0 है। अशुद्धि के कारण प्राप्त अन्य K_α रेखा की तरंगदैर्ध्य $\frac{784}{625}\lambda_0$ प्राप्त होती है। अशुद्धि तत्व का परमाणु क्रमांक होगा :-

(A) 22 (B) 23 (C) 24 (D) 26

$$\frac{hc}{\frac{784}{625}\lambda_0} = 13.6\,(z-1)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right]$$

Ans. (D)

$$\begin{array}{r} 28 \\ \times\ 28 \\ \hline 224 \\ 56 \\ \hline 784 \end{array}$$

$$\frac{625}{784} = \frac{(z-1)^2}{(28)^2}$$

$$\frac{25}{28} = \frac{(z-1)}{(28)}$$

$$z = 26$$

23. A graph of $\sqrt{\nu}$ (where ν is the frequency of K_α line of the characteristic X-ray spectrum) is plotted against the atomic number Z of the elements emitting the characteristic X-ray .The intercept of the graph on the Z-axis is 1 and the slope of the graph is 0.5×10^8 S.I. units. The frequency of the K_α line for an element of atomic number 41 is given as $\alpha \times 10^{16}$ Hz. Find the value of α.

अभिलाक्षणिक X-किरण उत्सर्जित कर रहे तत्व के परमाणु क्रमांक Z व $\sqrt{\nu}$ (जहां ν अभिलाक्षणिक X-किरण स्पेक्ट्रम की K_α रेखा की आवृत्ति है) के मध्य आरेख खींचा जाता है। इस आरेख का Z-अक्ष पर अन्तखण्ड 1 तथा इस आरेख की ढाल 0.5×10^8 S.I. इकाई प्राप्त होती है। परमाणु क्रमांक 41 वाले तत्व की K_α रेखा की आवृत्ति $\alpha \times 10^{16}$ (Hz में) है तो α ज्ञात कीजिए।

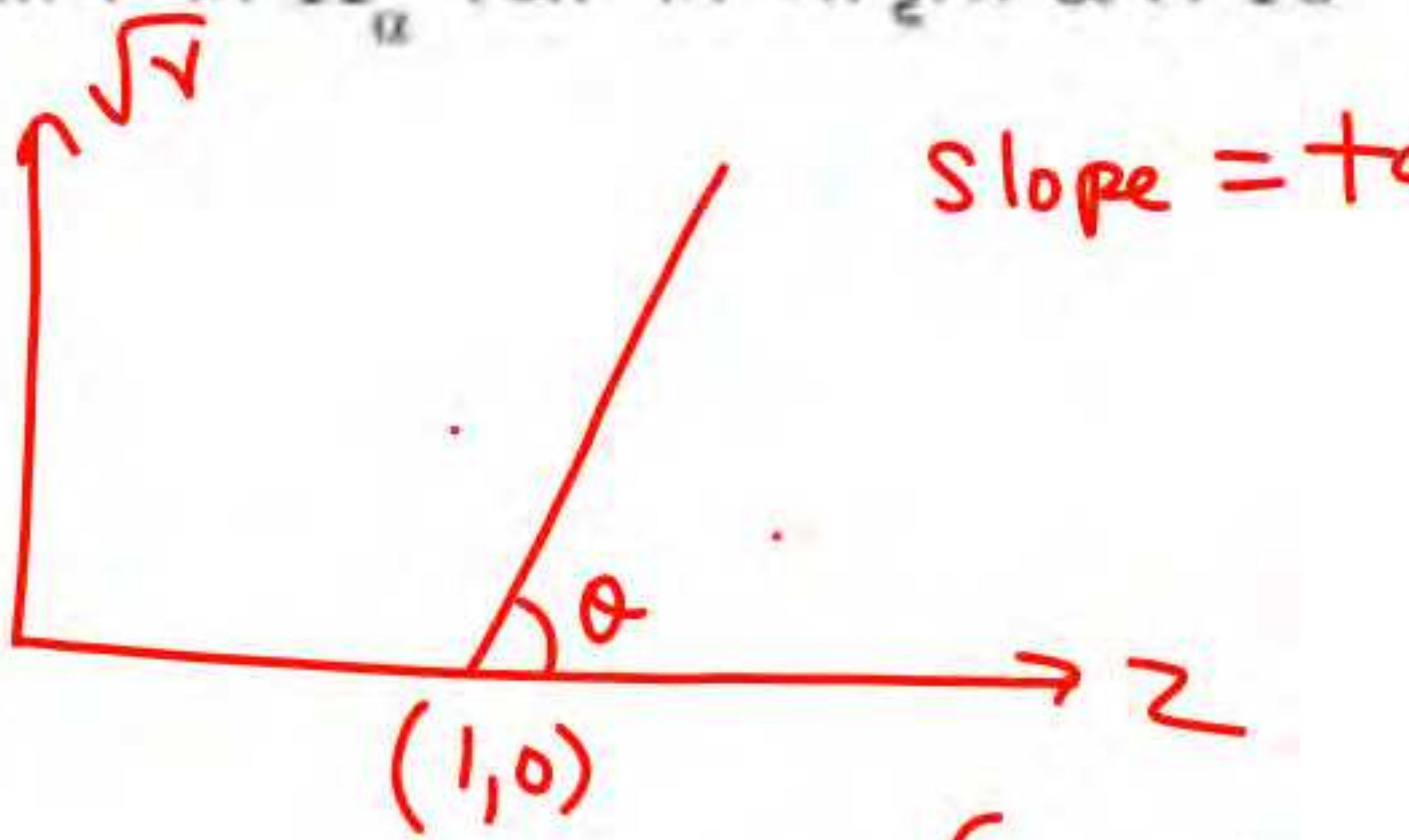
Ans. 400

slope

$\sqrt{\nu} = a(Z-b)$

$\sqrt{\nu} = 5\times10^7(41-1)$

$\sqrt{\nu} = 200\times10^7 = 2\times10^9$



Slope = tanθ = 5×10^7

Z = b = 1 , (K series)

$\nu = 4\times10^{18} = 400\times10^{16}$

25. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is [JEE 2007]

डी-ब्रोग्ली तरंगदैर्ध्य λ वाले इलेक्ट्रॉन एक X-किरण नलिका के लक्ष्य पर आपतित होते है। उत्सर्जित X-किरणों की अन्तक तरंगदैर्ध्य सीमा है-

(A) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (B) $\lambda_0 = \frac{2h}{mc}$ (C) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (D) $\lambda_0 = \lambda$

Ans. (A)

49. The wavelength of K_α X-ray of an element having atomic number $Z = 11$ is λ. The wavelength of K_α X-ray of another element of atomic number Z' is 4λ. Then Z' is **[JEE' 2005 (Scr)]**

परमाणु क्रमांक $Z = 11$ वाले तत्व द्वारा उत्सर्जित की गई K_α X-किरण की तरंगदैर्ध्य λ है। 4λ तरंगदैर्ध्य वाले K_α विकिरण उत्सर्जित कर रहे तत्व का परमाणु क्रमांक होगा–

(A) 11 (B) 44 (C) 6 (D) 4

Ans. (C)

20. The wavelength of characteristic K_α-line emitted by a hydrogen like element is 2.5 Å. Find the wavelength of the K_γ-line emitted by the same element (in Å). [Assume the shielding effect to be same as of K_α]

एक हाइड्रोजन सदृश तत्व द्वारा उत्सर्जित अभिलाक्षणिक K_α-रेखा की तरंगदैर्ध्य 2.5 Å है। इसी तत्व द्वारा उत्सर्जित K_γ-रेखा की तरंगदैर्ध्य (Å में) ज्ञात कीजिए। (माना कि परिरक्षण प्रभाव K_α के समान ही है)

$$\frac{12400}{2.5} = 13.6\,(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$\frac{12400}{\lambda} = 13.6\,(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{4^2}\right)$$

Ans. 2


THANK
YOU