PRAYAS
JEE 2025

Lecture - 07 | **Physics**

Capacitor

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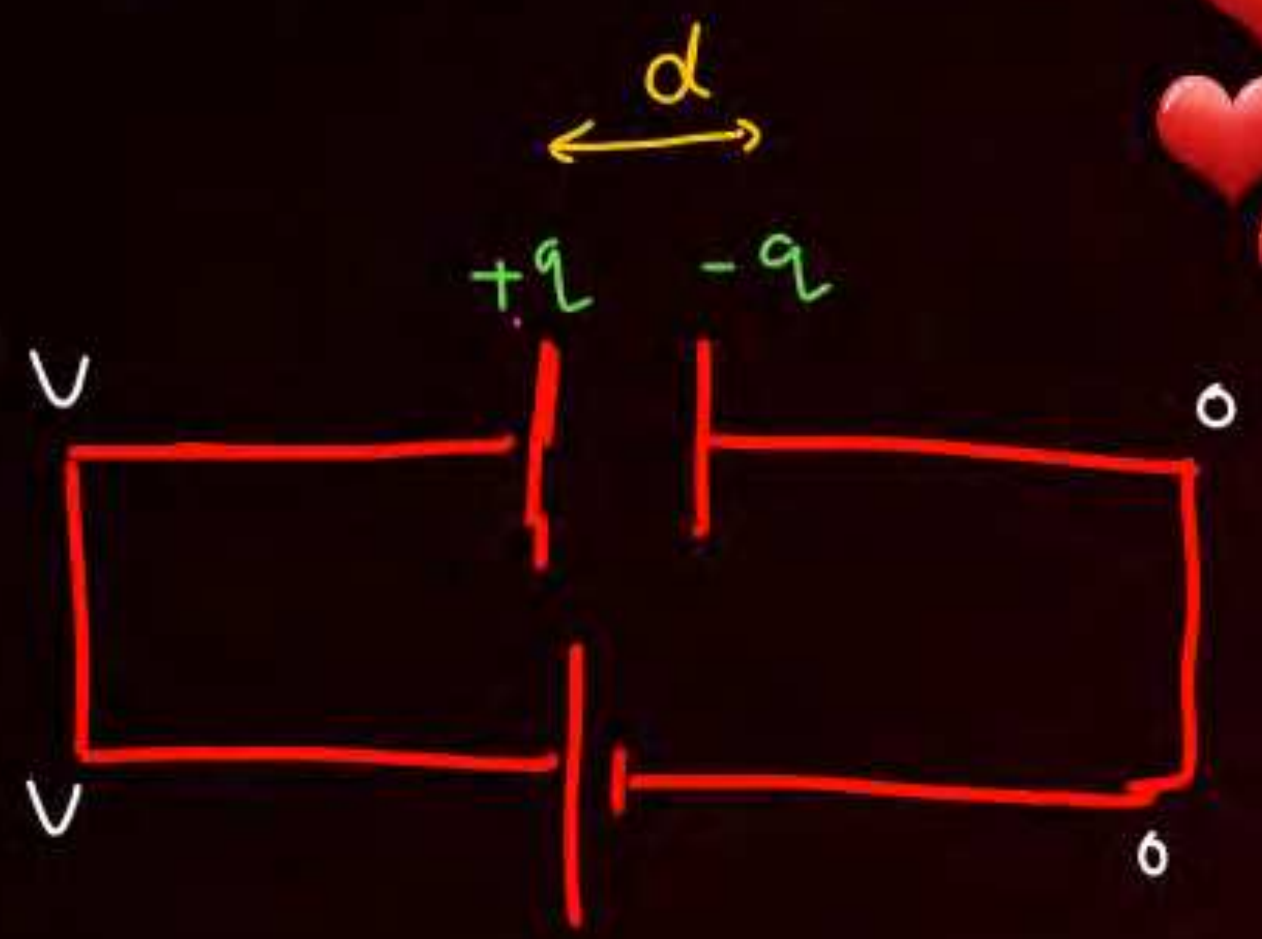
Topics *to be covered*

1. RC Circuit
2.
3.
4.

Battry is Connected

$V \rightarrow$ same



$C = \frac{A\epsilon_0}{d}$

$q = CV$

$U = \frac{1}{2}CV^2$

$E = \frac{V}{d}$

$d \longrightarrow$ half, $\frac{d}{2}$

$C \longrightarrow 2C$

$V \longrightarrow$ Same

$q \longrightarrow 2q$

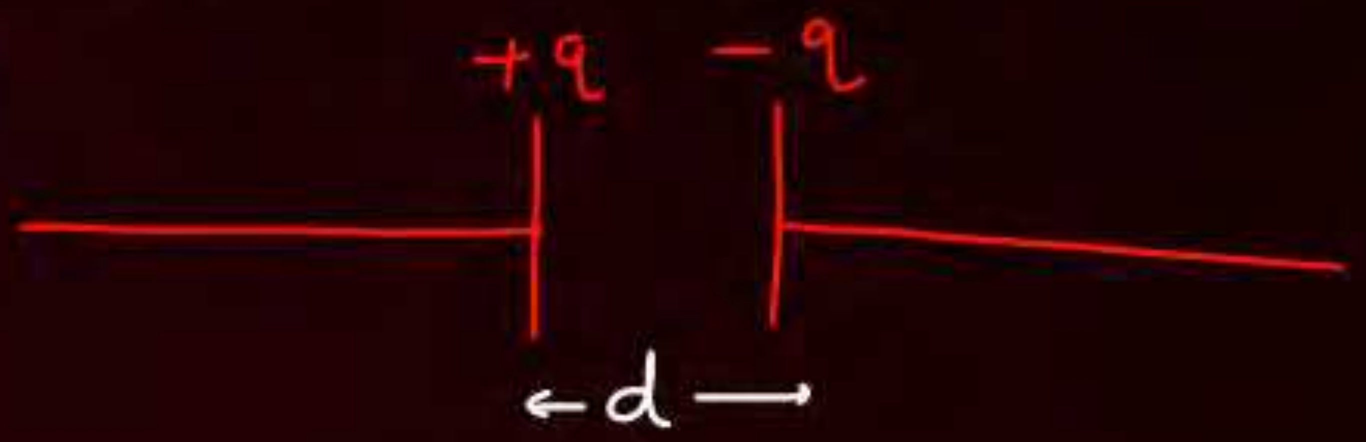
$U \longrightarrow 2U$

$E \longrightarrow 2E$

$(WD)_{battry} = (2q - q)E$

Battry is Dis-Connected

$q \rightarrow$ Same



$q \longrightarrow$ Same

$d \longrightarrow \frac{d}{2}$

$C \longrightarrow 2C$

$V \longrightarrow$ half

$U \longrightarrow \frac{U}{2}$

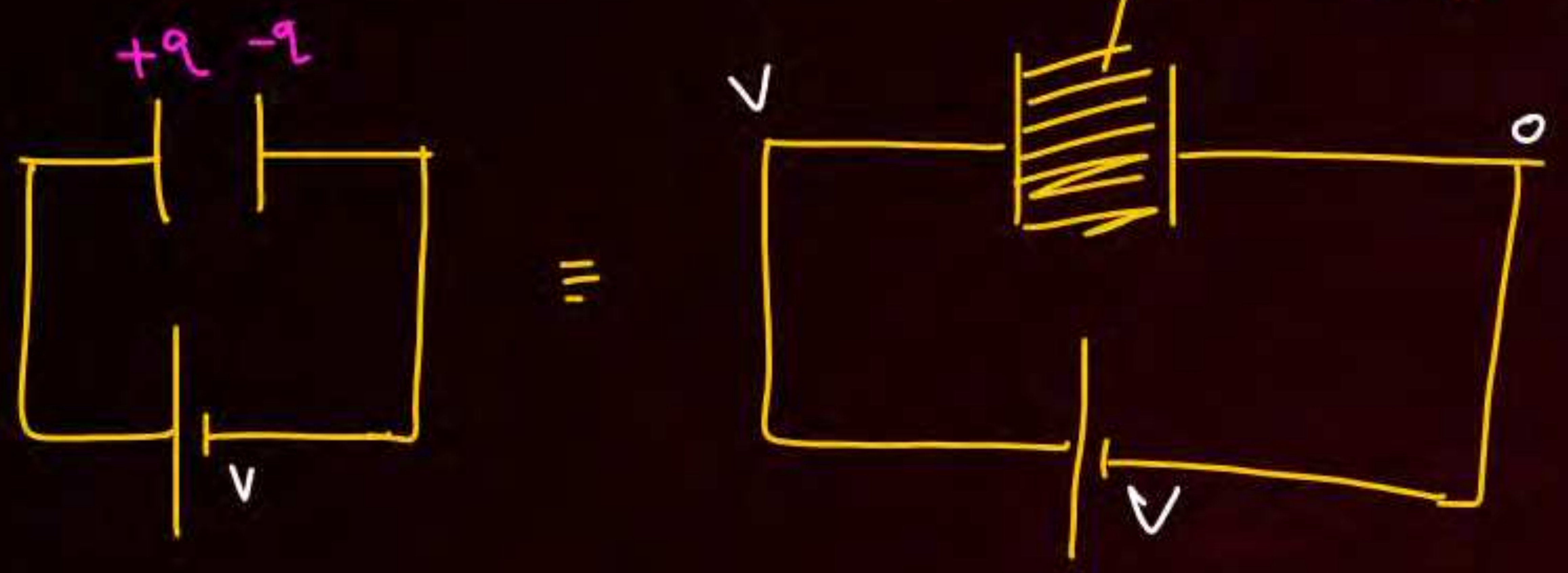
$E \longrightarrow$ Same

$q = CV$

$U = \frac{1}{2}\frac{q^2}{C}$

Battery is connected

$V \rightarrow$ same



$C \longrightarrow KC$

$V \longrightarrow$ Same

$q \longrightarrow Kq$

$q_i = q \longrightarrow q_f = Kq$

$C = \frac{A\epsilon_0}{d}$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$

$q = CV$

$(WD)_{\text{by battery}} = (Kq - q)E = qE(K-1)$

$E \longrightarrow$ Same

$U \longrightarrow KU$

Force on one plate $\longrightarrow K^2$ (times)

Battery is disconnected

$C \longrightarrow KC$

$q \longrightarrow$ same

$V \longrightarrow \frac{V}{K}$

$U \longrightarrow \frac{U}{K}$

$E \longrightarrow \frac{E}{K}$

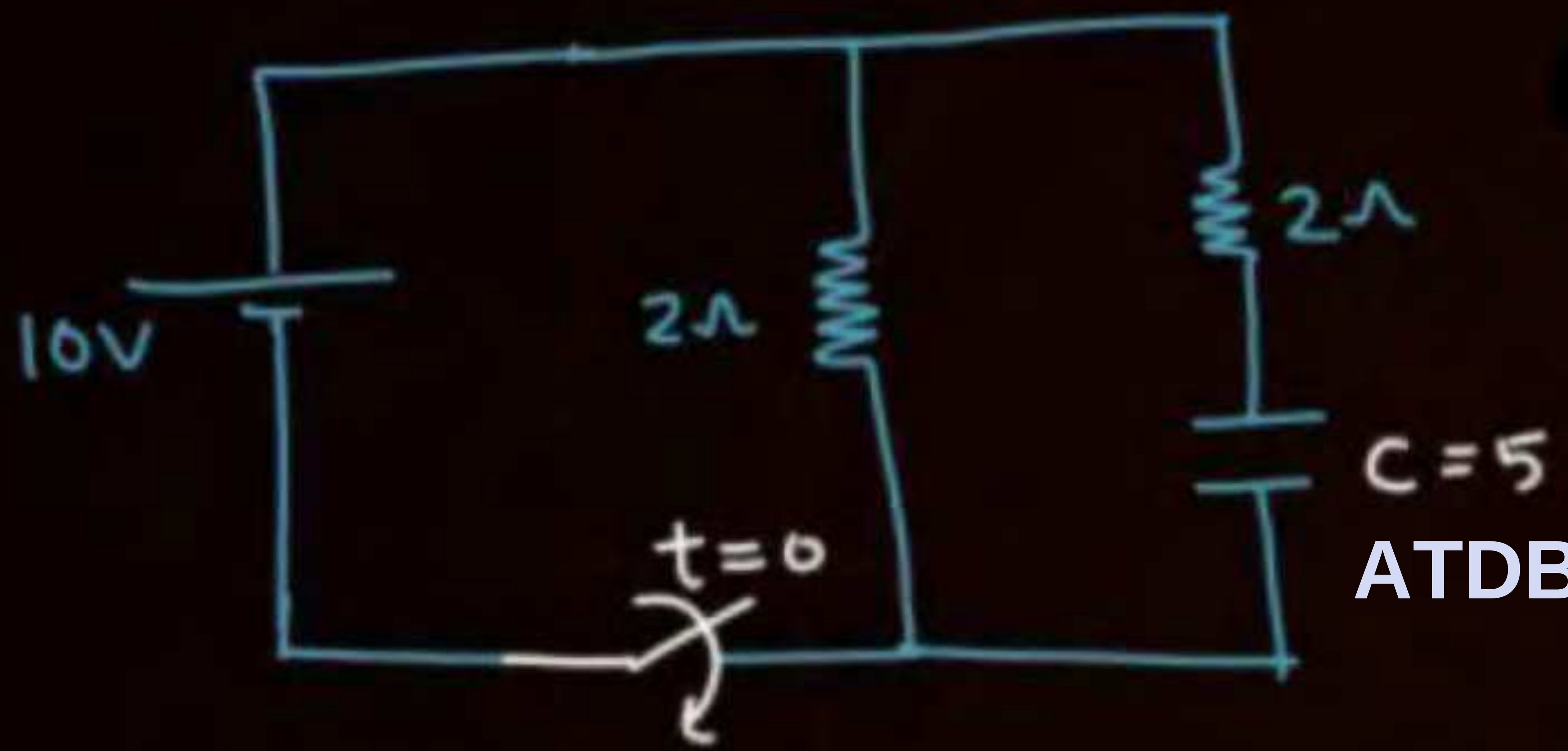
$U = \frac{1}{2}\frac{q^2}{C}$

Force on one plate $\longrightarrow$ Same

ATDB.uno

Q Find current in each resistance at $t=0$ & $t=\infty$

1



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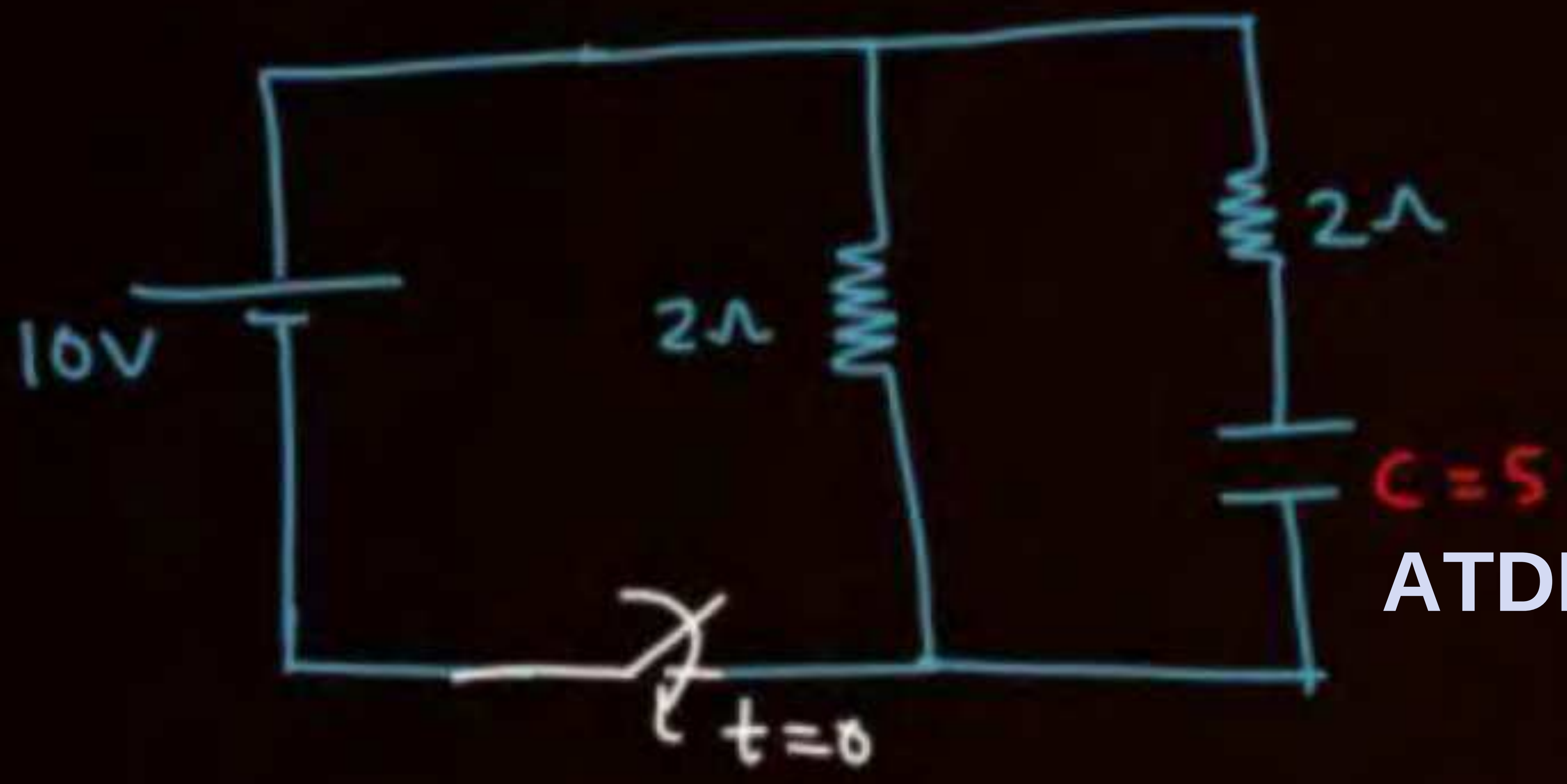
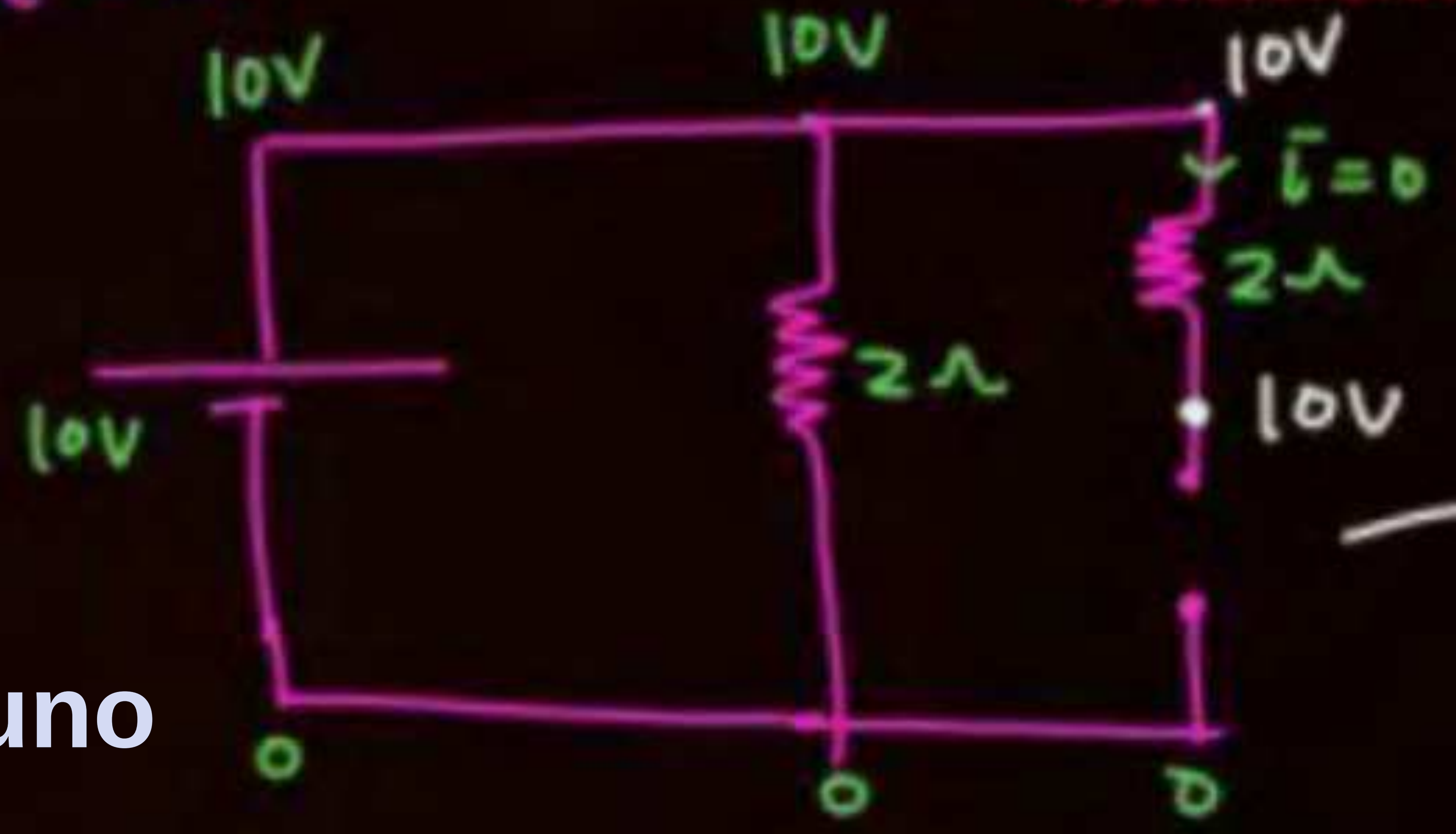
(b) Find charge on capacitor at $t=\infty$

$t=\infty \rightarrow$ After very long time

$\rightarrow$ At steady state

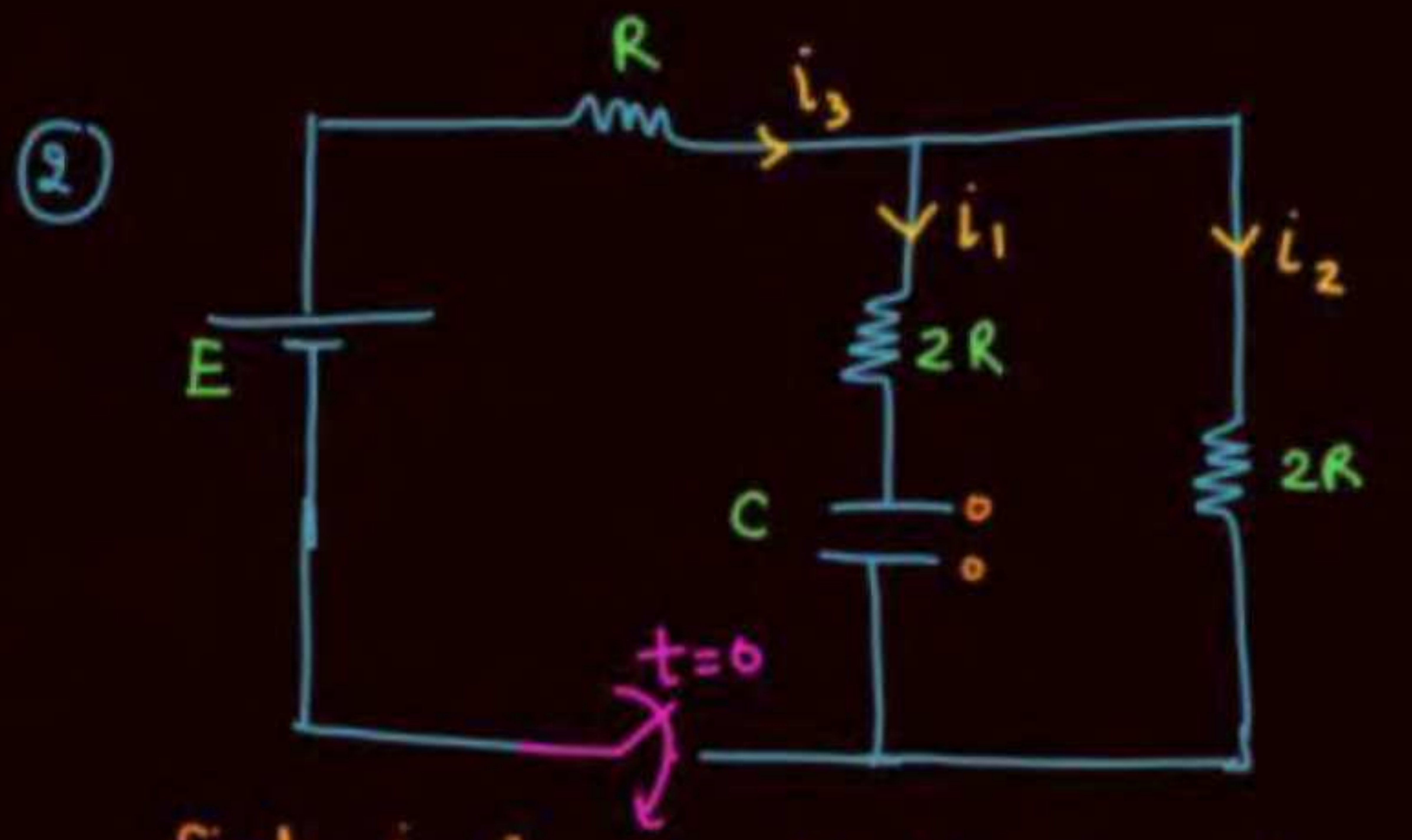


$t=\infty$



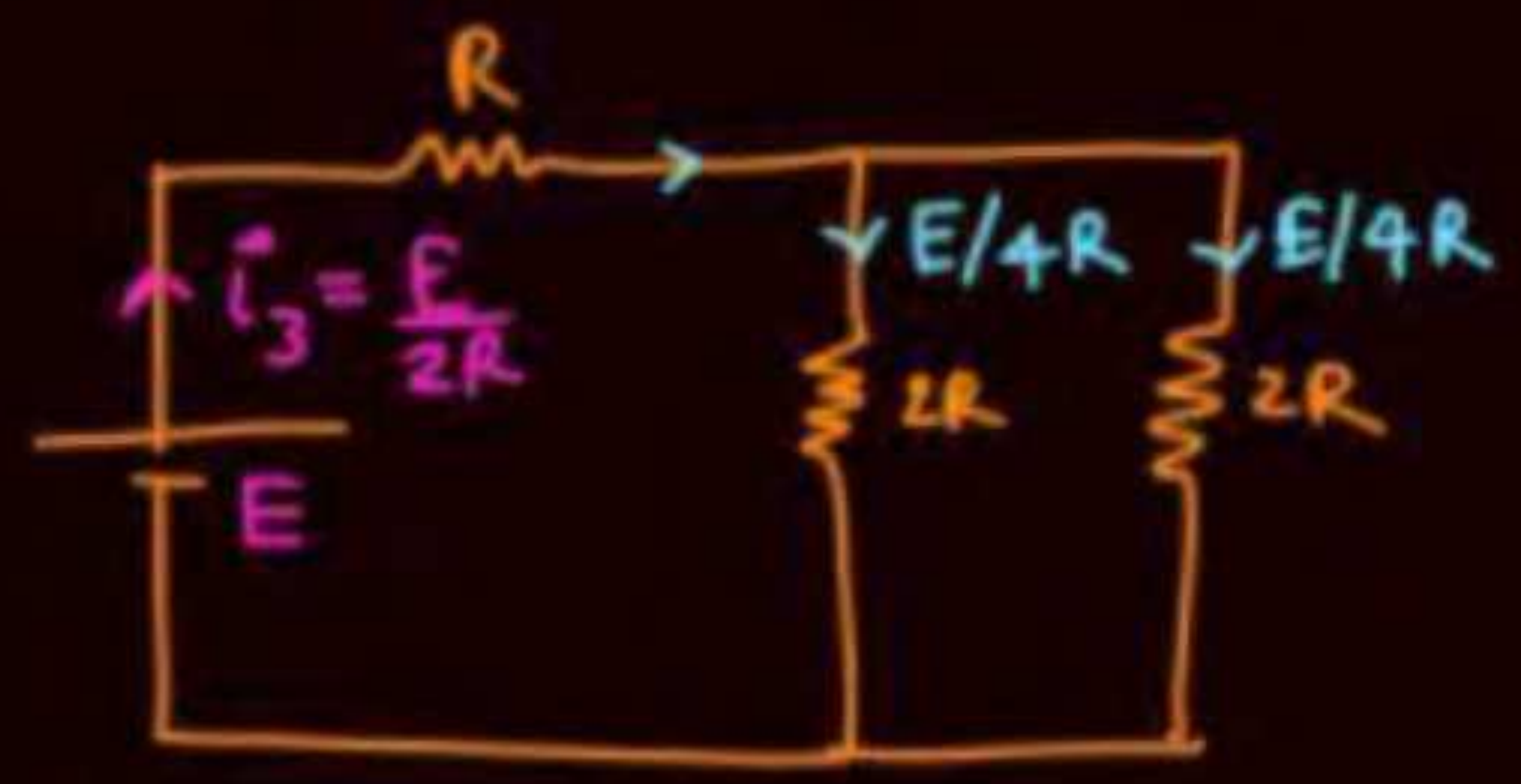
यहाँ भी Cap है

$$q = CV = 5\times 10 = 50$$

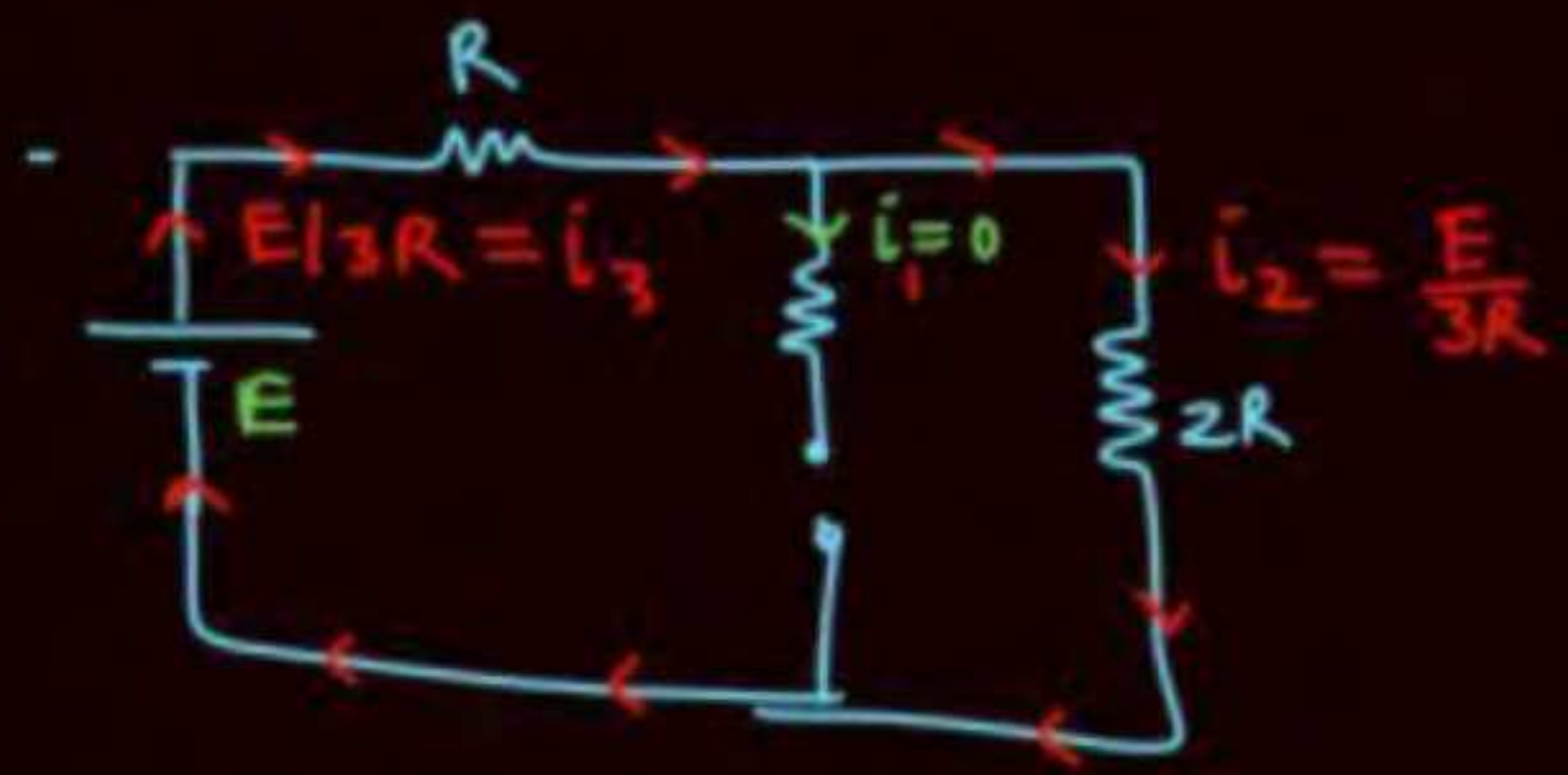


find i_1, i_2, i_3, q on cap at $t=0$ & $t=\infty$

t=0

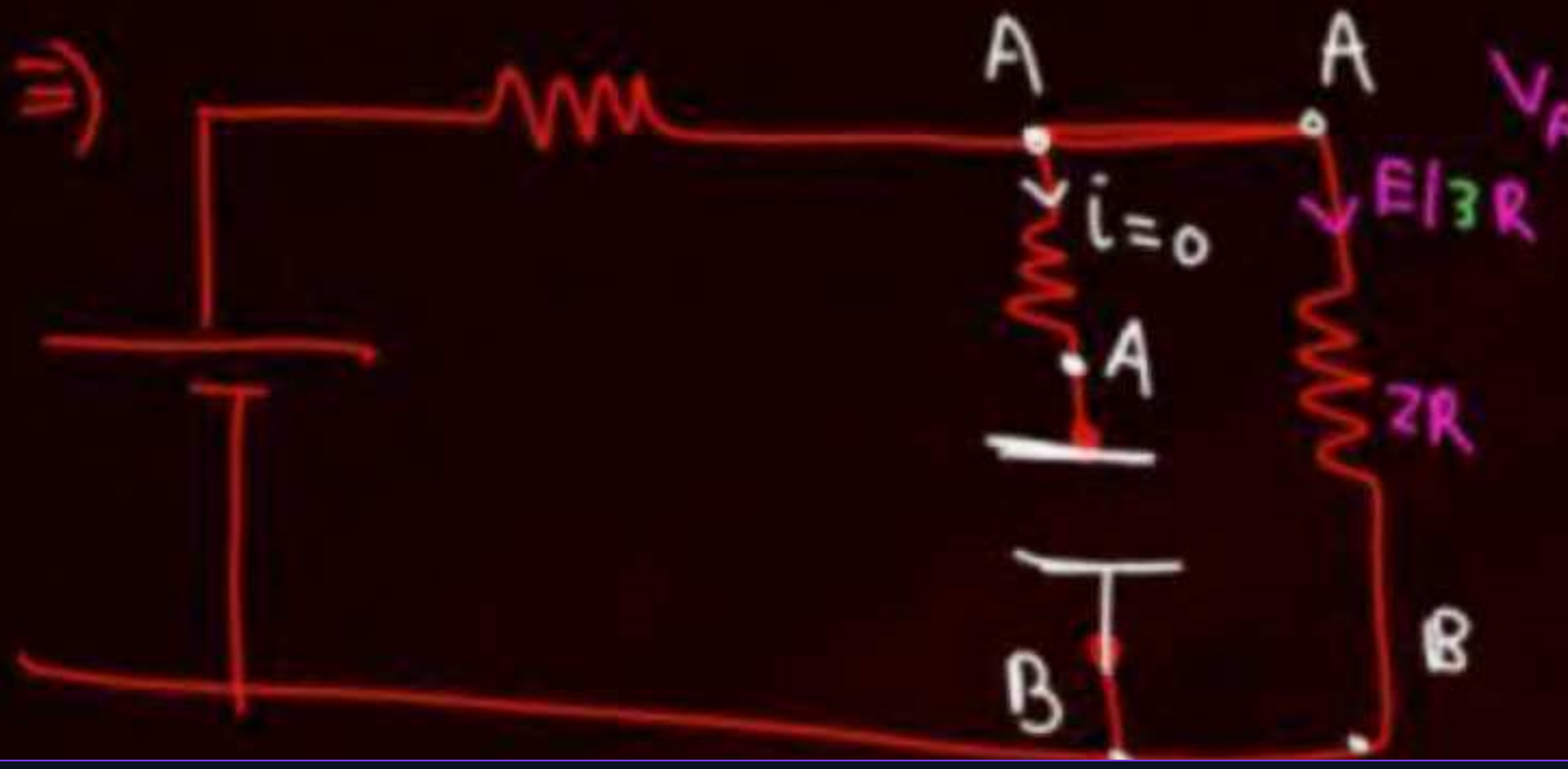


t=∞

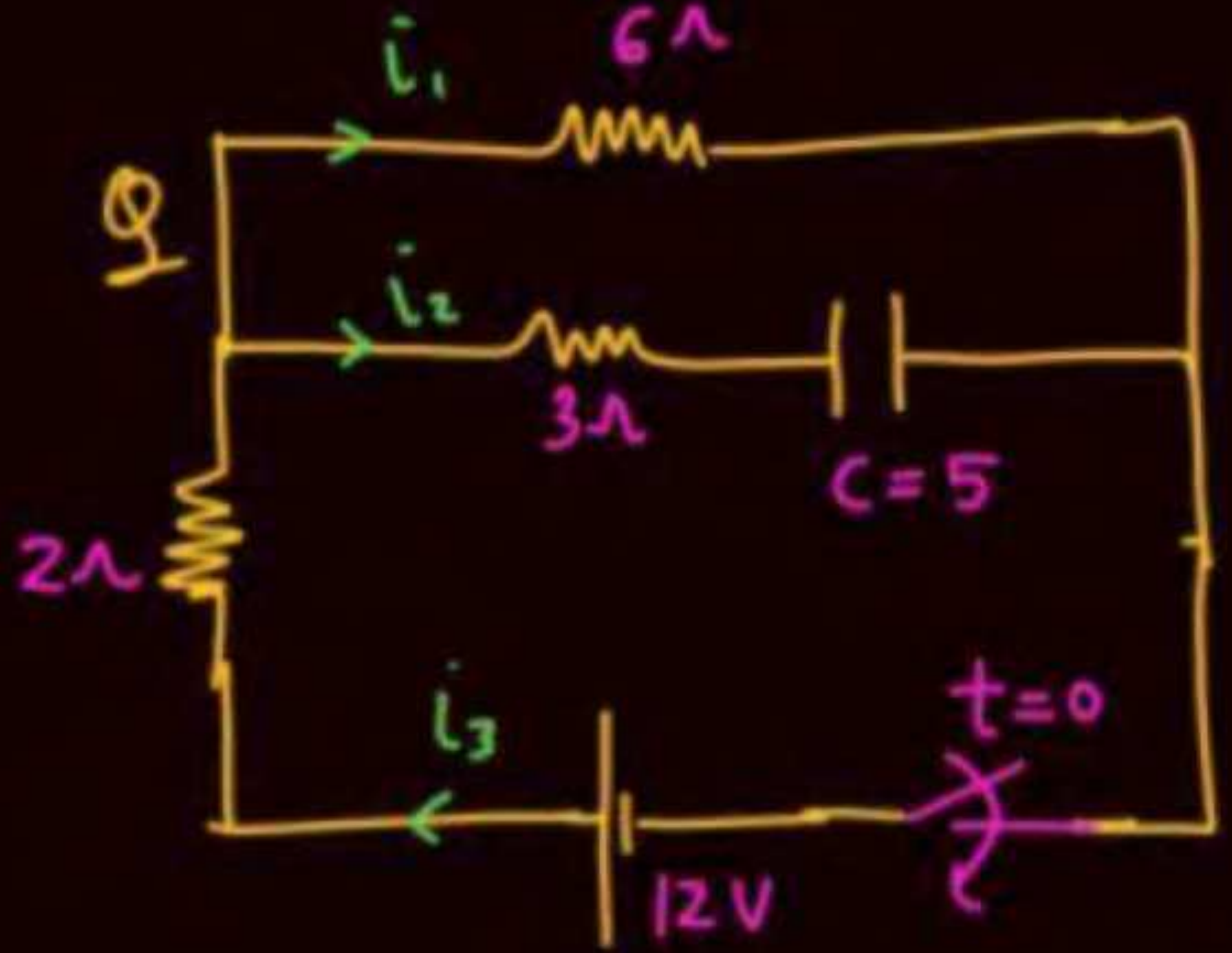


$t=\infty$ पर charge on Capacitor ⇒

$$q = CV_{AB} = C \cdot \frac{2E}{3}$$

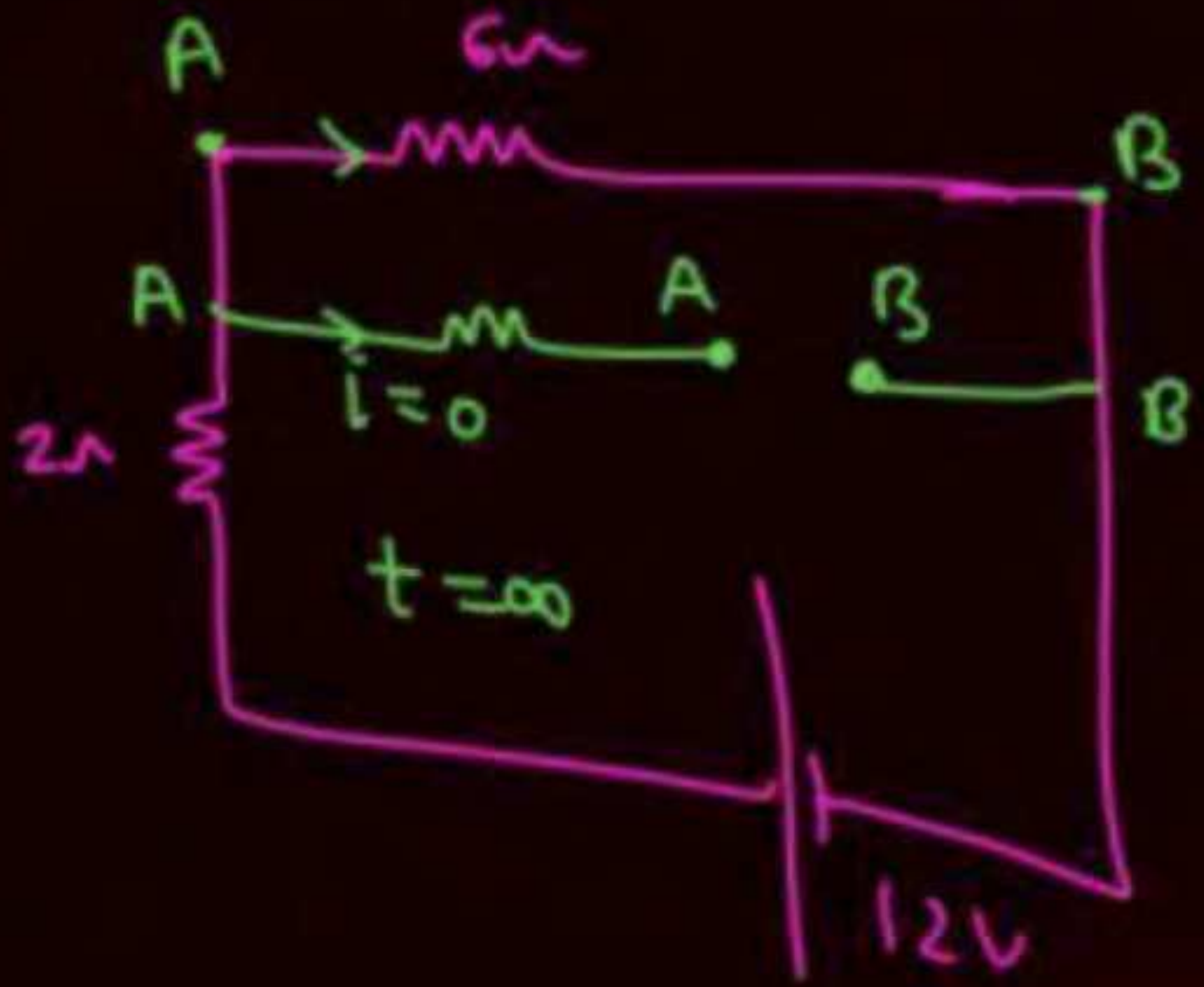


$$V_{AB} = \frac{E}{3R} \cdot 2R = \frac{2E}{3}$$



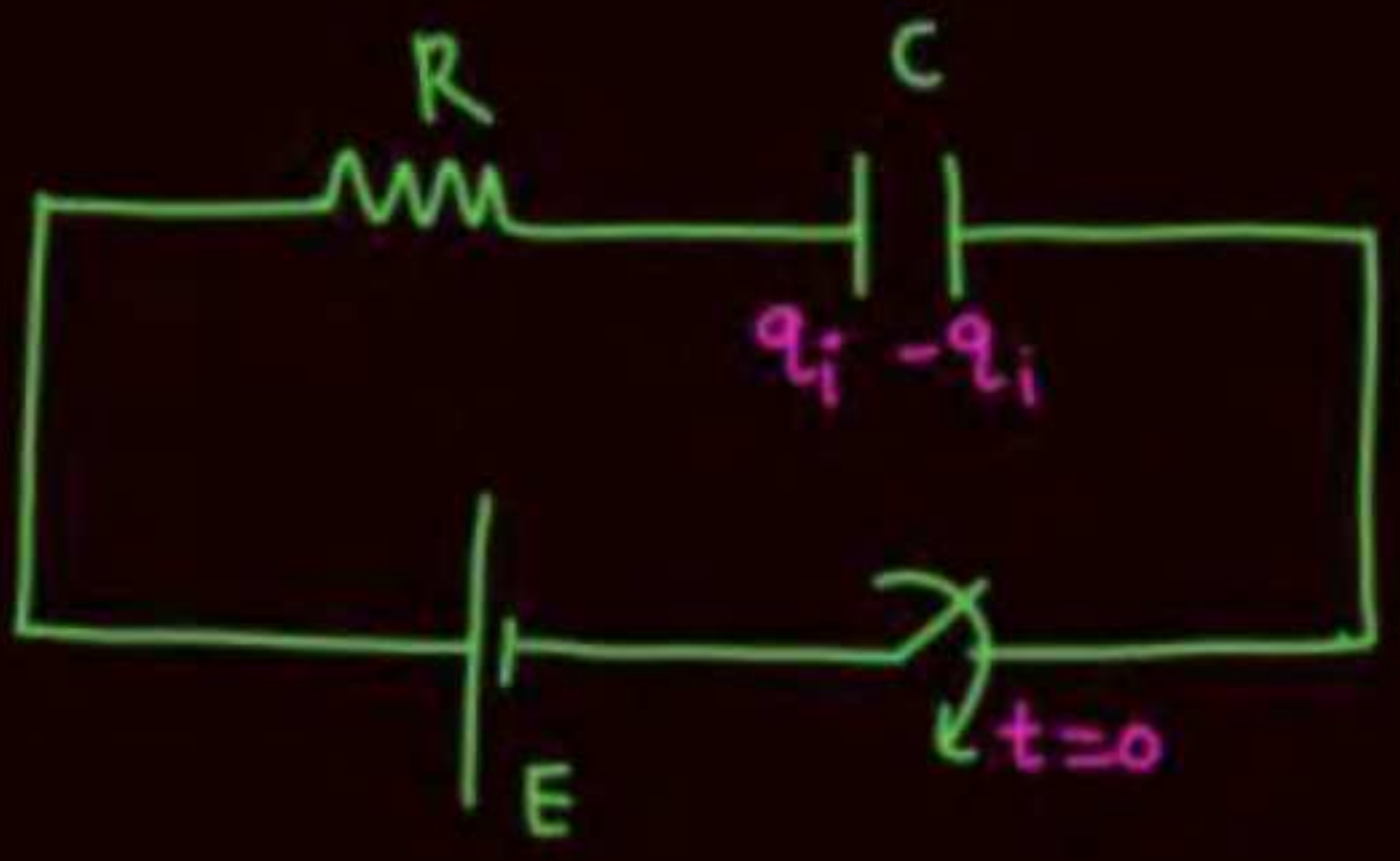
	i_1	i_2	$i_{net} = i_3$
$t=0$	1	2	3
$t=\infty$	$\frac{12}{8}$	0	$\frac{12}{8}$

Charge on cap. at $t=\infty$

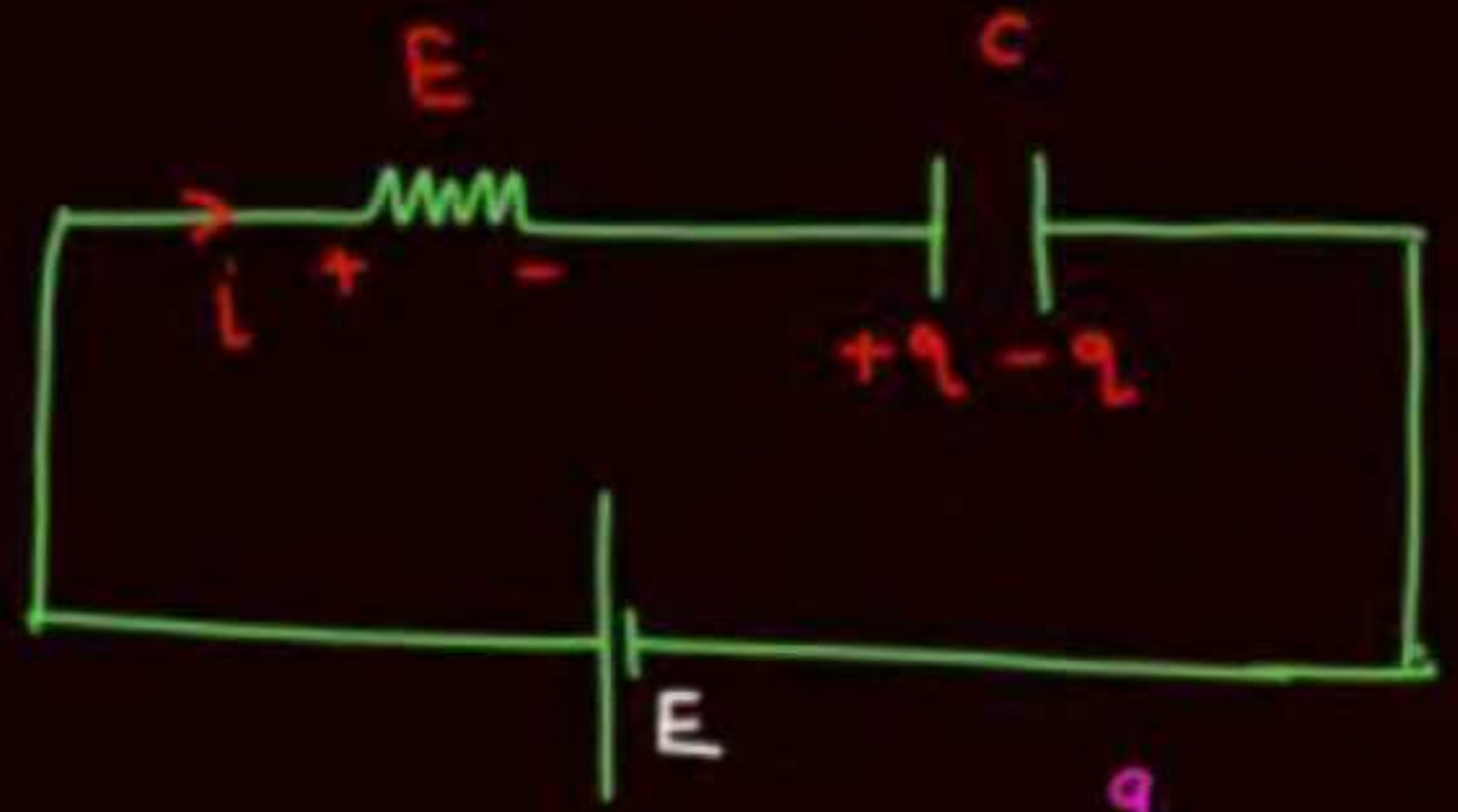


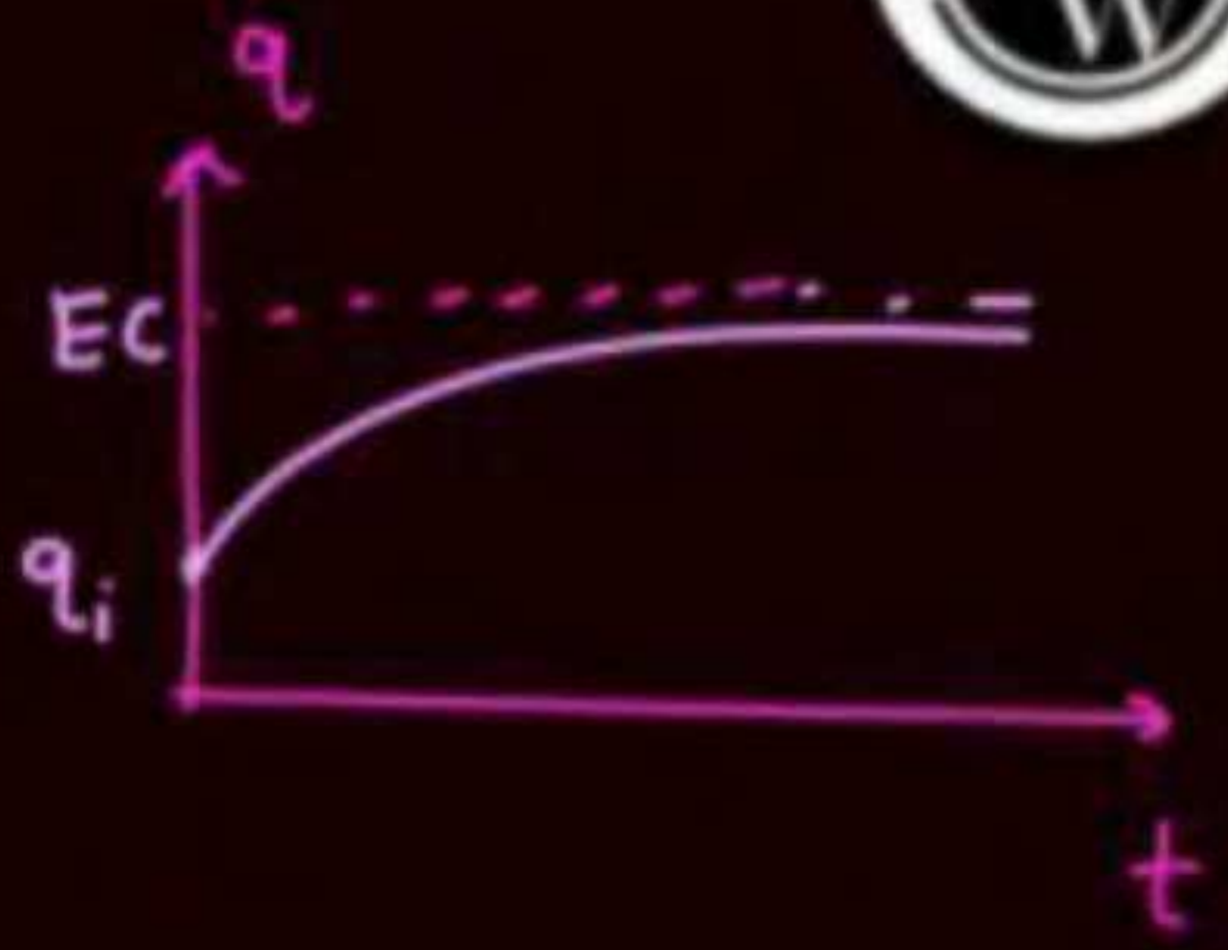
$$q = 5 \times 9 = 45$$

$$V_{AB} = \frac{12}{8} \times 6 = 9$$



$\Rightarrow$





$$E - iR - \frac{q}{C} = 0$$

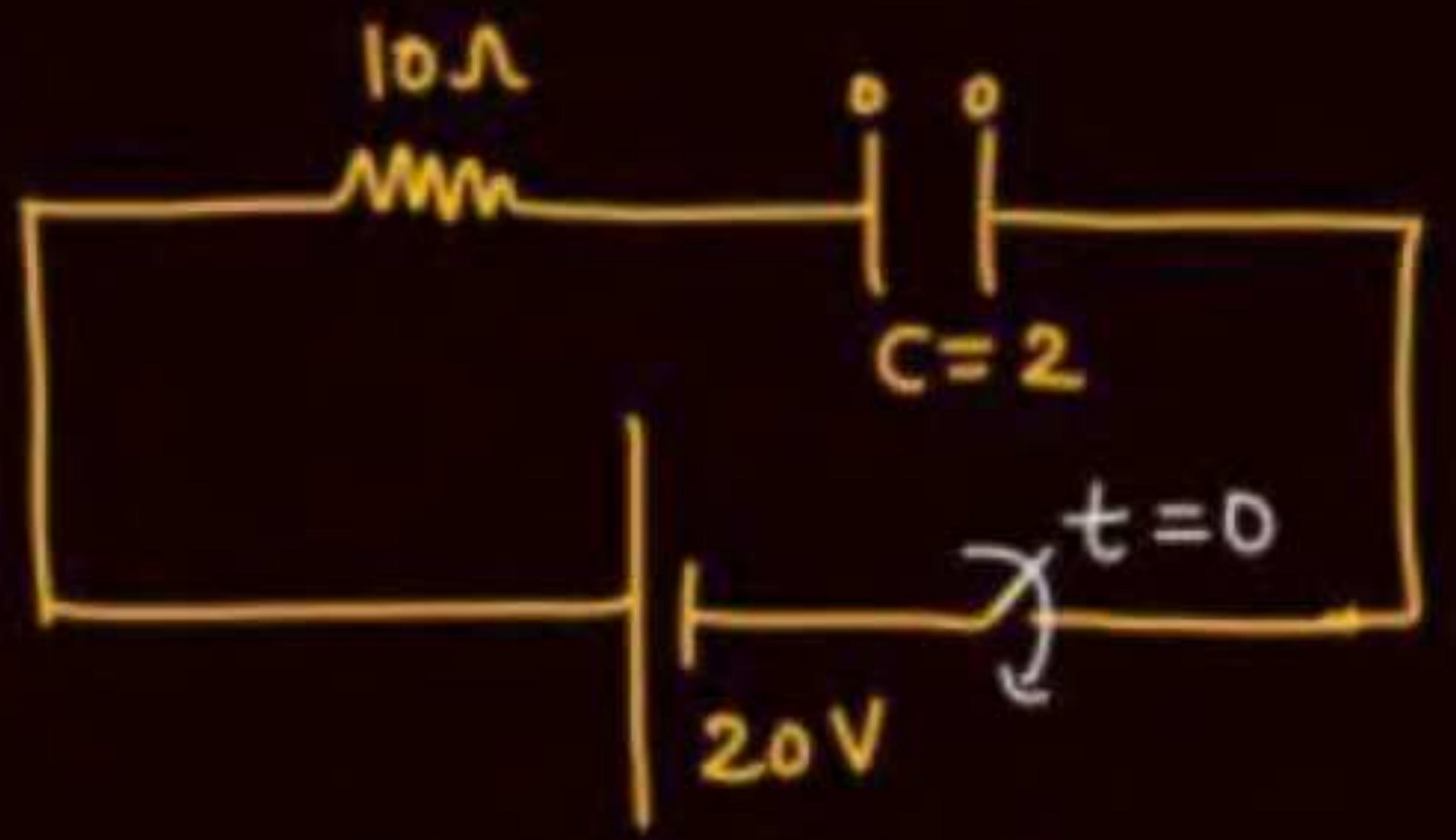
$$EC - iRC - q = 0$$

$$EC - q = \frac{dq}{dt} RC$$

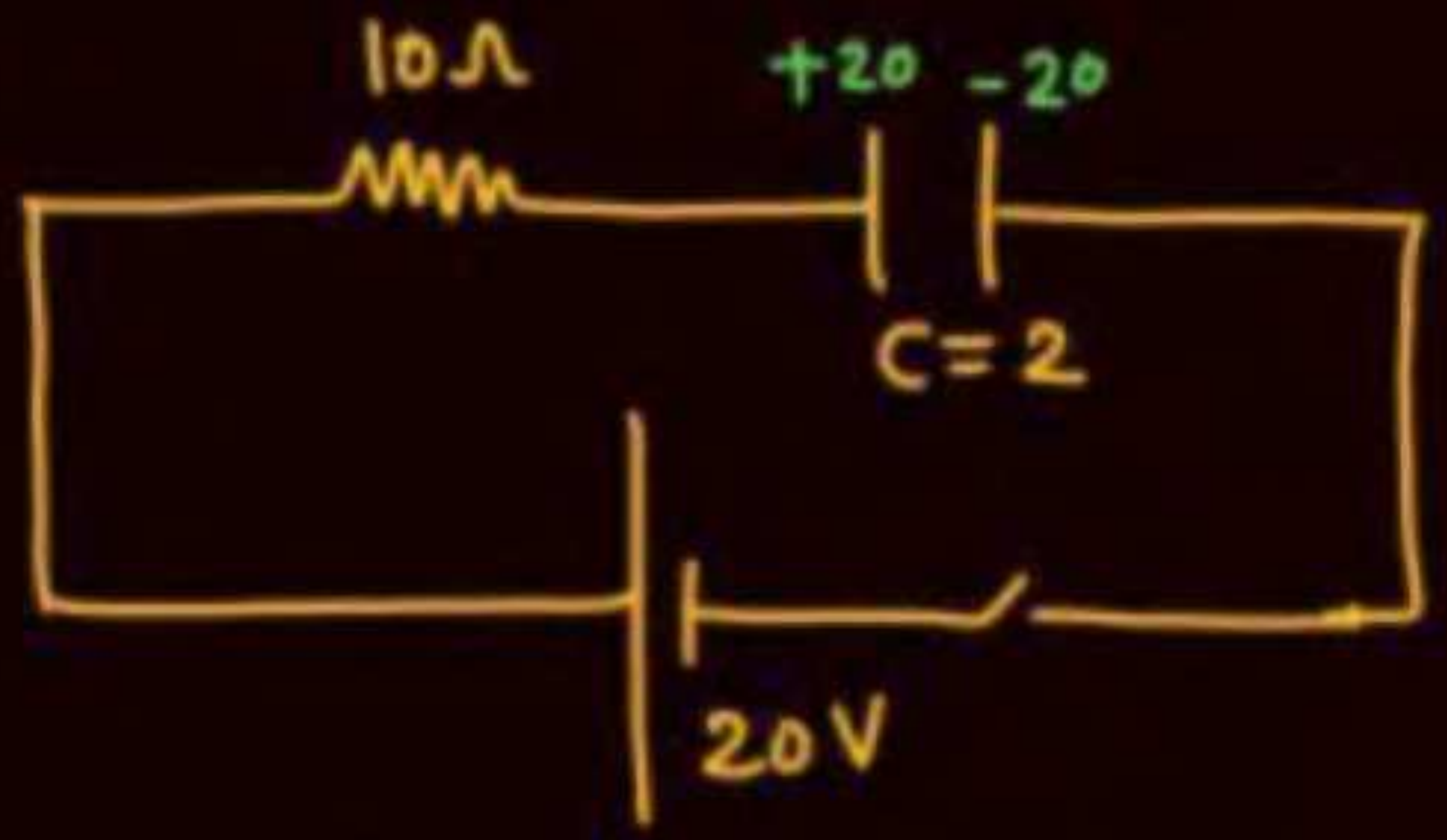
$$\int_{q_i}^{q} \frac{dq}{EC-q} = \int_{0}^{t} \frac{dt}{RC}$$

$$\boxed{q - q_i = (EC - q_i)\left(1 - e^{-t/RC}\right)}$$

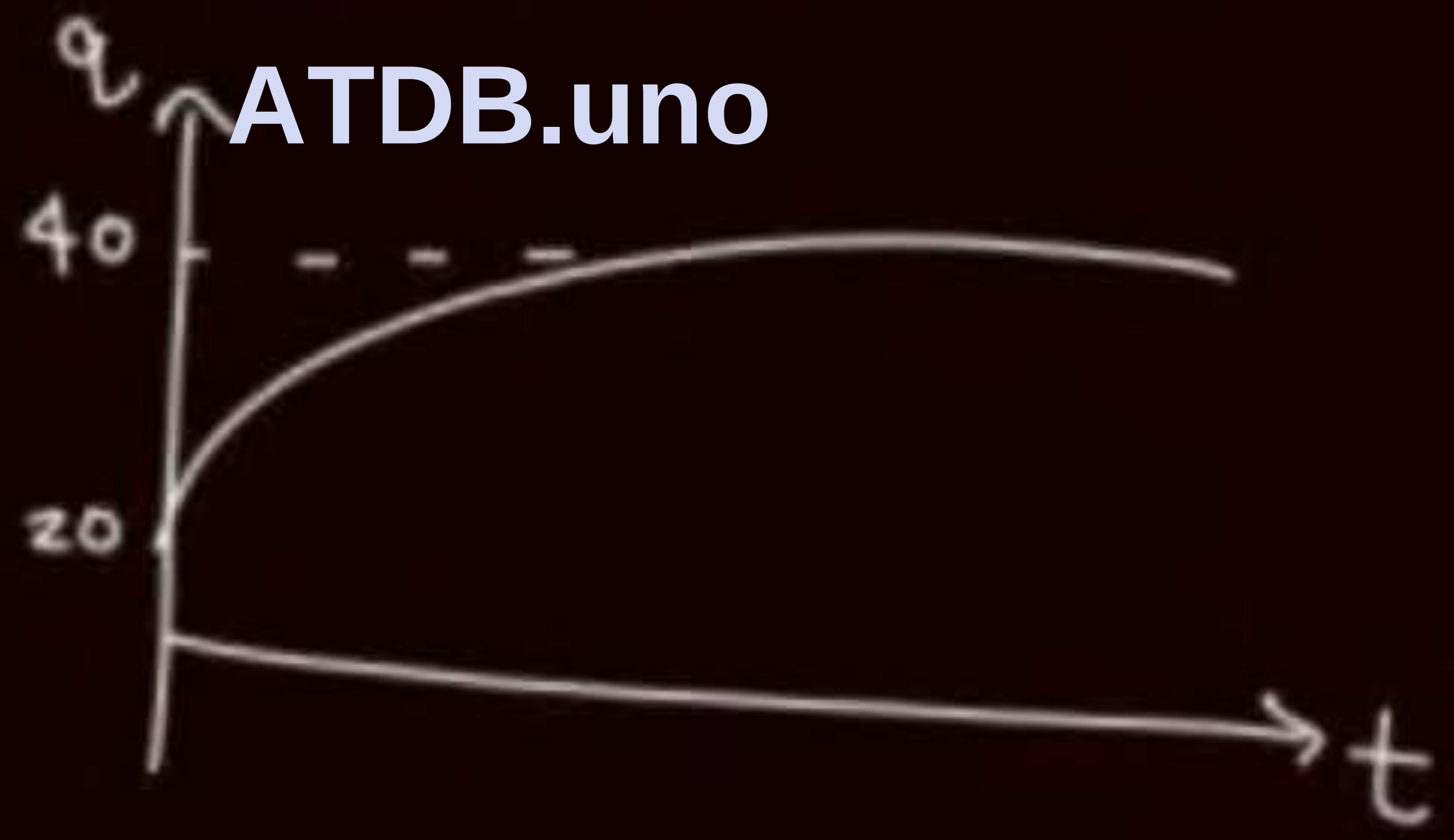
Q

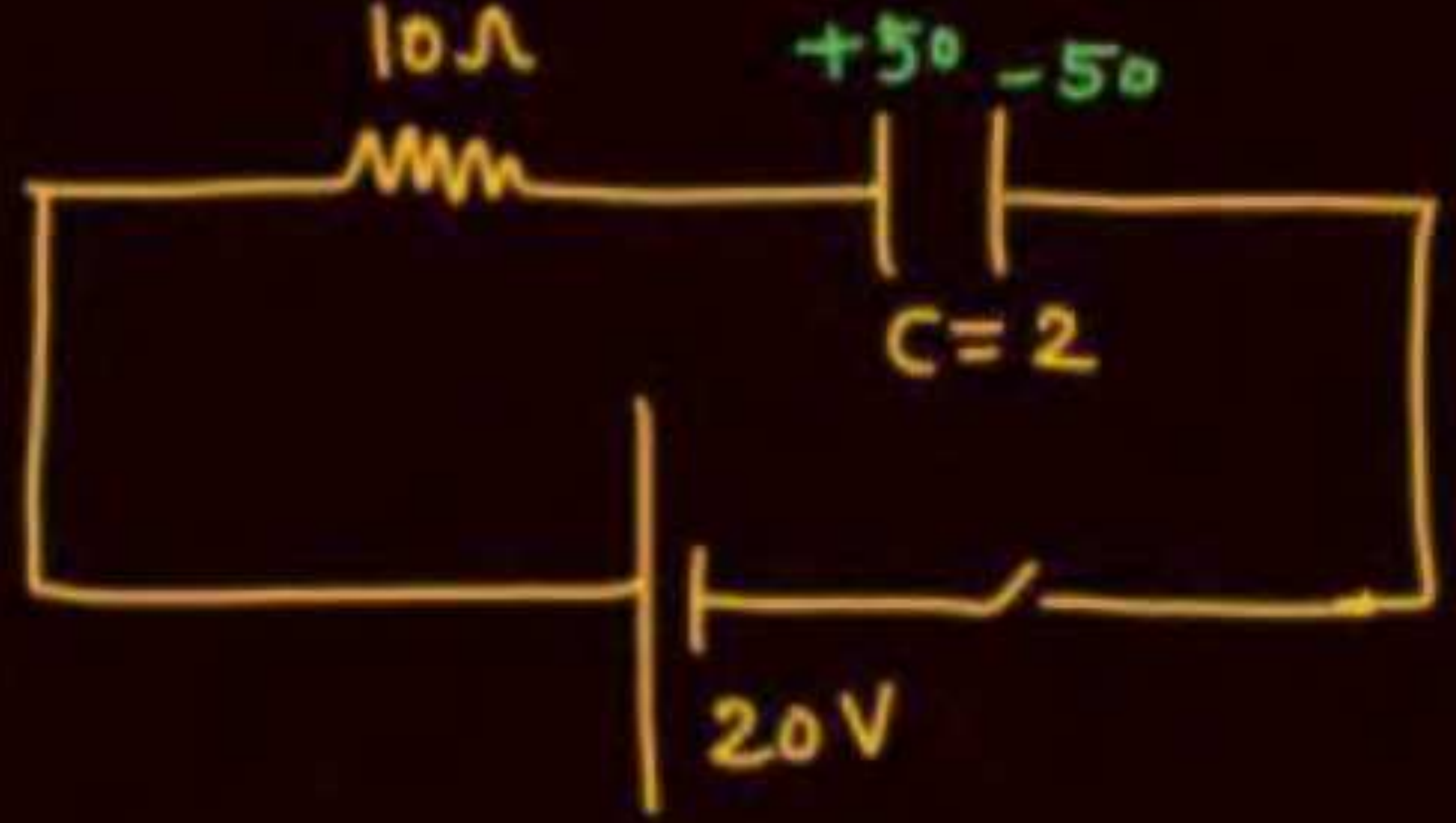


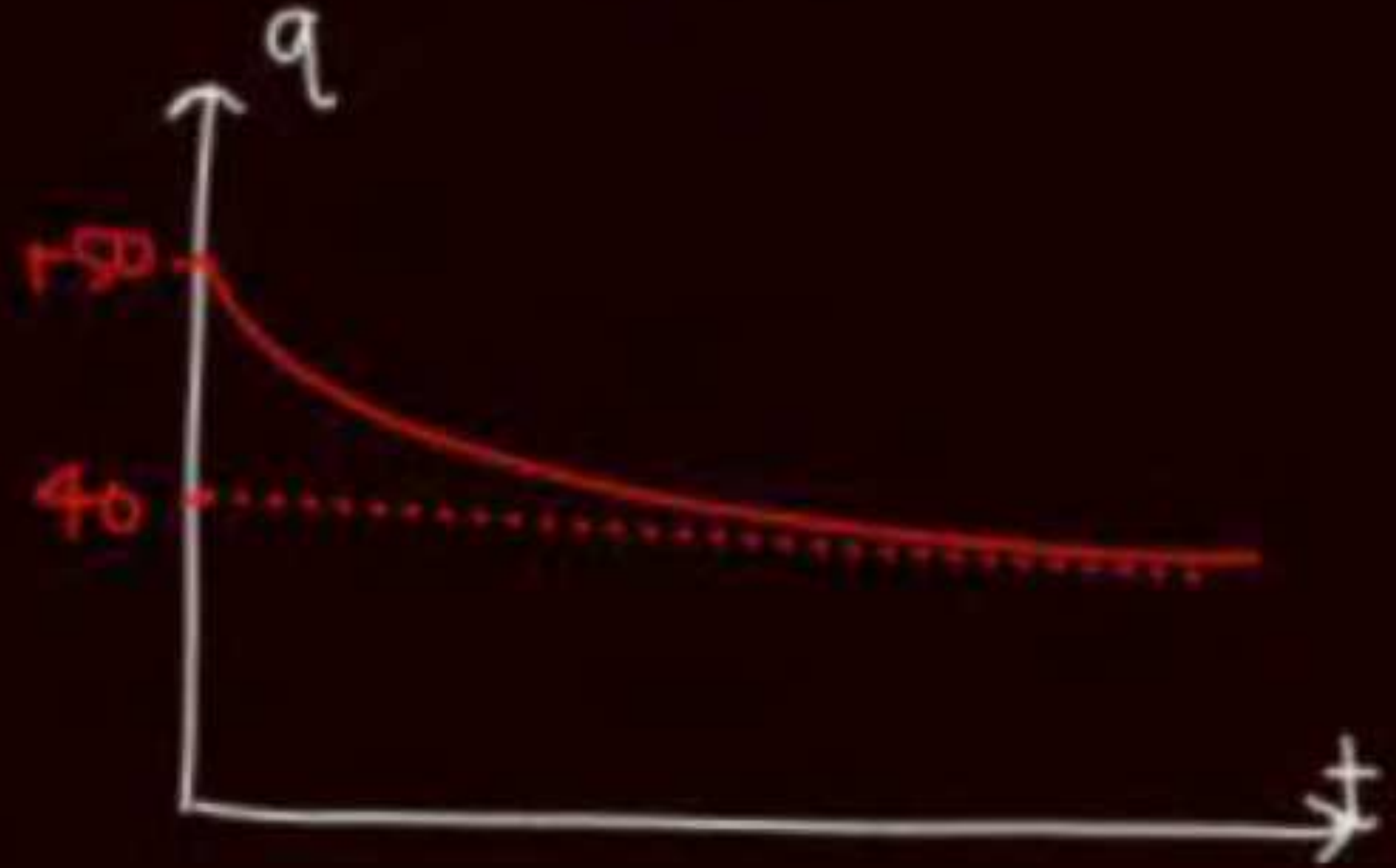
$Q_{max} = EC = 40$



$Q_{max} = 40$

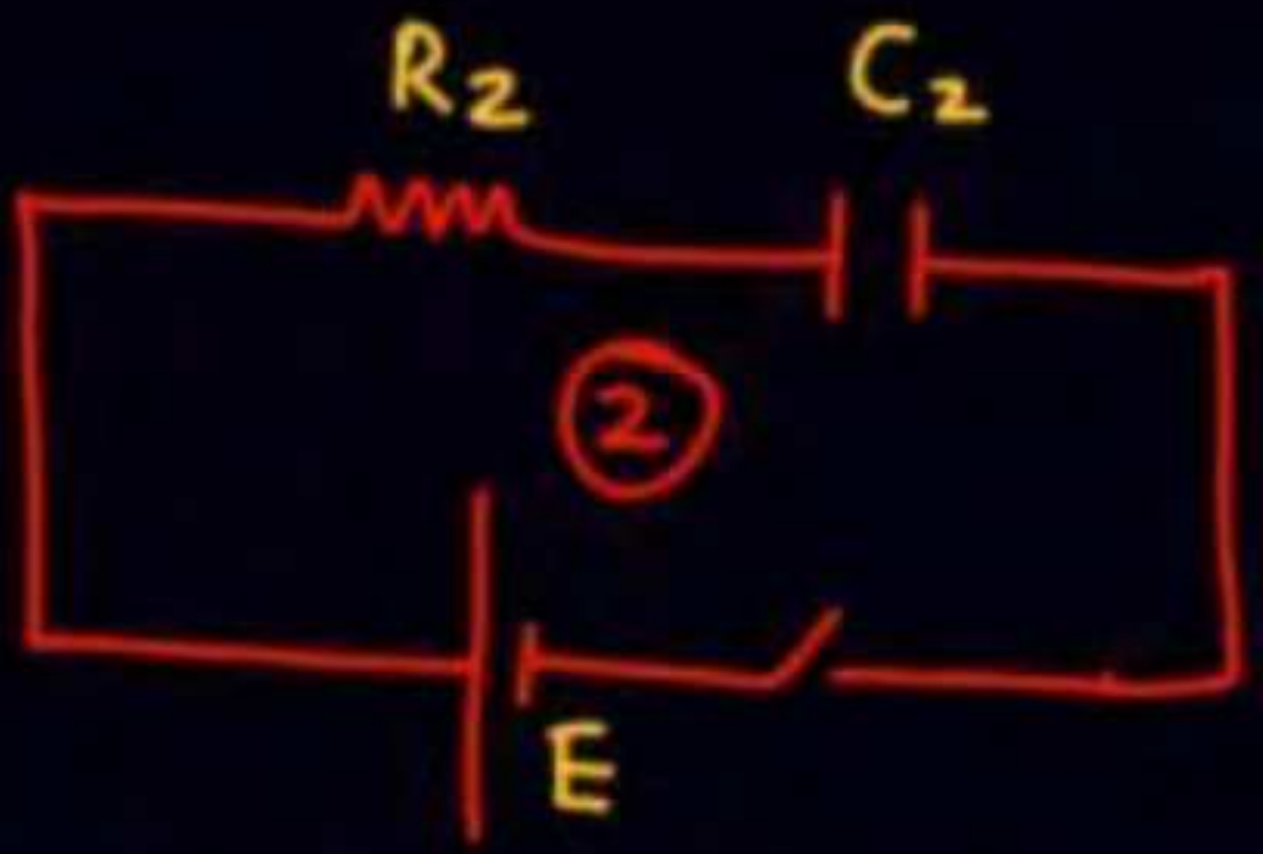


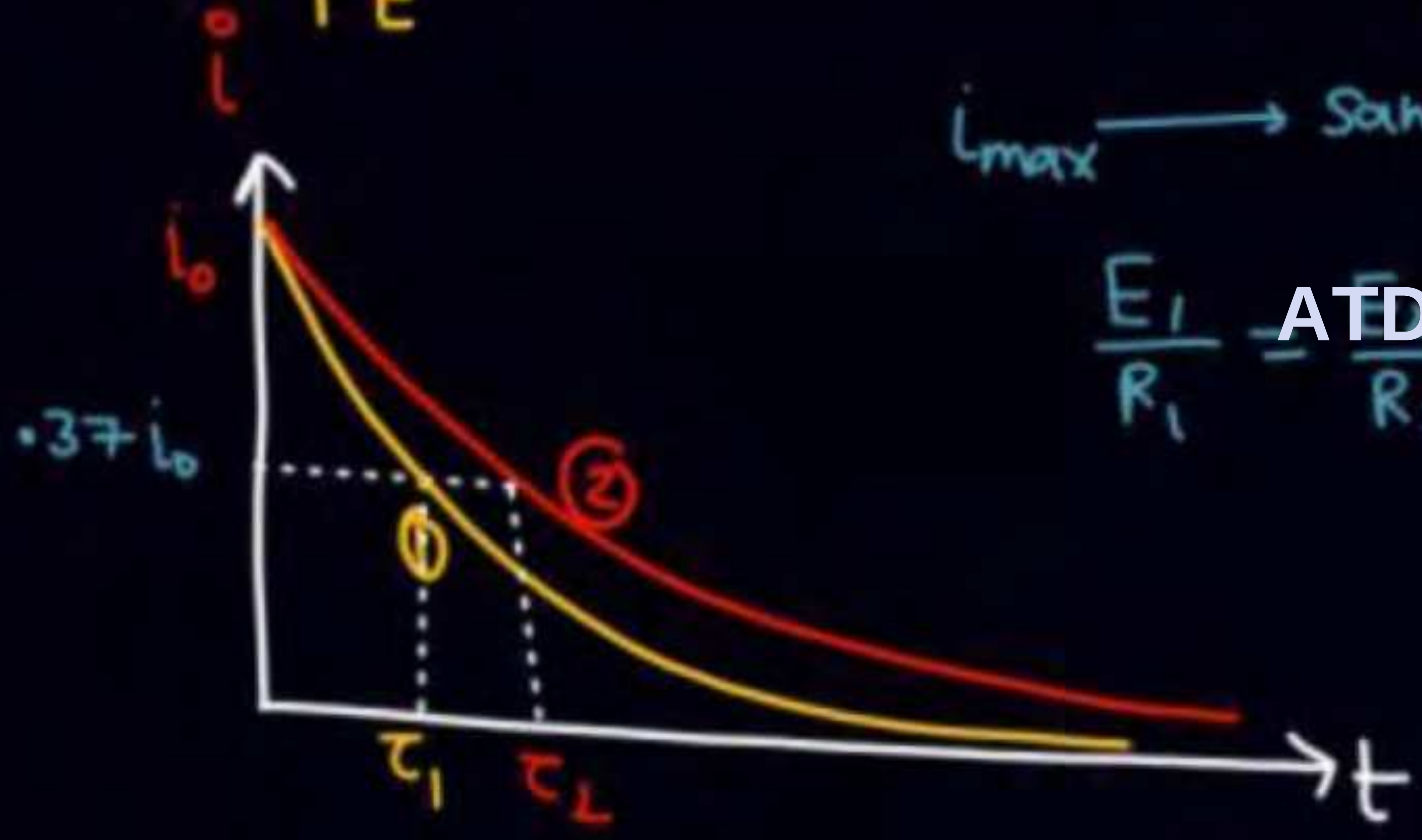




$$q - 10 = (40 - 10)\left(1 - e^{-t/10}\right)$$

Q





$i_{max} \longrightarrow$ same $= \frac{E}{R}$

$\frac{E_1}{R_1} = \frac{E_2}{R_2}$ $(E_1 = E_2)$

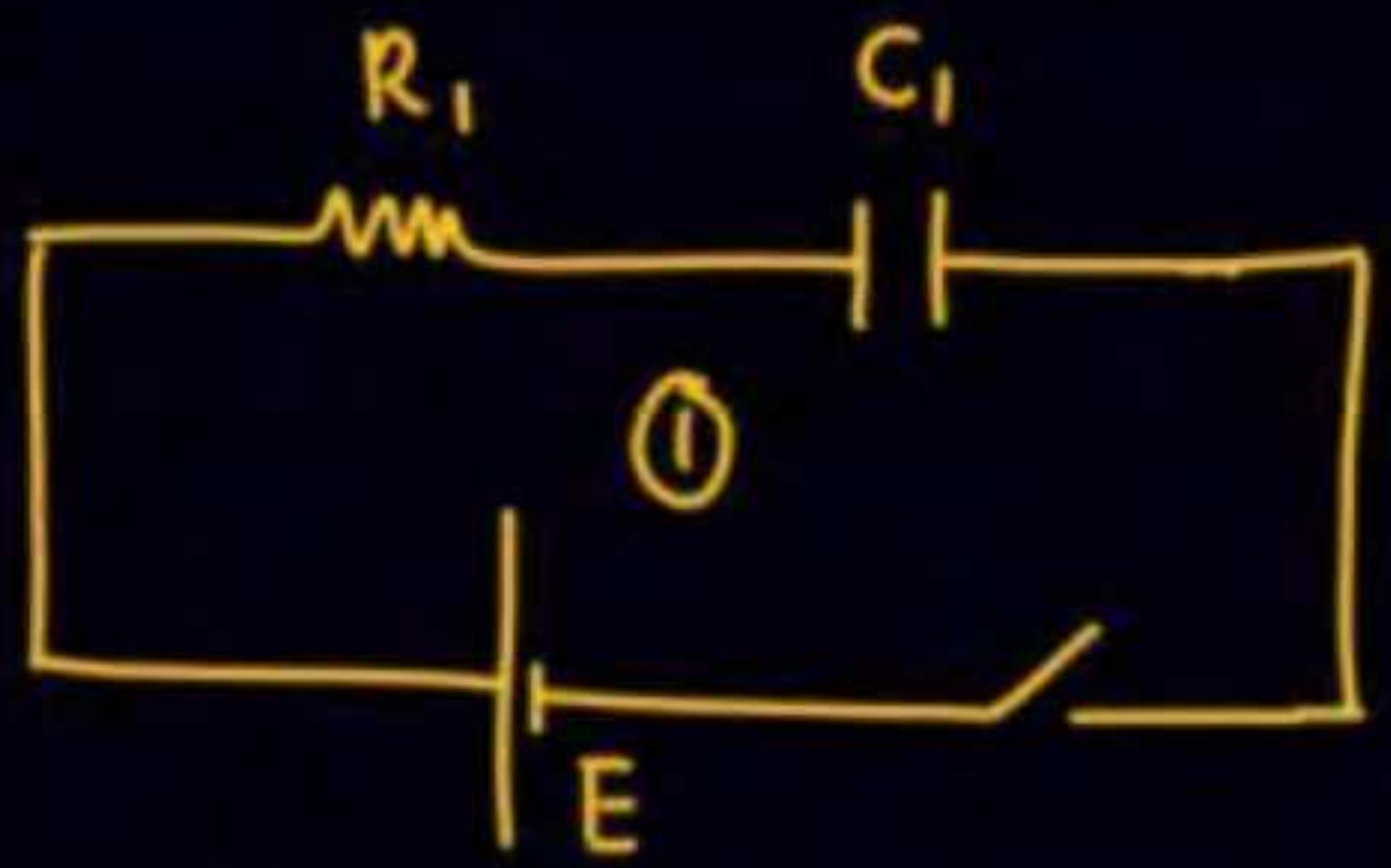
$\boxed{R_1 = R_2}$

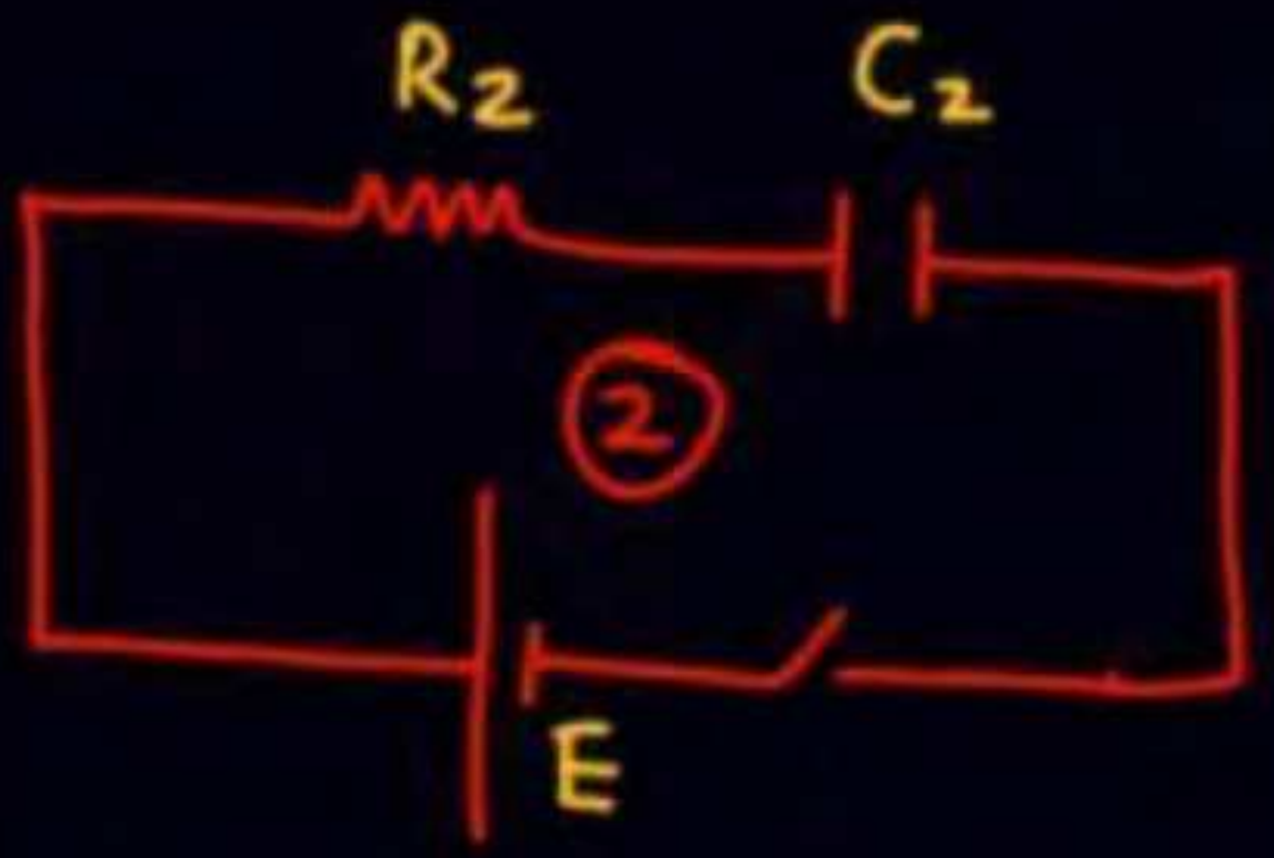
$\tau_2 > \tau_1$,

$R_2C_2 > R_1C_1$

$C_2 > C_1$

Q



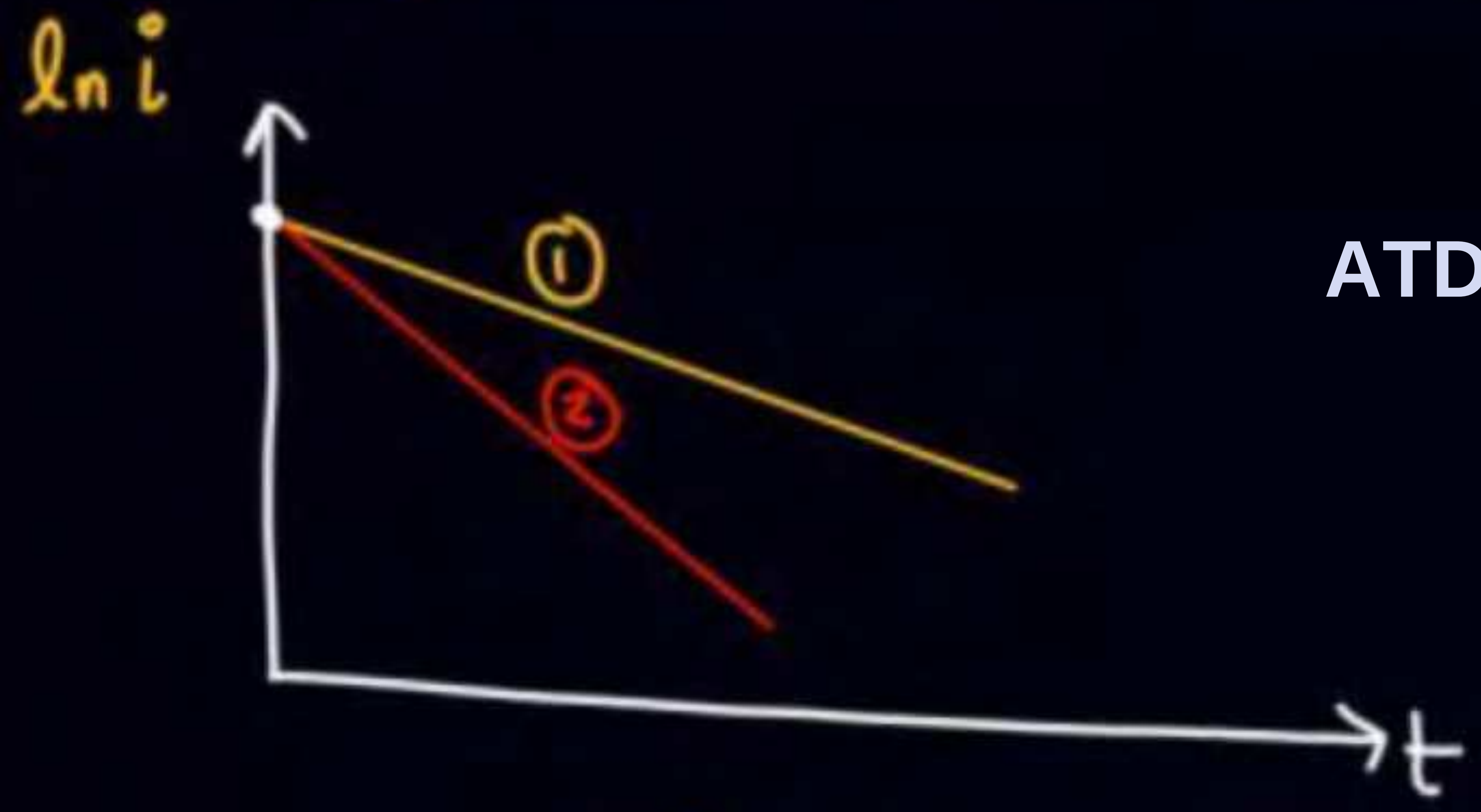


$$i = \frac{E}{R} e^{-t/\tau}$$

$$\ln i = \ln \frac{E}{R} - \frac{t}{\tau}$$

$$\ln i = -\frac{1}{\tau} t + \ln \frac{E}{R}$$

$$y = -mx + C$$



$t = 0$, $\ln i \longrightarrow$ same

$\frac{E}{R} \longrightarrow$ same $(R_1 = R_2)$

$E_1 = E_2$ (same)

$|\text{Slope}|_2 > |\text{Slope}|_1$

$\left(\frac{1}{\tau_2}\right) > \left(\frac{1}{\tau_1}\right)$

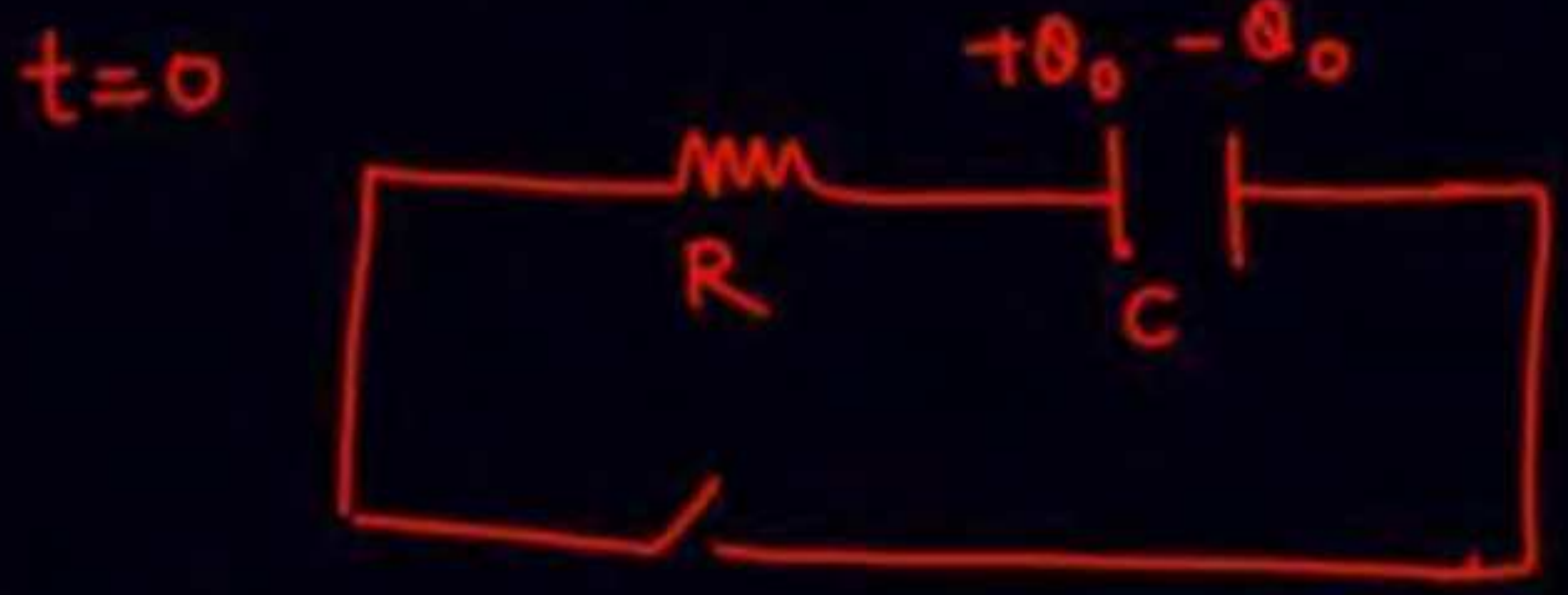
$\tau_2 < \tau_1$

$\boxed{C_2 < C_1}$

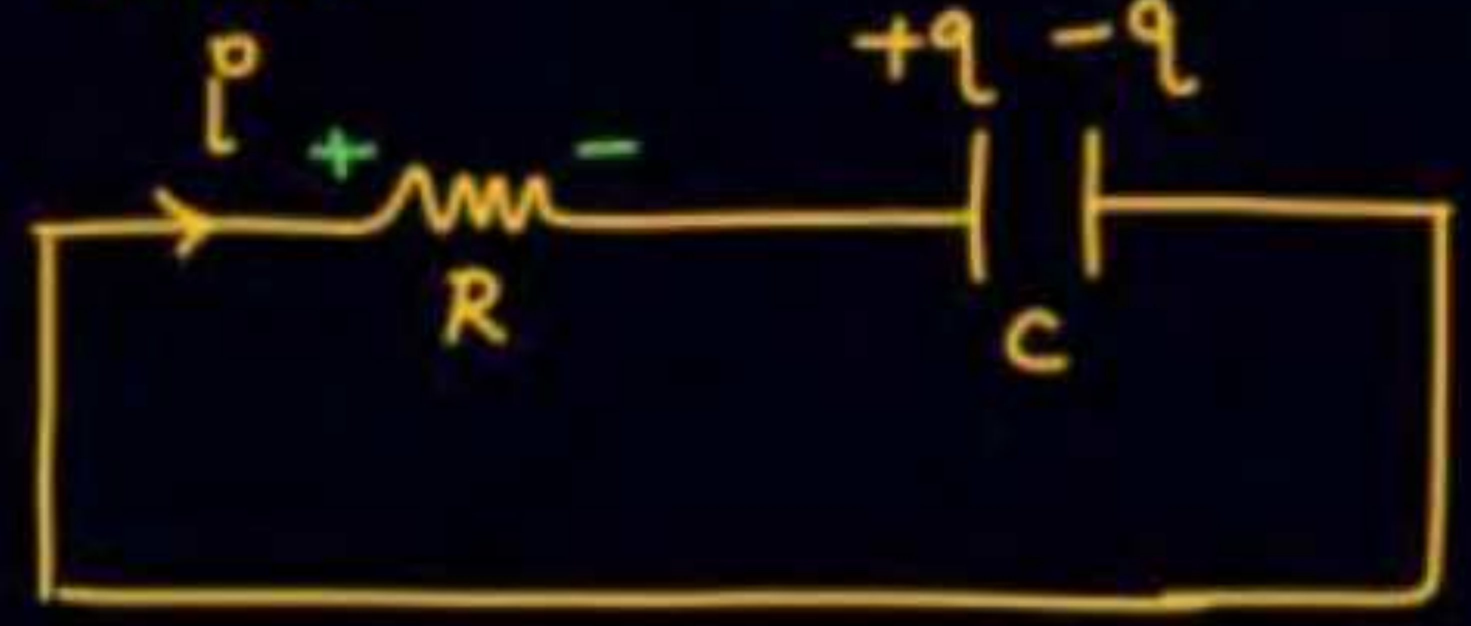
Discharging of Capacitor

$$q = Q_0 e^{-t/RC}$$

$$V_C = V_R$$



⇒ (let)



$$i = \left|\frac{dq}{dt}\right| = \left|\frac{Q_0}{RC} e^{-t/RC}\right|$$

magnitude

find $q = f(t)$ on capacitor

$$\boxed{\begin{aligned} q &= Q_0 e^{-t/RC} = Q_0 e^{-t/\tau} \\ i &= i_0 e^{-t/RC} \end{aligned}}$$

$$-iR - \frac{q}{C} = 0$$

$$iR = -\frac{q}{C}$$

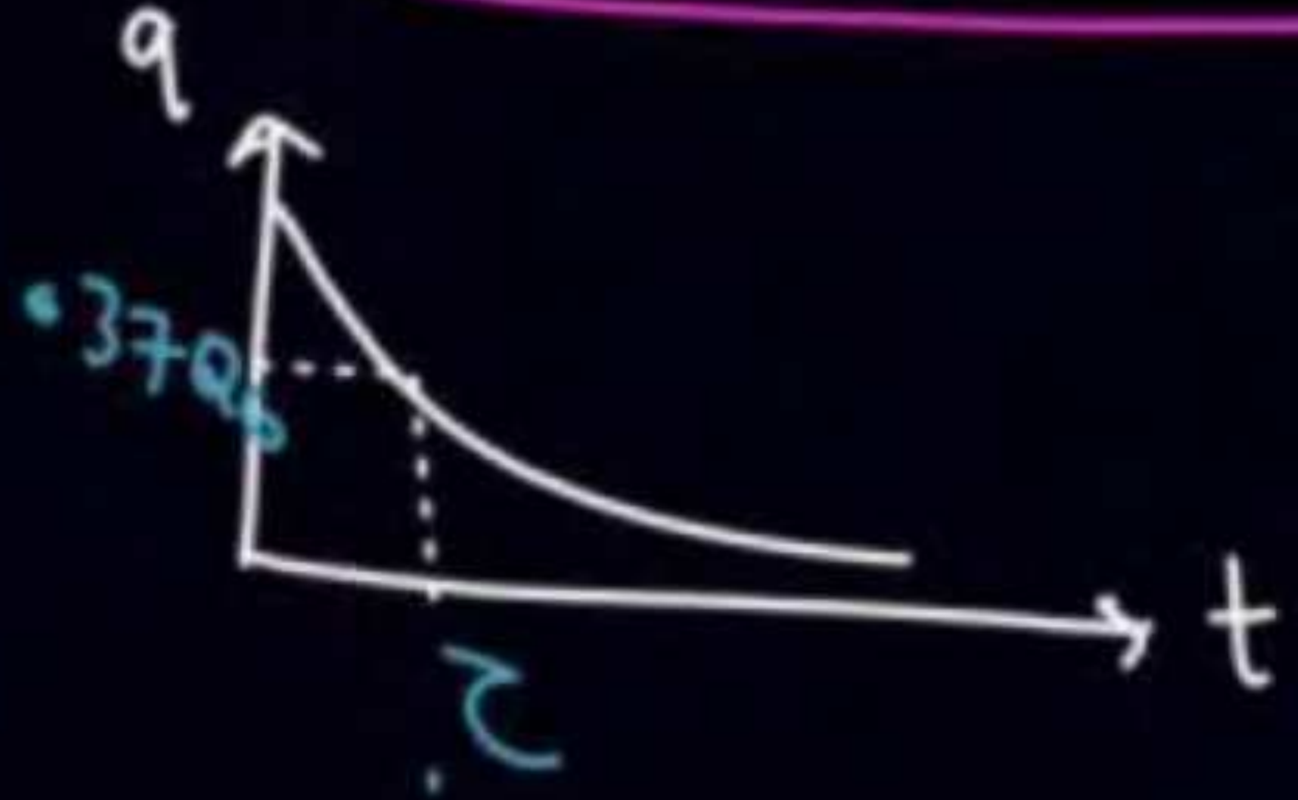
$$\frac{dq}{dt} = -\frac{q}{RC}$$

$$\int_{Q_0}^{q} \frac{dq}{q} = \int_0^t \frac{-dt}{RC}$$

$$\ln\frac{q}{Q_0} = \frac{-t}{RC}$$

$$\boxed{q = Q_0 e^{-t/RC}}$$

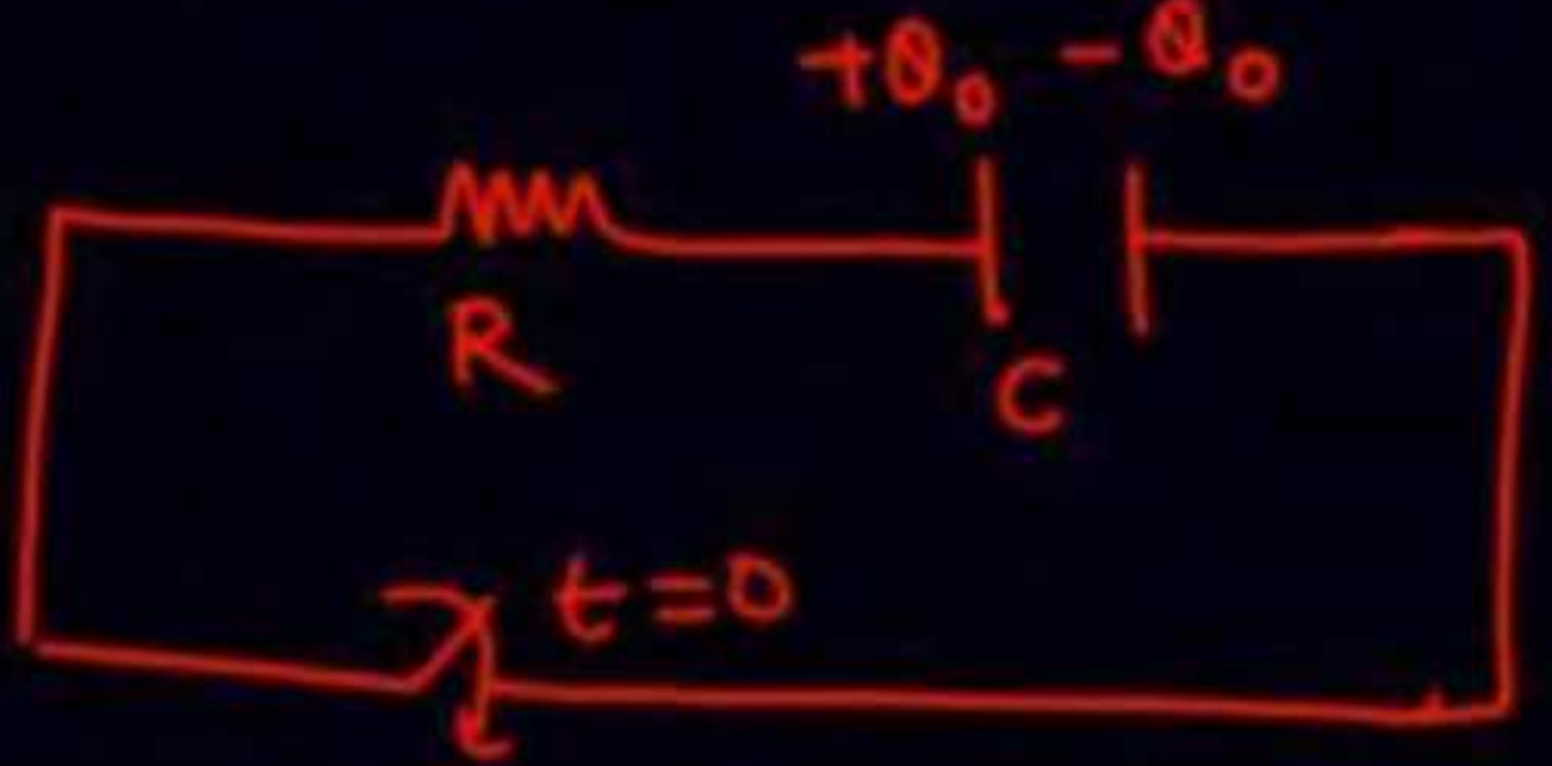
$t\uparrow \Rightarrow q\downarrow,\ U\downarrow$

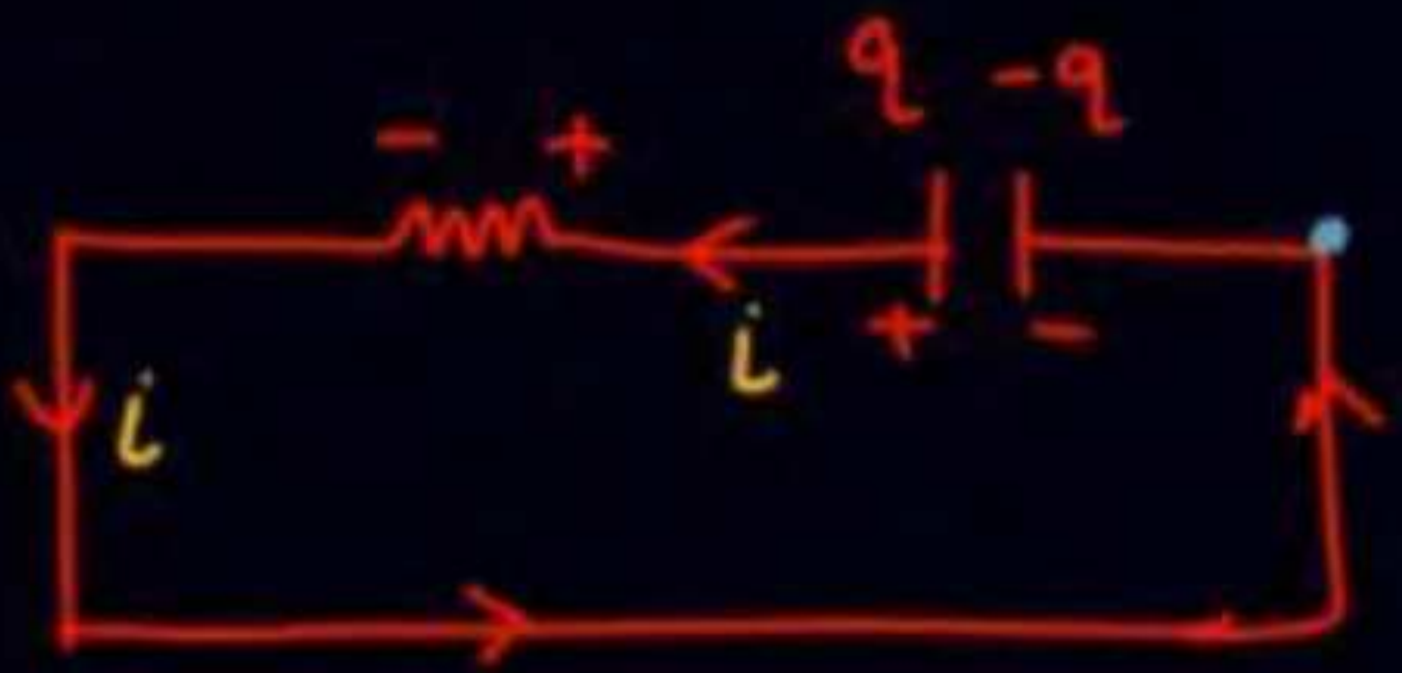


$$\tau = RC$$

Discharging of Capacitor

t=0





find $q = f(t)$ on capacitor

$$\frac{q}{C} - iR = 0$$

$$\frac{q}{C} = iR \qquad i = -\frac{dq}{dt}$$

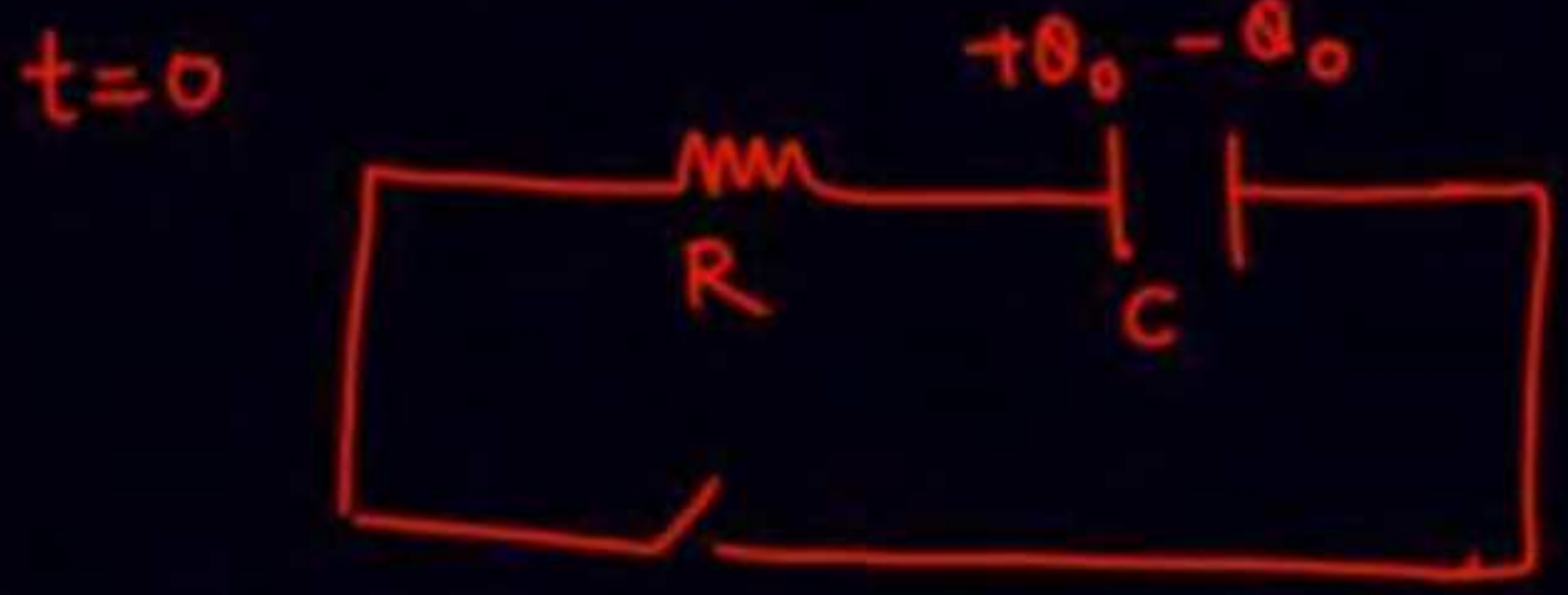
$$\frac{q}{C} = -\frac{dq}{dt} \cdot R$$

$$\int_{Q_0}^{q} \frac{dq}{q} = \int_0^t -\frac{dt}{RC}$$

$$\ln \frac{q}{Q_0} = -\frac{t}{RC}$$

$$\boxed{q = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}}$$

Discharging of Capacitor



find $q = f(t)$ on capacitor

$$-iR - \frac{q}{C} = 0$$

$$iR = -\frac{q}{C}$$

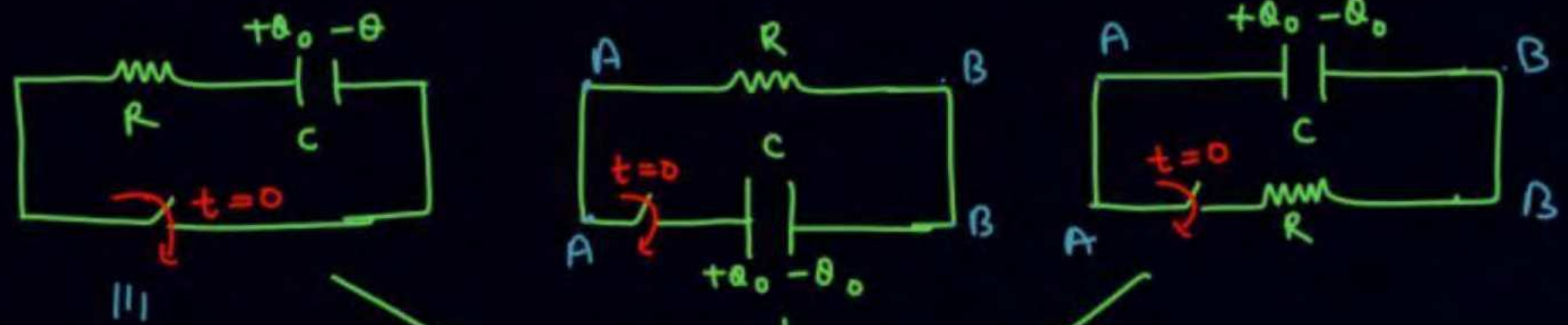
$$\frac{dq}{dt} \cdot R = -\frac{q}{C}$$

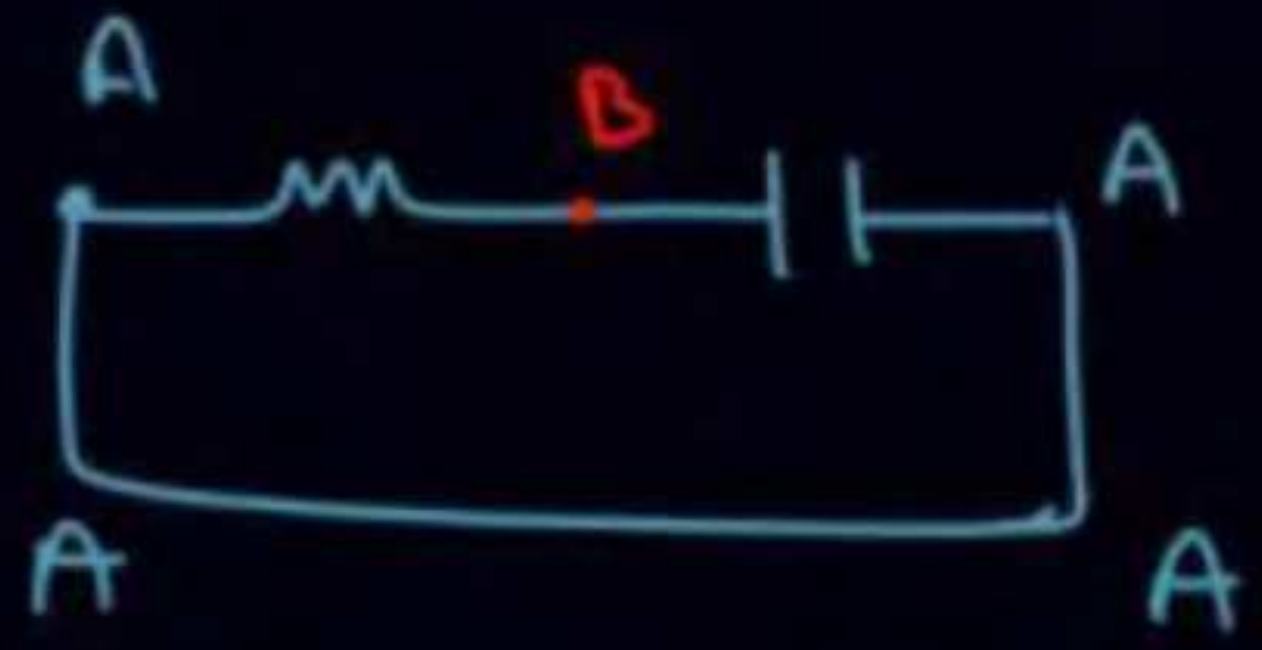
$$\int_{Q_0}^{q} \frac{dq}{q} = \int_0^t -\frac{1}{RC}\,dt$$

$$\ln \frac{q}{Q_0} = -\frac{t}{RC}$$

$$\frac{q}{Q_0} = e^{-t/RC}$$

$$\boxed{q = Q_0 e^{-t/RC}}$$





All are Same

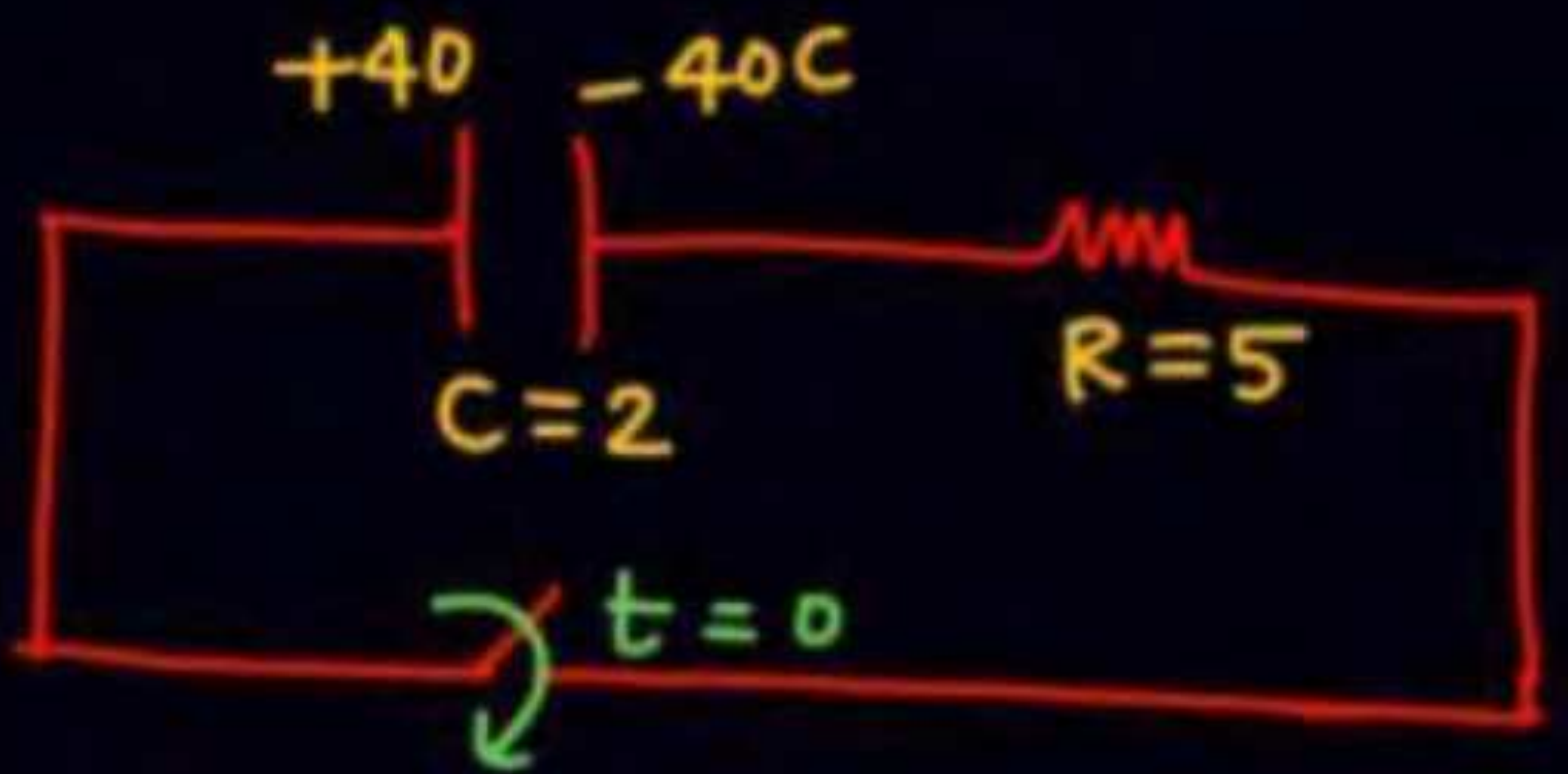
t=0, switch closed

$q = Q_0 e^{-t/RC}$

$i = i_0 e^{-t/RC}$

$$\boxed{V_C = V_R}$$

Q



① $\tau = RC = 5 \times 2 = 10$

$t = \tau$ पर charge

$q = Q_0 e^{-t/\tau}$

$q = Q_0 e^{-1} = \frac{Q_0}{e}$

$q = .37 Q_0$

② Charge as function of time on capacitor

$q = Q_0 e^{-t/RC}$

$= 40 e^{-t/10}$

③ $i, V_r, V_c, (P_{loss})_{resistance}$ at time 't'

$i = \left|\frac{dq}{dt}\right| = \frac{40}{10} e^{-t/10}$

$V_r = \checkmark$

$V_c = \checkmark$

$P = i^2 R$

$= (4 e^{-t/10})^2 \times 5$

④ heat loss in 10 sec

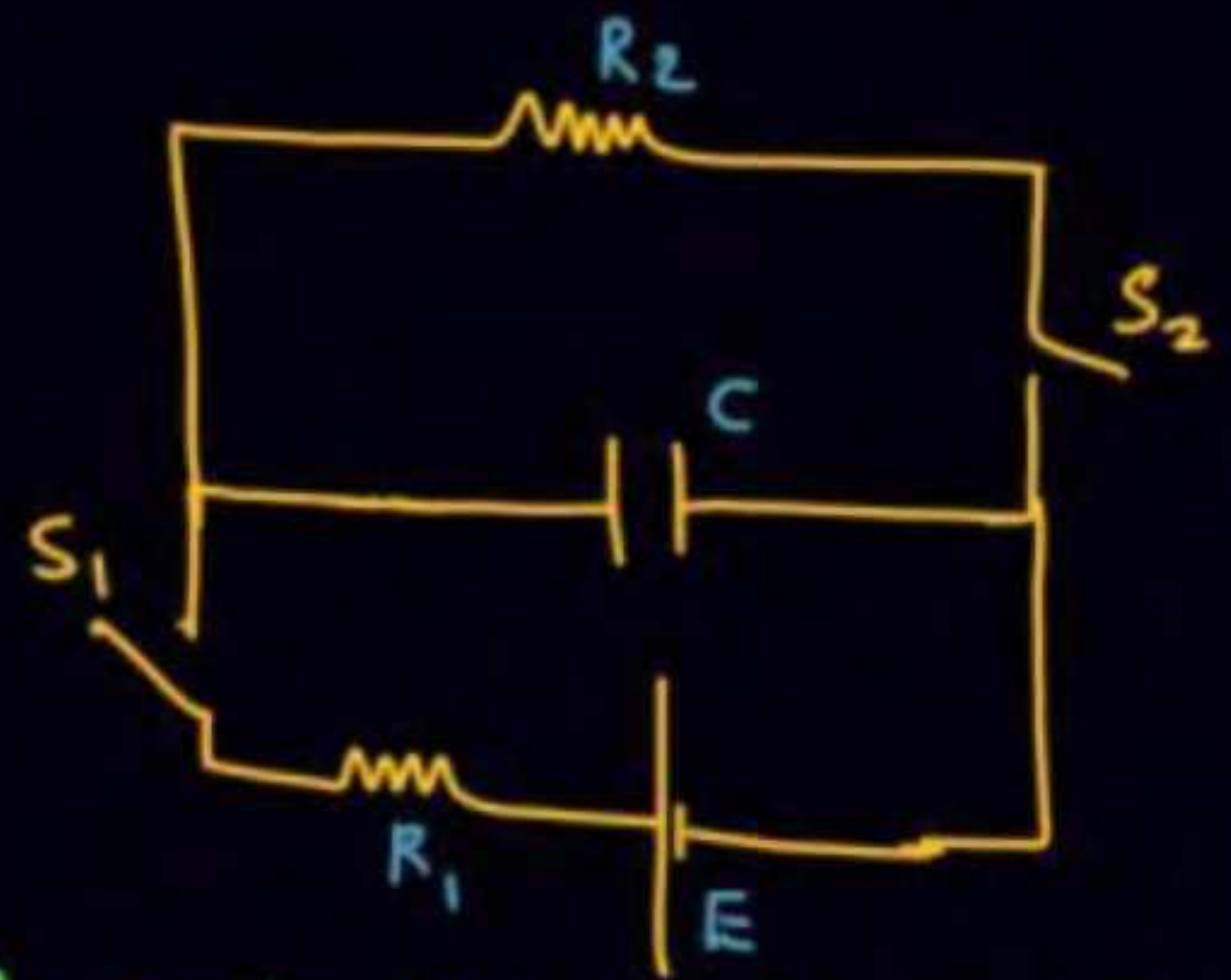
$H = \int_0^{10} i^2 R\, dt$

⑤ total heat loss.

$\frac{1}{2}\frac{Q_0^2}{C} \longrightarrow$ heat loss.

$\text{Heat loss} = \int_0^{\infty} i^2 R\, dt = \frac{Q_0^2}{2C}$

Q



① Initially S_1 is closed & S_2 is open at $t=0$

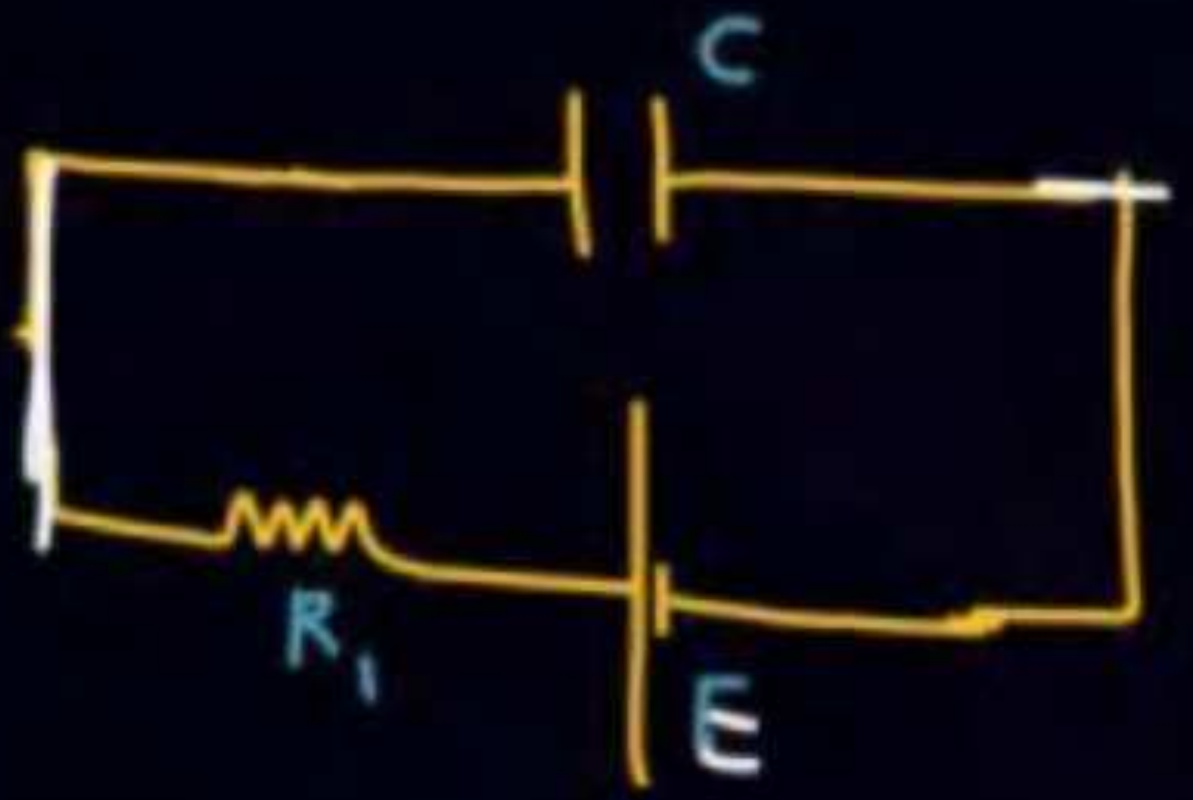
find $q = f(t)$ $\quad t=0$

$$q = EC\left(1 - e^{-t/R_1C}\right)$$

$$t=\infty \Rightarrow q = Q_0 = EC$$

$$i = 0$$

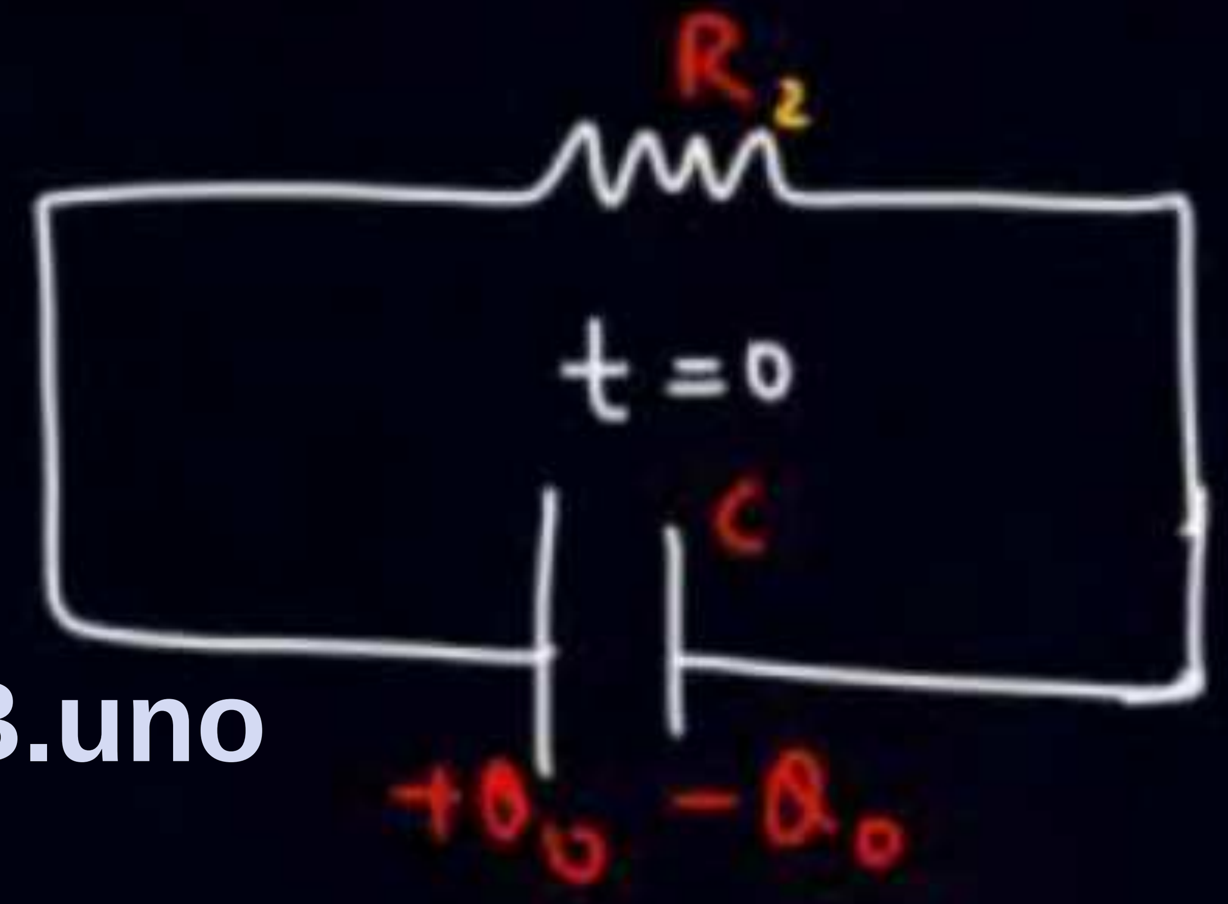
$$i = \frac{E}{R_1} e^{-t/R_1C}$$



② After very long time S_1 is open and S_2 is closed and another new stopwatch is start at $t=0$

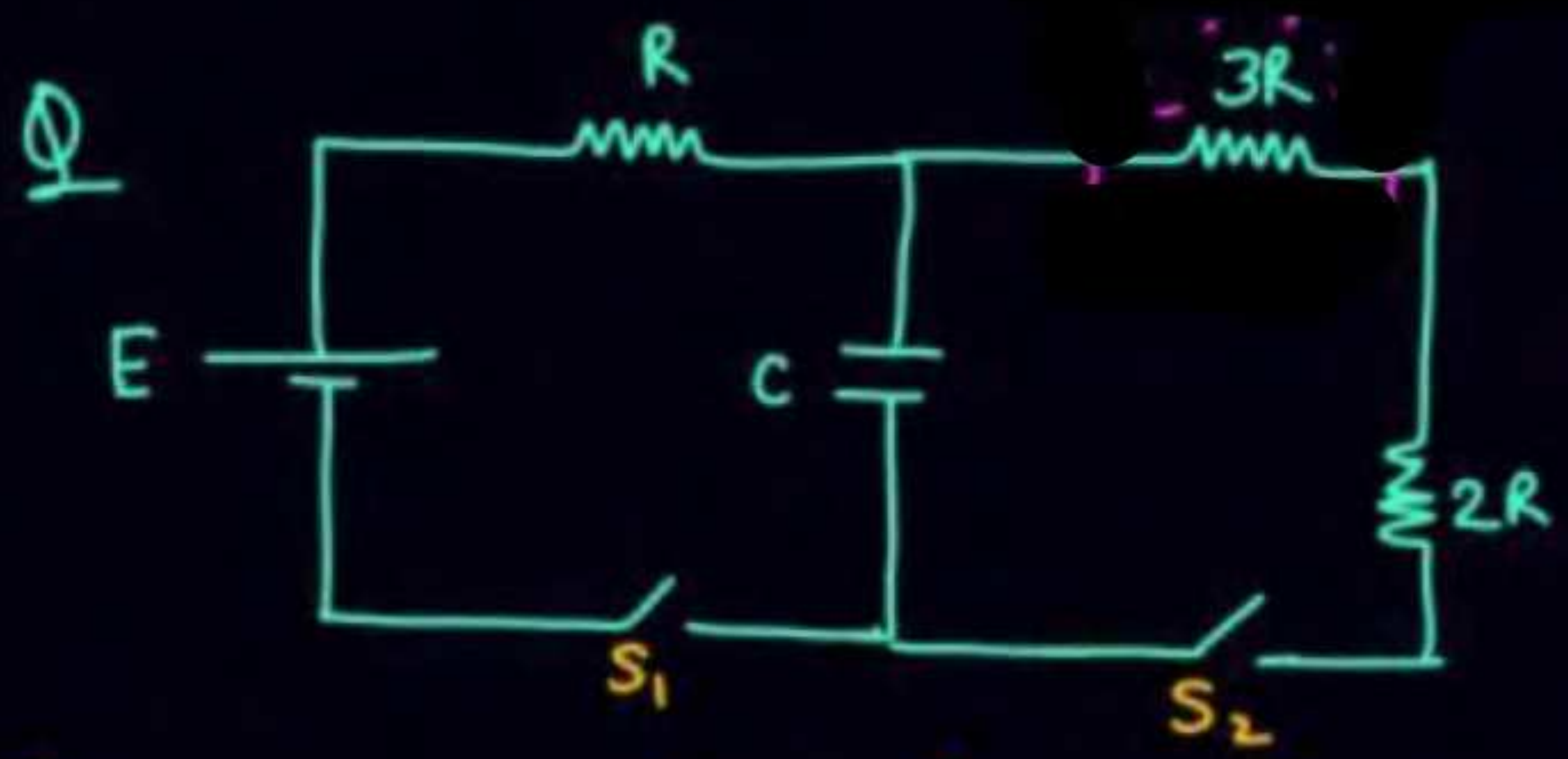
find $q=f(t)$ according to new observation

total heat loss in resistor.



Discharging $\longrightarrow$ $q = Q_0 e^{-t/R_2C}$

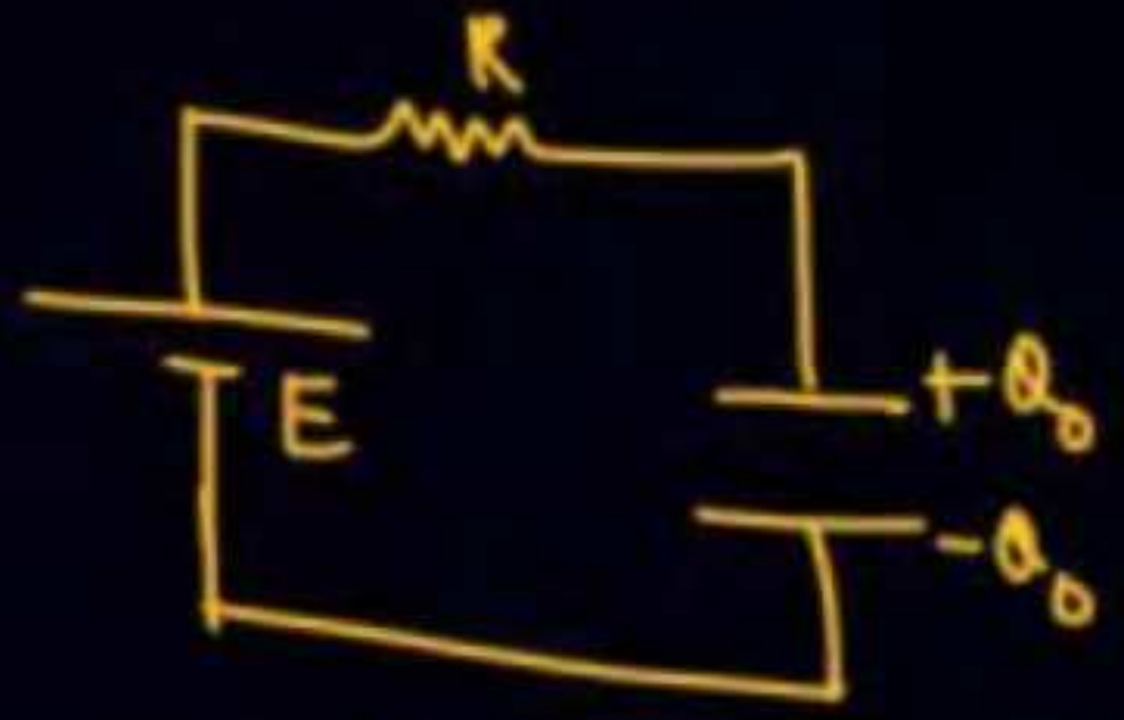
$$\boxed{q = EC\, e^{-t/R_2C}}$$



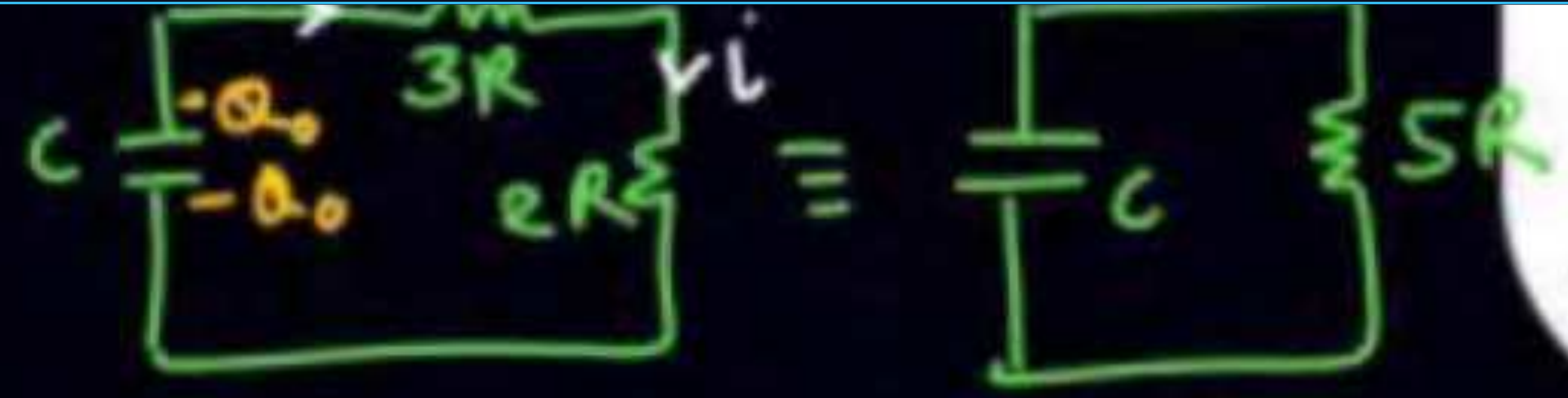
① Initially $S_1 \rightarrow$ closed $S_2 \rightarrow$ open

$$q = EC\left(1 - e^{-t/RC}\right)$$

$t = \infty$



$$Q_0 = EC$$



② After very long time for new observation S_1 open S_2 closed at $t = 0$

$$q = Q_0 e^{-t/5RC}$$

③ In new observation total heat loss (In above part)

$$H = \frac{1}{2}\frac{Q_0^2}{C} = U_i$$

④ In above part total heat loss in 3R resistance.

$$\frac{3H}{5}$$

[redacted]

3R

$$\frac{H_1}{H_2} = \frac{3R}{2R} = \frac{3}{2}$$

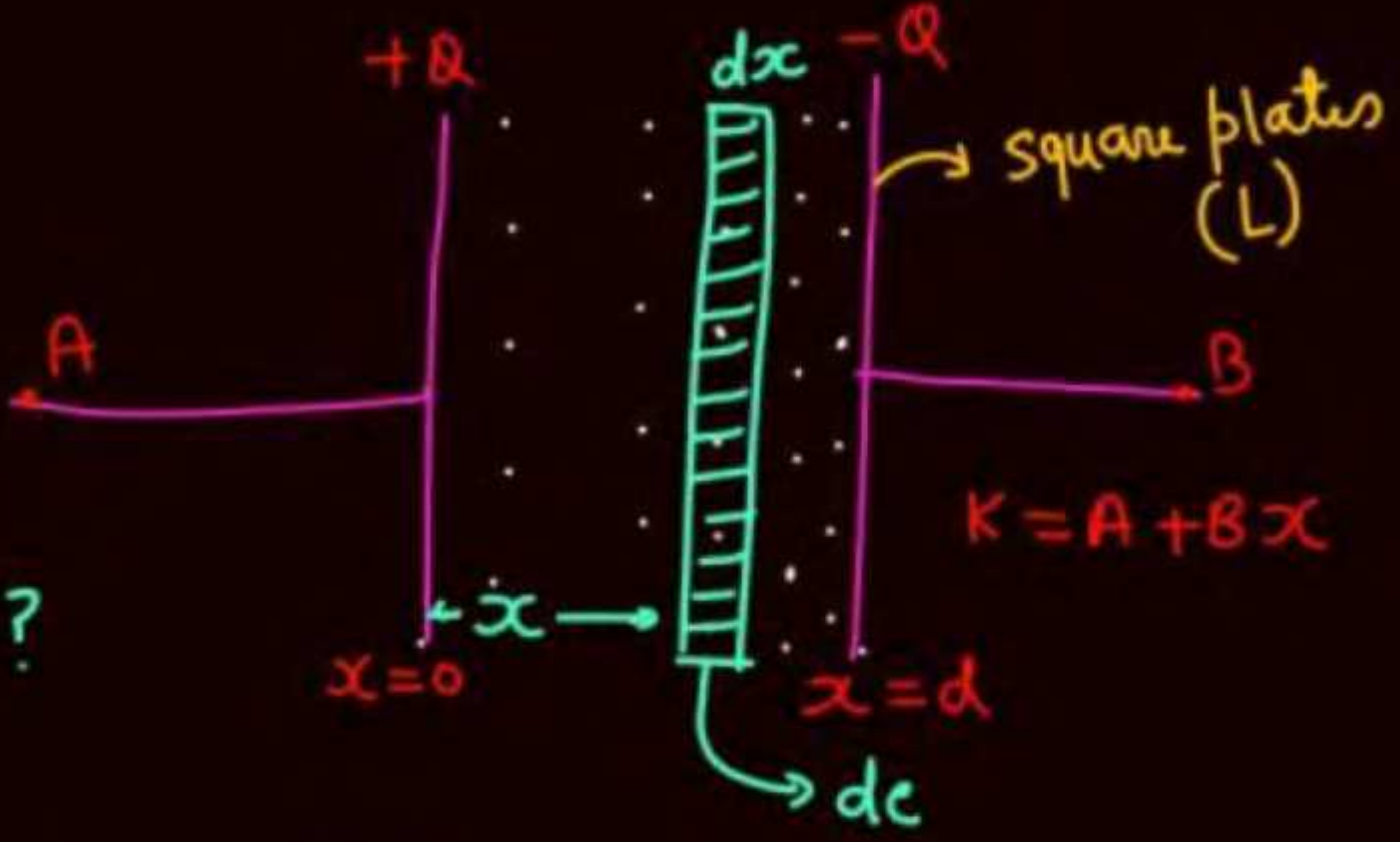
$$H_1 + H_2 = H$$

$$H_1 + \frac{2}{3}H_1 = H$$

$$\frac{5H_1}{3} = H$$

$$H_1 = \frac{3H}{5}, \quad H_2 = \frac{2H}{5}$$

Q



find $C_{AB} = ?$

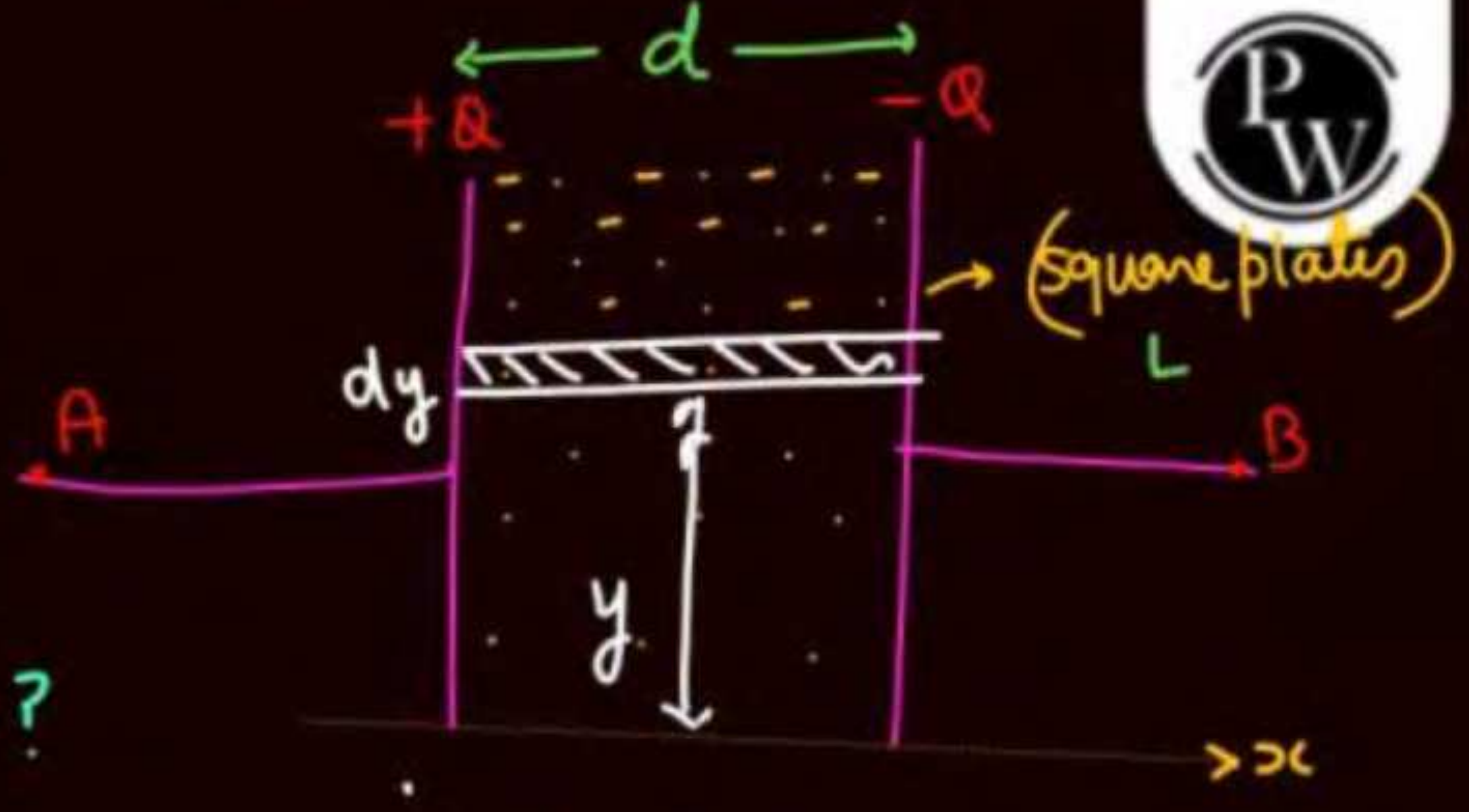
$$dc = \frac{A\epsilon_0(A+Bx)}{dx}$$

$$\frac{1}{C_{eq}} = \int \frac{1}{dC} = \int_0^d \frac{dx}{A\epsilon_0(A+Bx)}$$

$$\frac{1}{C_{eq}} = \frac{1}{A\epsilon_0 B} \ln\left(\frac{A+Bd}{A}\right)$$

$$C_{eq} = \checkmark$$

Q



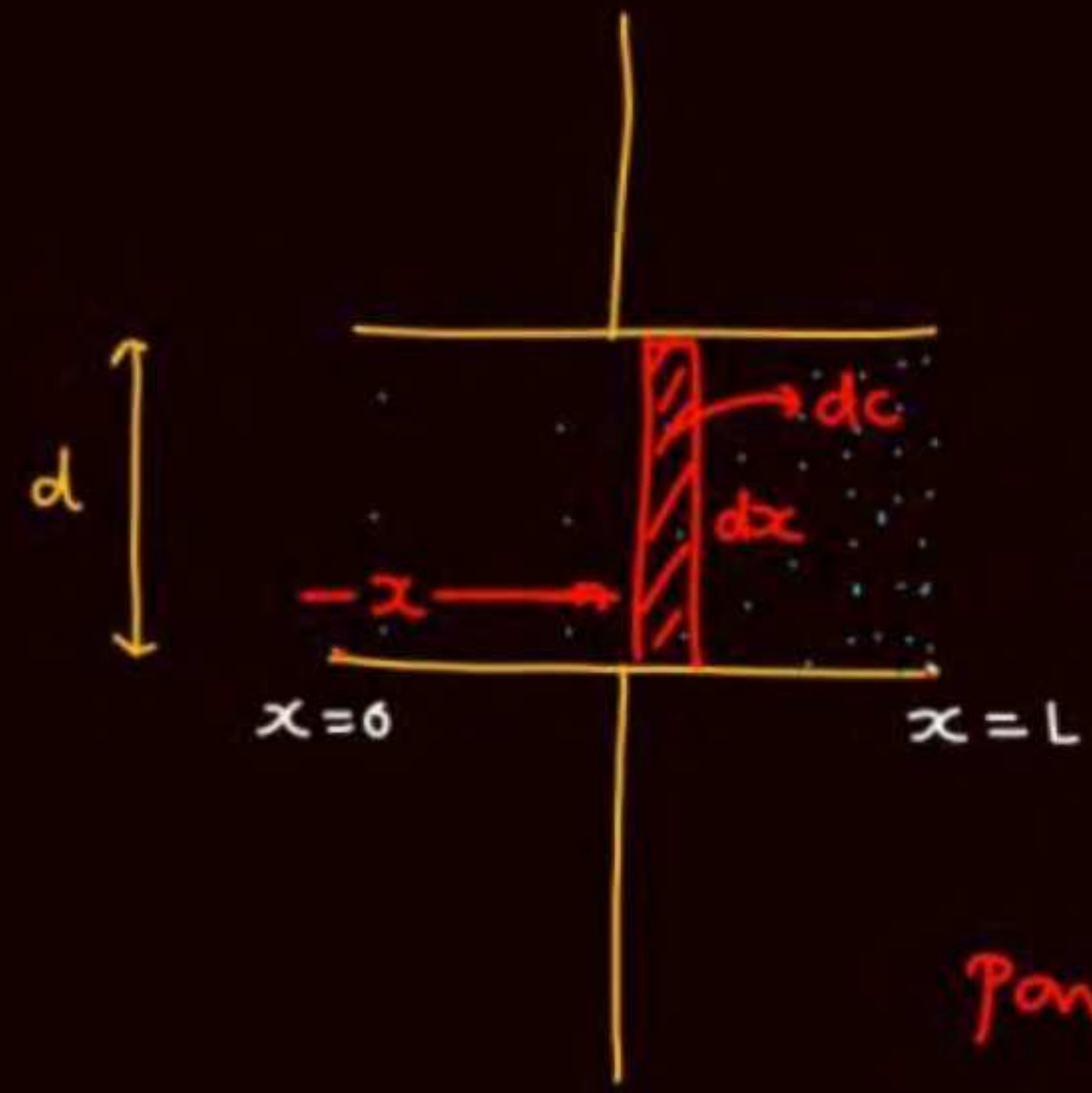
find $C_{AB} = ?$

$$K = A + By$$

$$dc = \frac{L\,dy \cdot \epsilon_0(A+By)}{d}$$

$$C_{eq} = \int_0^L dc = \int_0^L \frac{L\epsilon_0}{d}(A+By)\,dy$$

Q

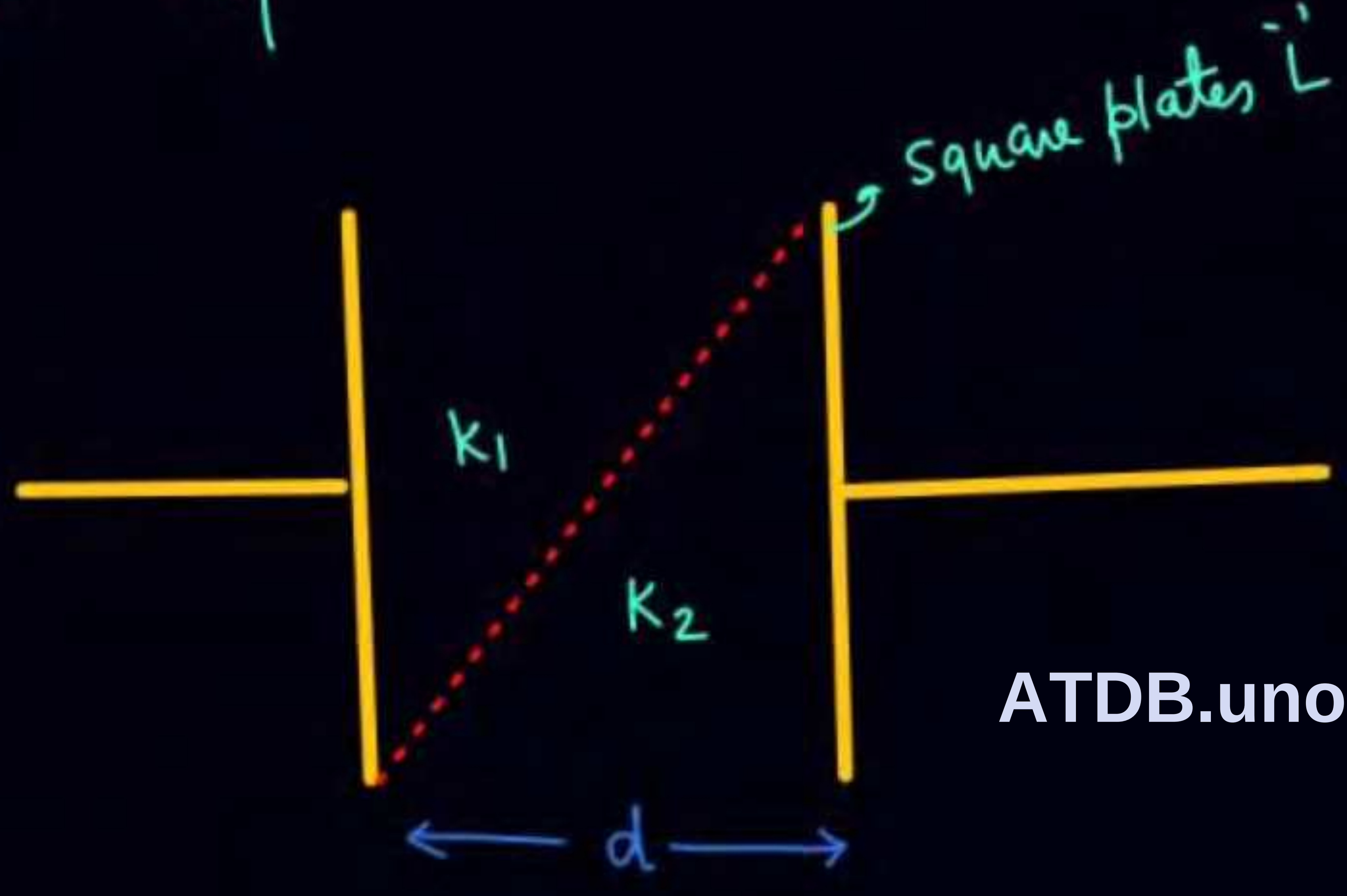


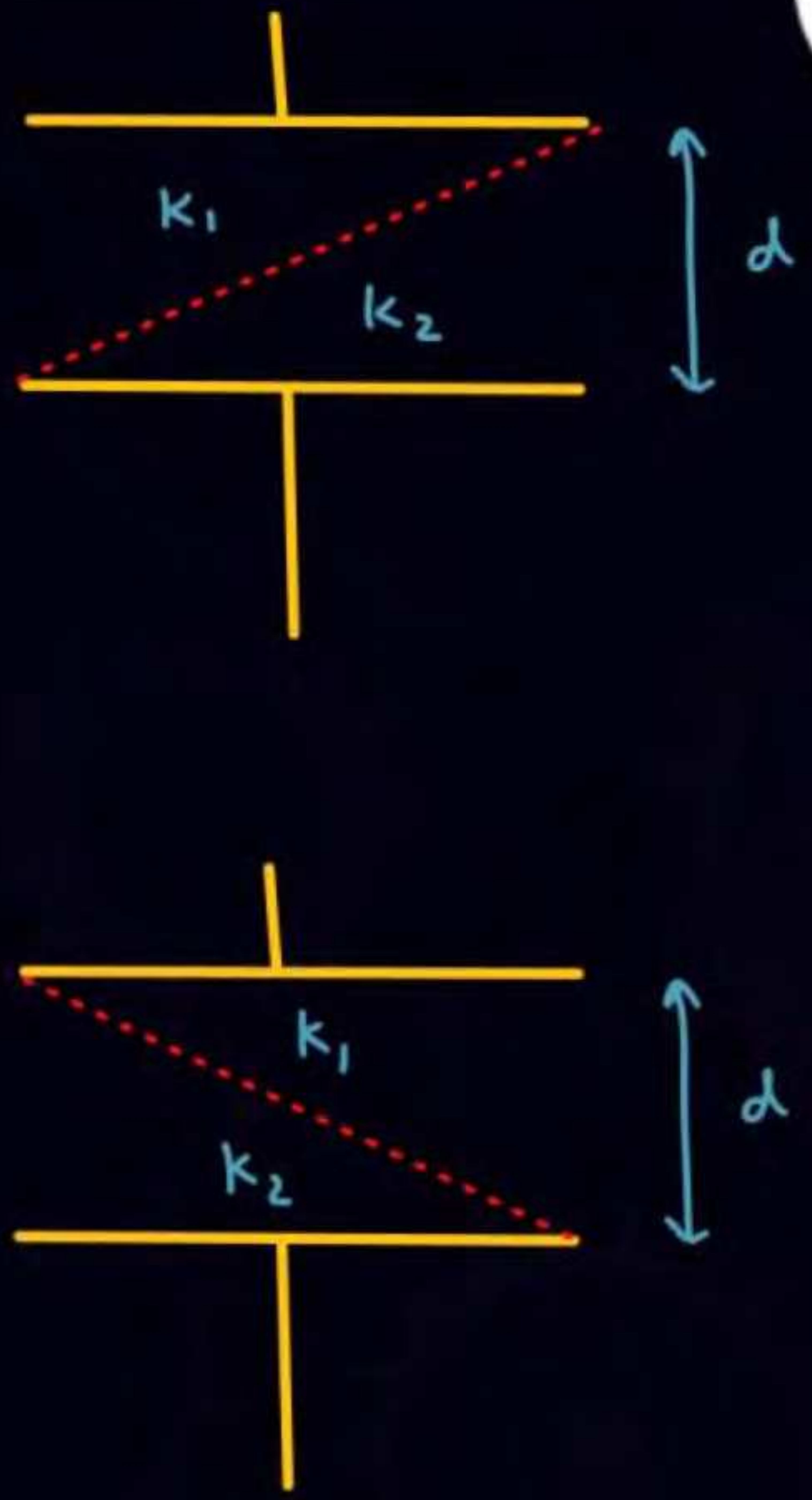
$K = A + Bx$

$$\int dc = \int_0^L \frac{L\,dx \cdot \epsilon_0 (A + Bx)}{d} = C_{eq}$$

Parallel

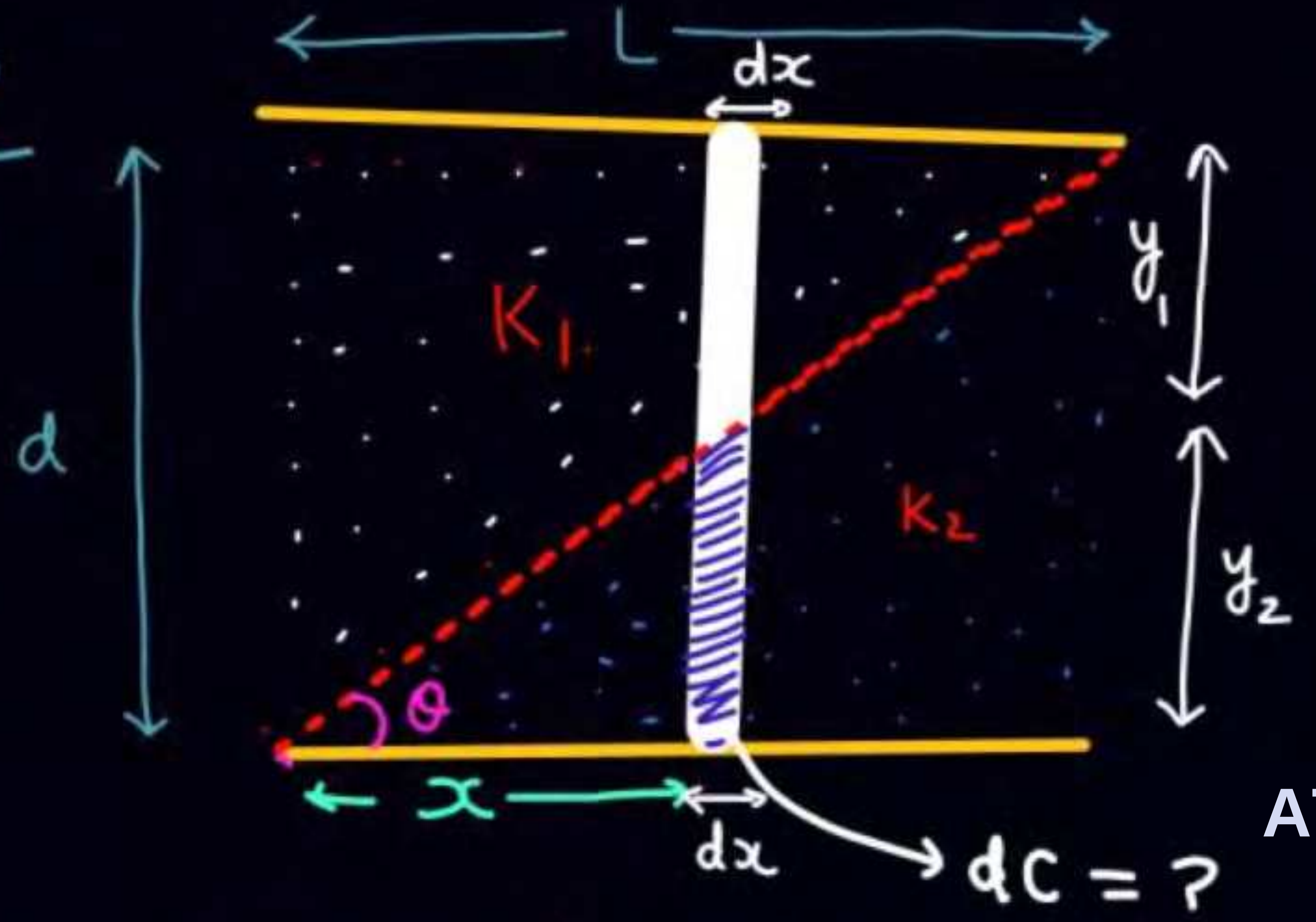
Q find Ceq

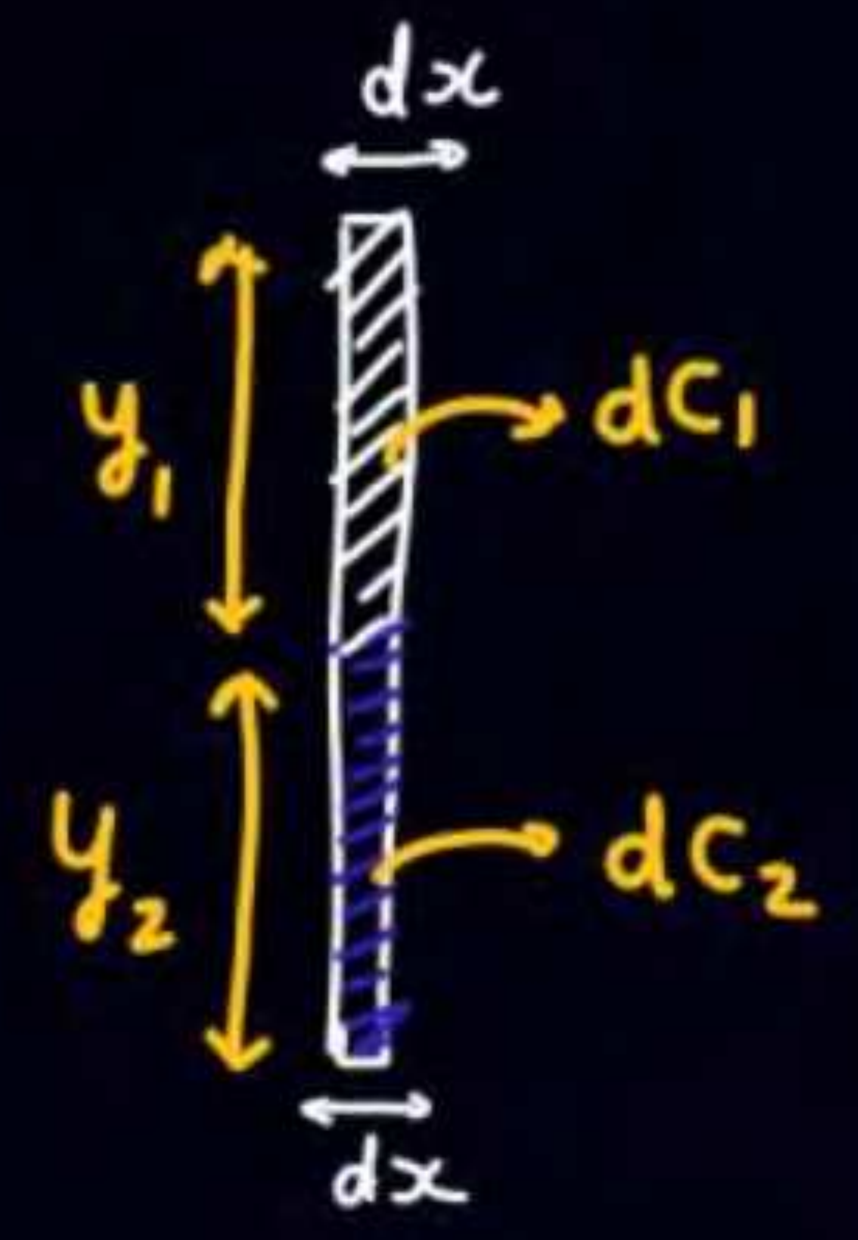




(square plate L)

sol





$$dC_1 = \frac{L\,dx.\,\epsilon_0 K_1}{y_1}$$

$$dC_2 = \frac{L\,dx\,\epsilon_0 . K_2}{y_2}$$

dC_1 & $dC_2 \longrightarrow$ Series

$$\tan\theta = \frac{d}{L} \qquad \tan\theta = \frac{y_2}{x}$$

$$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2}$$

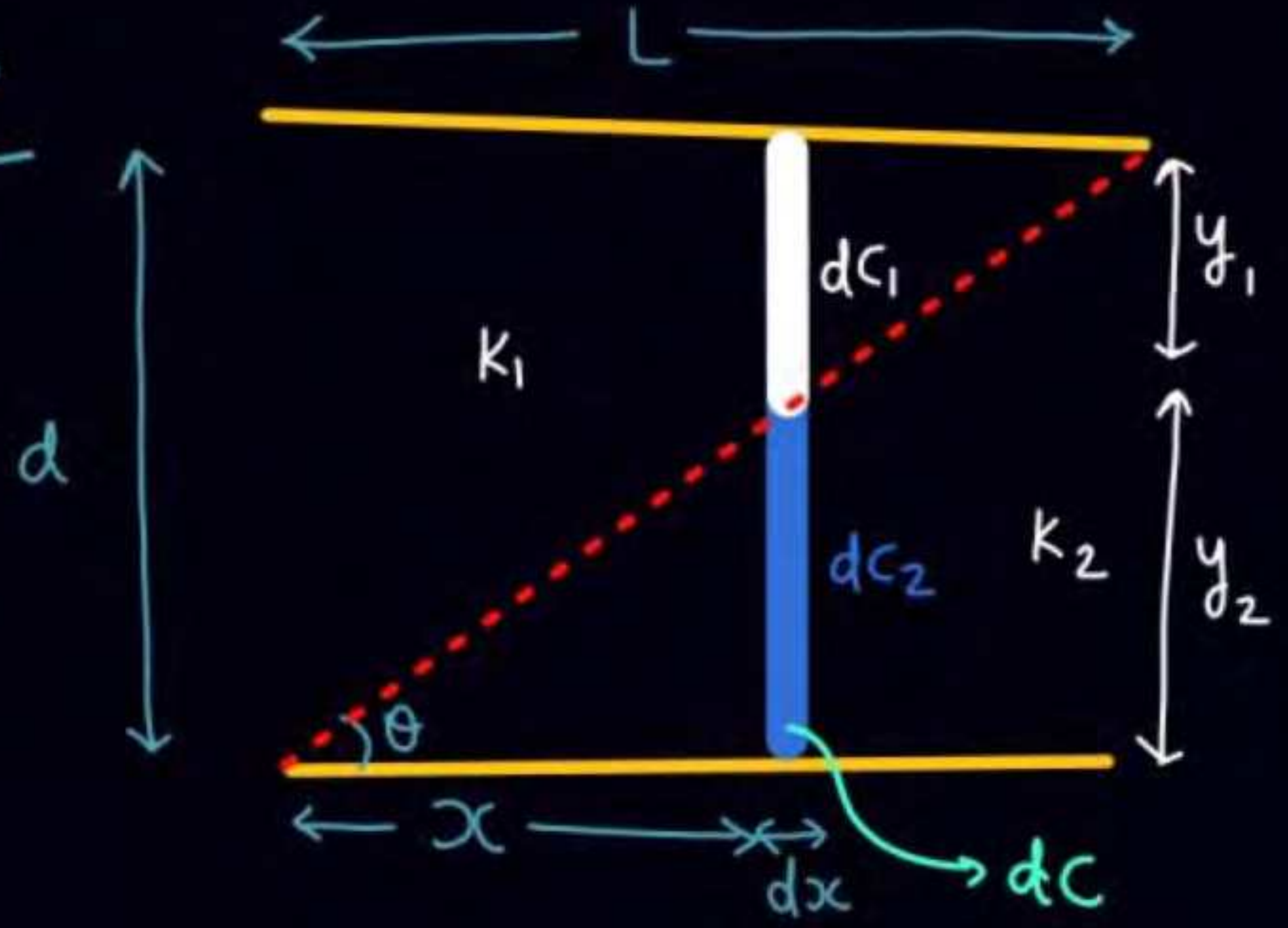
$$y_2 = x\tan\theta$$

$$y_1 = d - x\tan\theta$$

$dC = \checkmark \qquad \Rightarrow \int dC = C_{eq} = \underline{Ans}$

(square plate L)

Sol^n



$$\tan\theta = \frac{d}{L}$$

$$y_2 = x\tan\theta = \frac{xd}{L}$$

$$y_1 = d - x\tan\theta$$

$$y_1 = d - x\frac{d}{L}$$

$$dC_1 = \frac{(L\,dx)\,\epsilon_0 K_1}{y_1} \qquad dC_2 = \frac{L\,dx\ \epsilon_0 K_2}{y_2}$$

$$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2} = \frac{y_1}{L\,dx\,\epsilon_0 K_1} + \frac{y_2}{L\,dx\,\epsilon_0 K_2}$$

$$\frac{1}{dC} = \frac{1}{L\cdot dx\cdot \epsilon_0}\left[\frac{y_1}{K_1} + \frac{y_2}{K_2}\right]$$

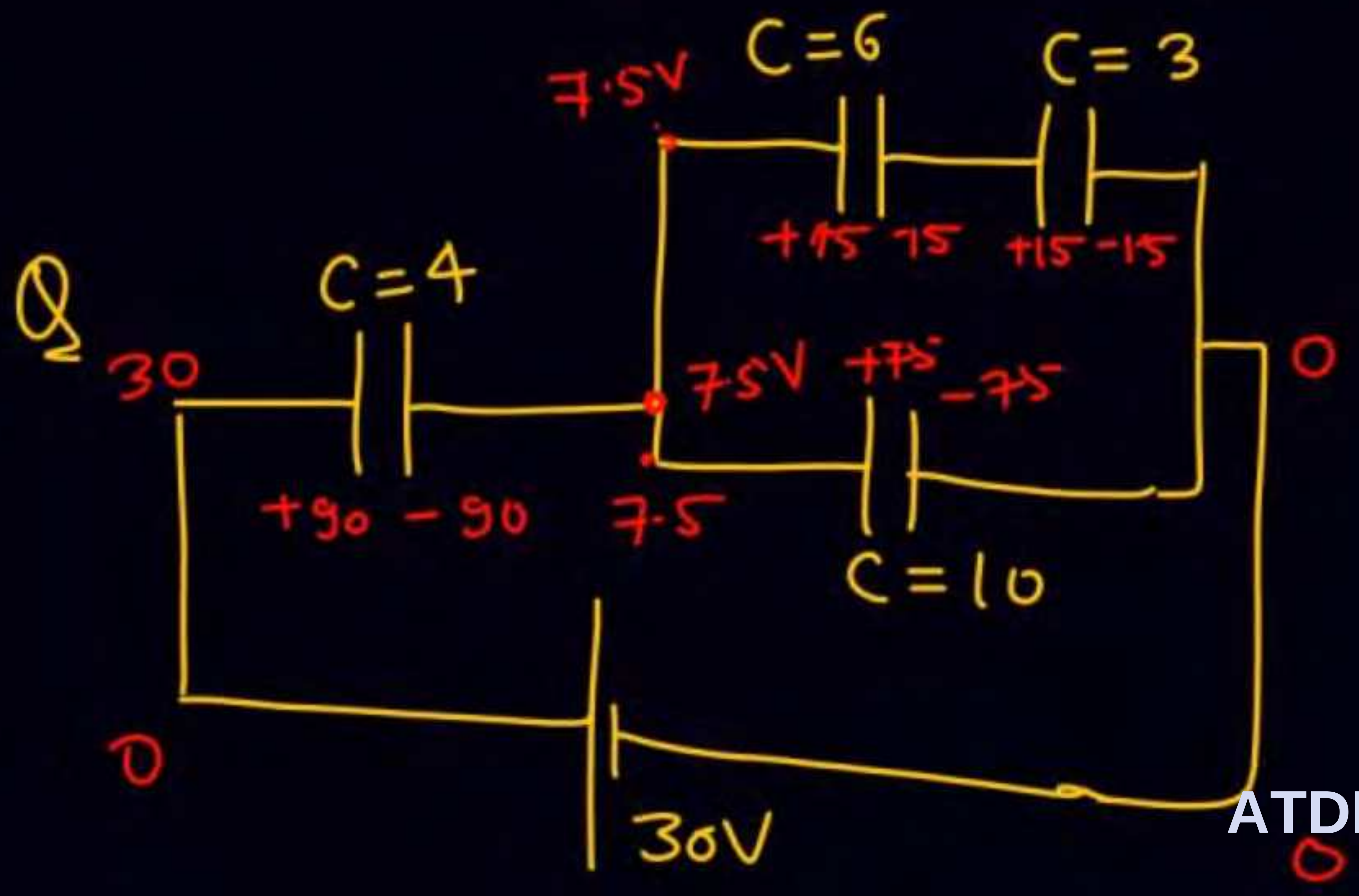
$$dC = \frac{L\epsilon_0\,dx}{\frac{y_1}{K_1} + \frac{y_2}{K_2}} = \frac{L\epsilon_0\,dx}{\frac{d - \frac{xd}{L}}{K_1} + \frac{xd}{L\cdot K_2}}$$

$$dC = \frac{L\epsilon_0\,dx}{\frac{d}{K_1} + x\left[\frac{d}{LK_2} - \frac{d}{LK_1}\right]}$$

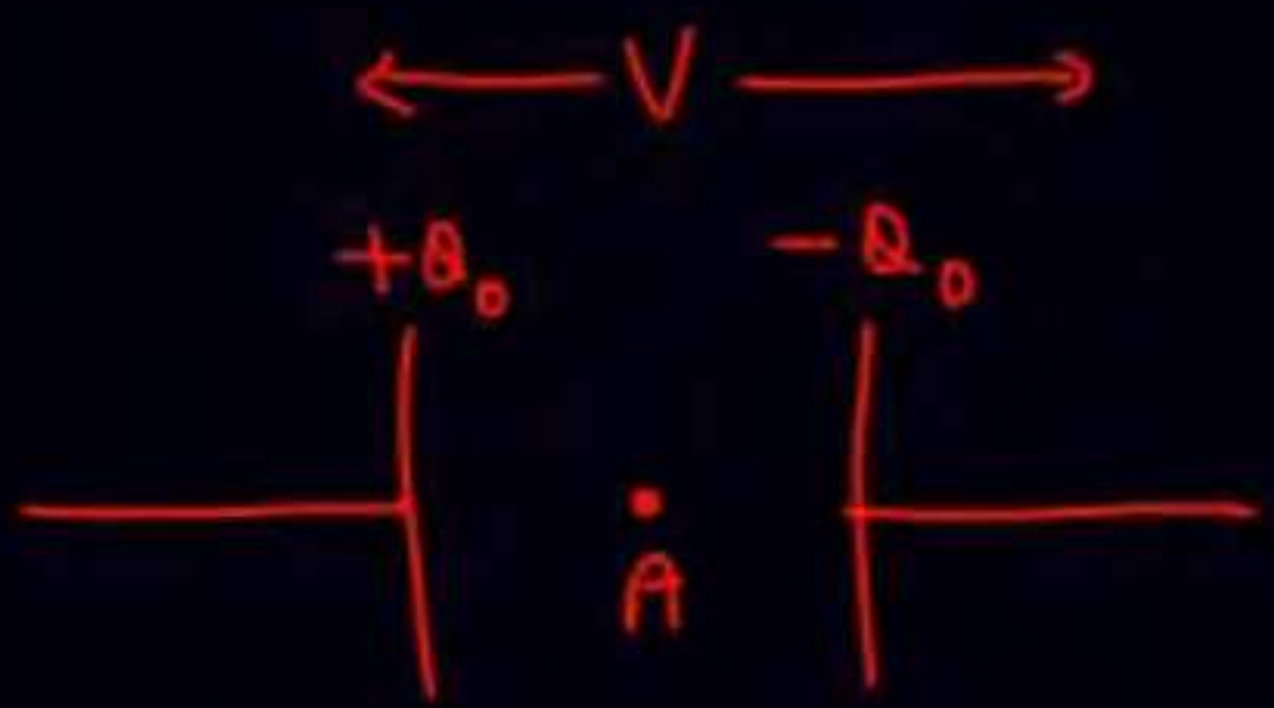
$$dc = \frac{L\epsilon_0\, dx}{\frac{d}{K_1} + x\left[\frac{d}{LK_2} - \frac{d}{LK_1}\right]}$$

$$\int dc = \int_0^L \frac{L\epsilon_0\, dx}{\frac{d}{K_1} + x\left[\frac{d}{LK_2} - \frac{d}{LK_1}\right]} \Rightarrow \boxed{\int \frac{A\,dx}{B + Cx} = \frac{A}{C}\ln(B + Cx)}$$

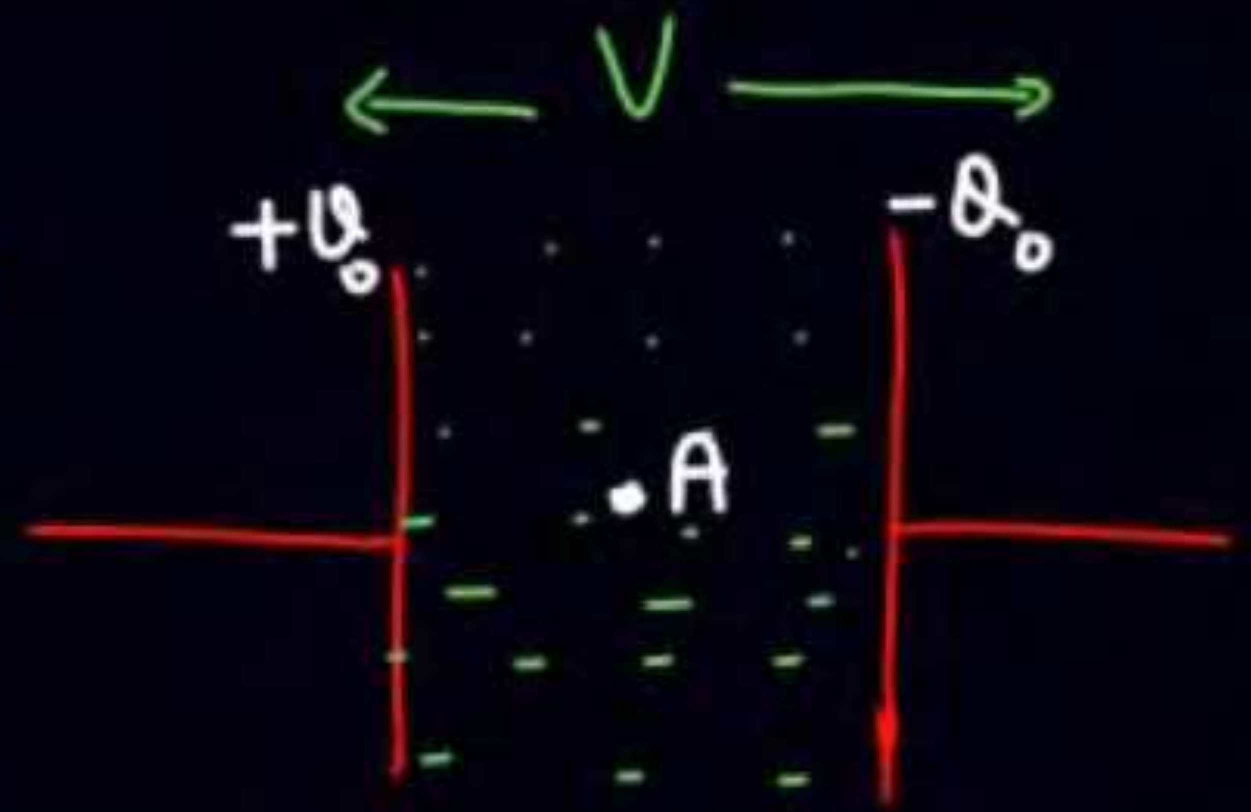
$$C_{eq} = \frac{L\epsilon_0}{-\left(\frac{d}{LK_1} - \frac{d}{LK_2}\right)} \ln\left\{\frac{\frac{d}{K_1} + L\left[\frac{d}{LK_2} - \frac{d}{LK_1}\right]}{\frac{d}{K_1}}\right\} = \frac{L^2\epsilon_0\, K_1 K_2}{d\cdot(K_2 - K_1)} \ln\left(\frac{K_1}{K_2}\right)$$



C=6
C=3
7.5V
+15 -15
+15 -15
Q
C=4
30
+90 -90
7.5
7.5V
+75
-75
C=10
0
0
30V
0



$$E_A = \frac{V}{d} \qquad E_0 = \frac{Q_0}{A\epsilon_0}$$



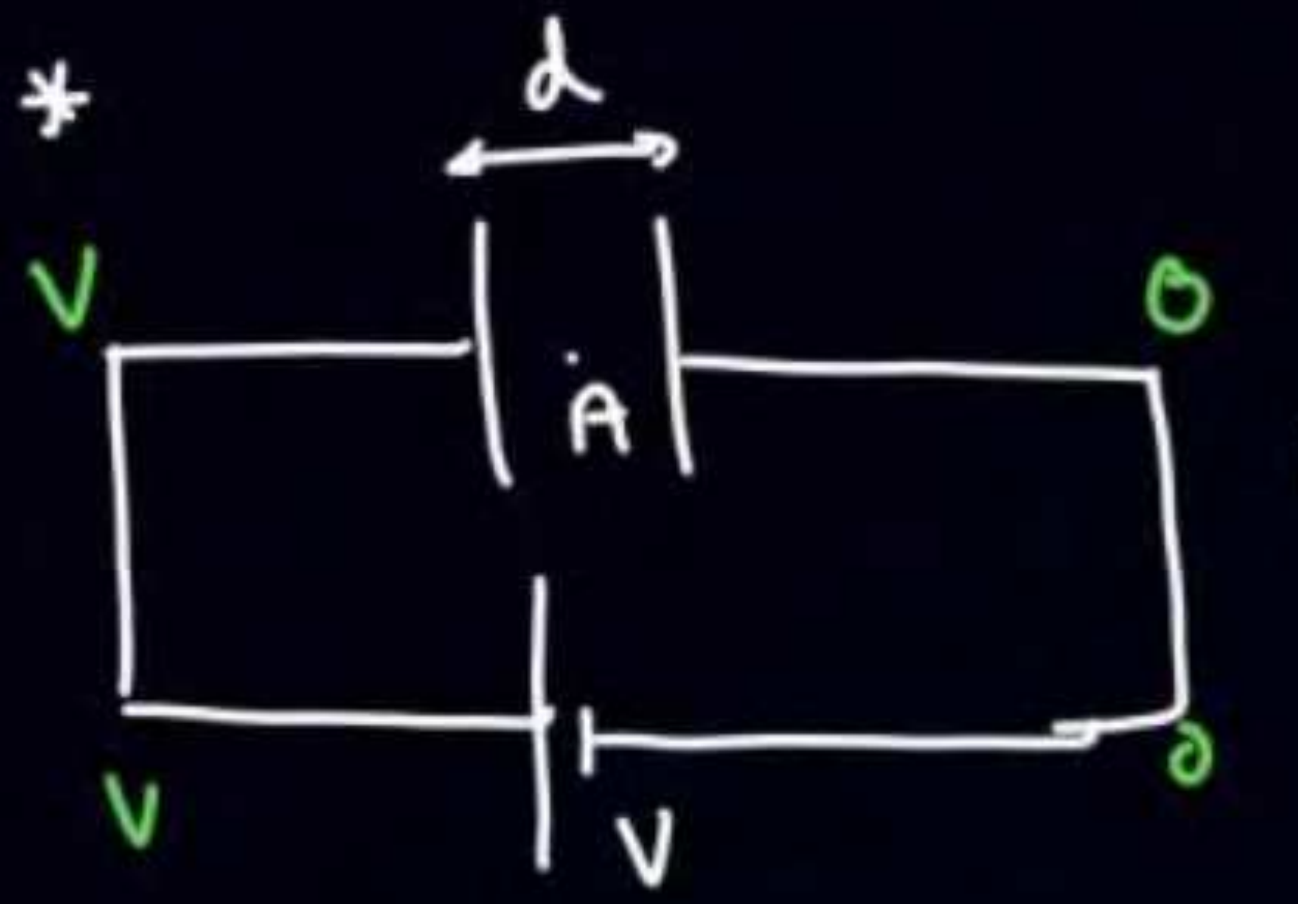
$Q = CV$

Q ↓ same, C ↓ k times

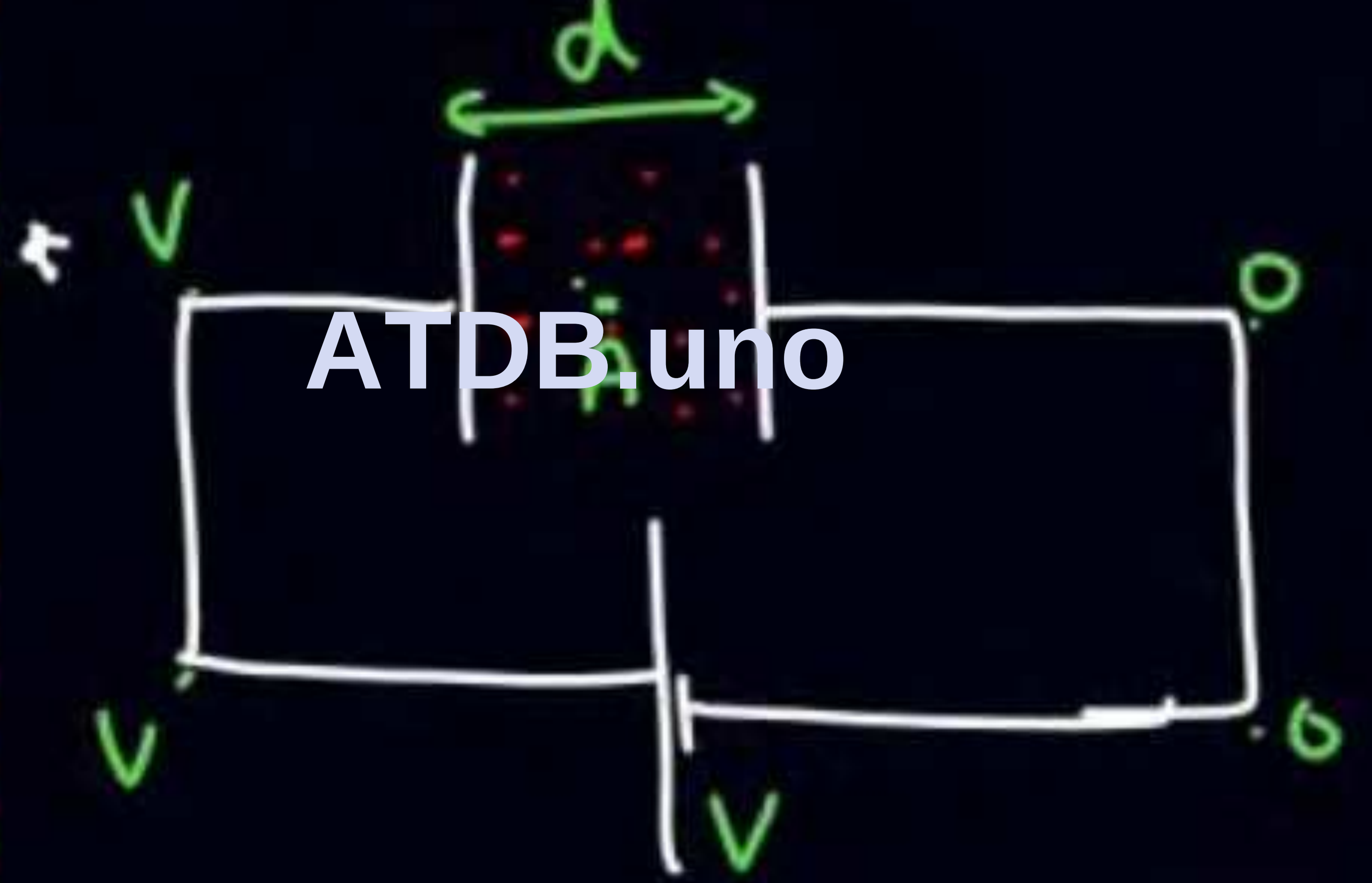
$V \longrightarrow \frac{V}{k}$

$E_0 \longrightarrow E_0/k$

$$E_A = \frac{V}{d} \qquad E_A = \frac{E_0}{K}$$



$$E_A = \frac{V}{d}$$

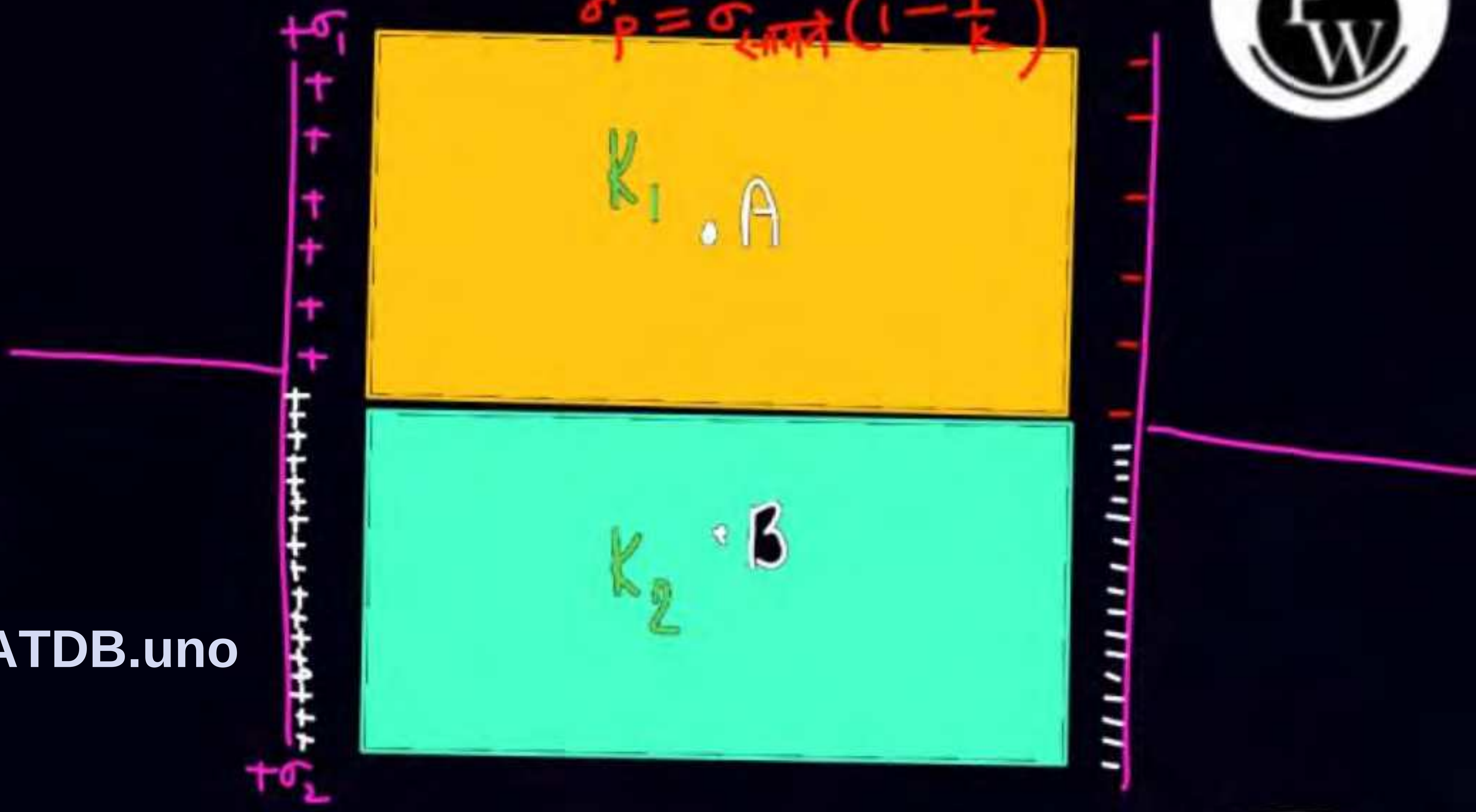


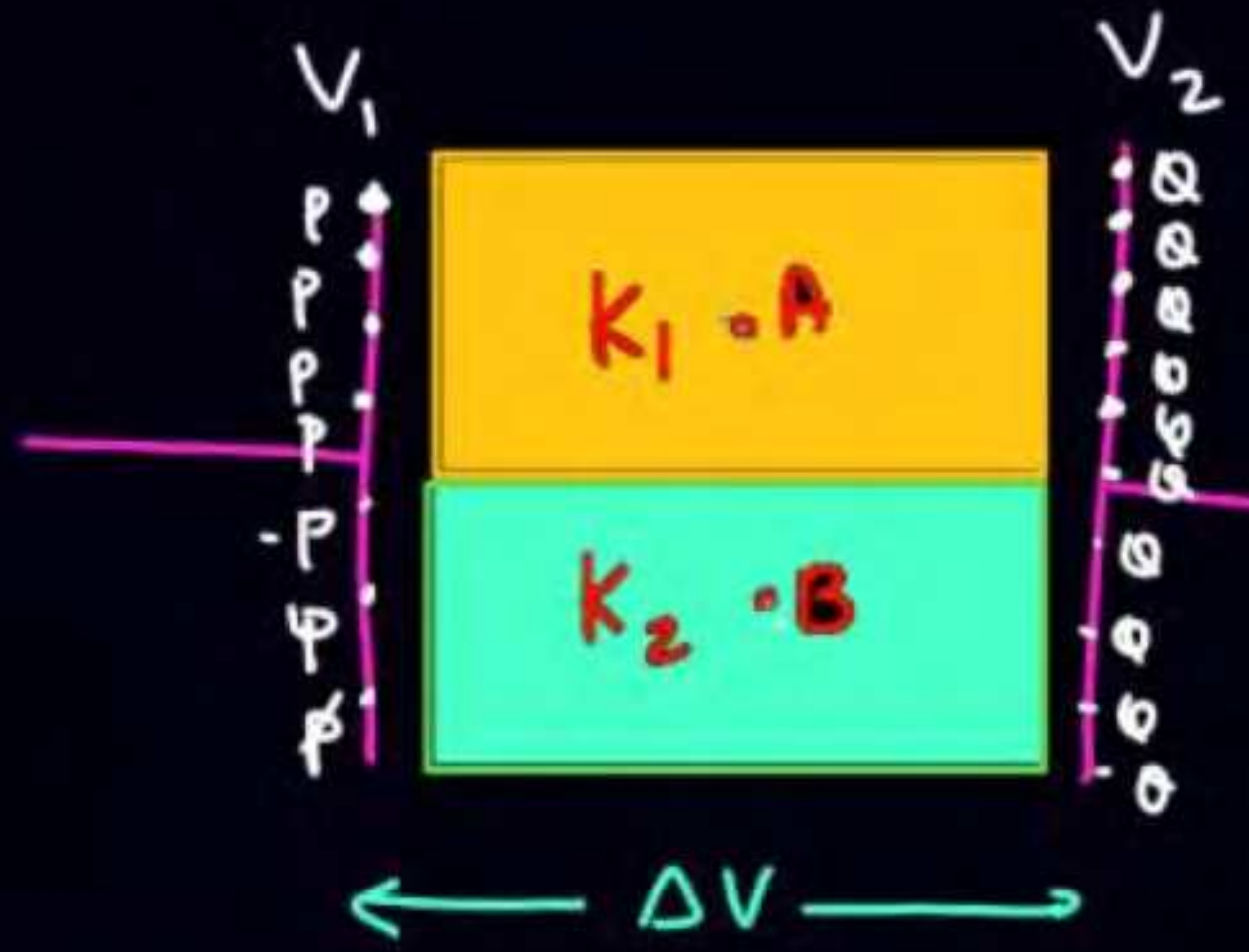
$$E_A = \frac{V}{d}$$

$$E_A = \frac{V}{d}$$

$\sigma_p = \sigma_0\left(1 - \frac{1}{K}\right)$

$\sigma_p = \sigma_{\text{लगाया}}\left(1 - \frac{1}{k}\right)$





Let $K_1 = K$

$K_2 = 2K$

$$\boxed{E_A = E_B}$$

$$E = \frac{\Delta V \rightarrow \text{same}}{d \rightarrow \text{same}}$$

$$(E_{inside})_A = \frac{\sigma_1}{\epsilon_0 K_1}$$

$$(E_{inside})_B = \frac{\sigma_2}{\epsilon_0 K_2}$$

$$E_A = E_B$$

$$\frac{\sigma_1}{\epsilon_0 K_1} = \frac{\sigma_2}{\epsilon_0 K_2}$$

$$\boxed{\frac{\sigma_1}{\sigma_2} = \frac{K_1}{K_2}}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{K}{2K} = \frac{1}{2}$$

QUESTION

$C_{\text{ऊपर}} = C_1 = \frac{A}{3}\frac{\epsilon_0 K}{d}$

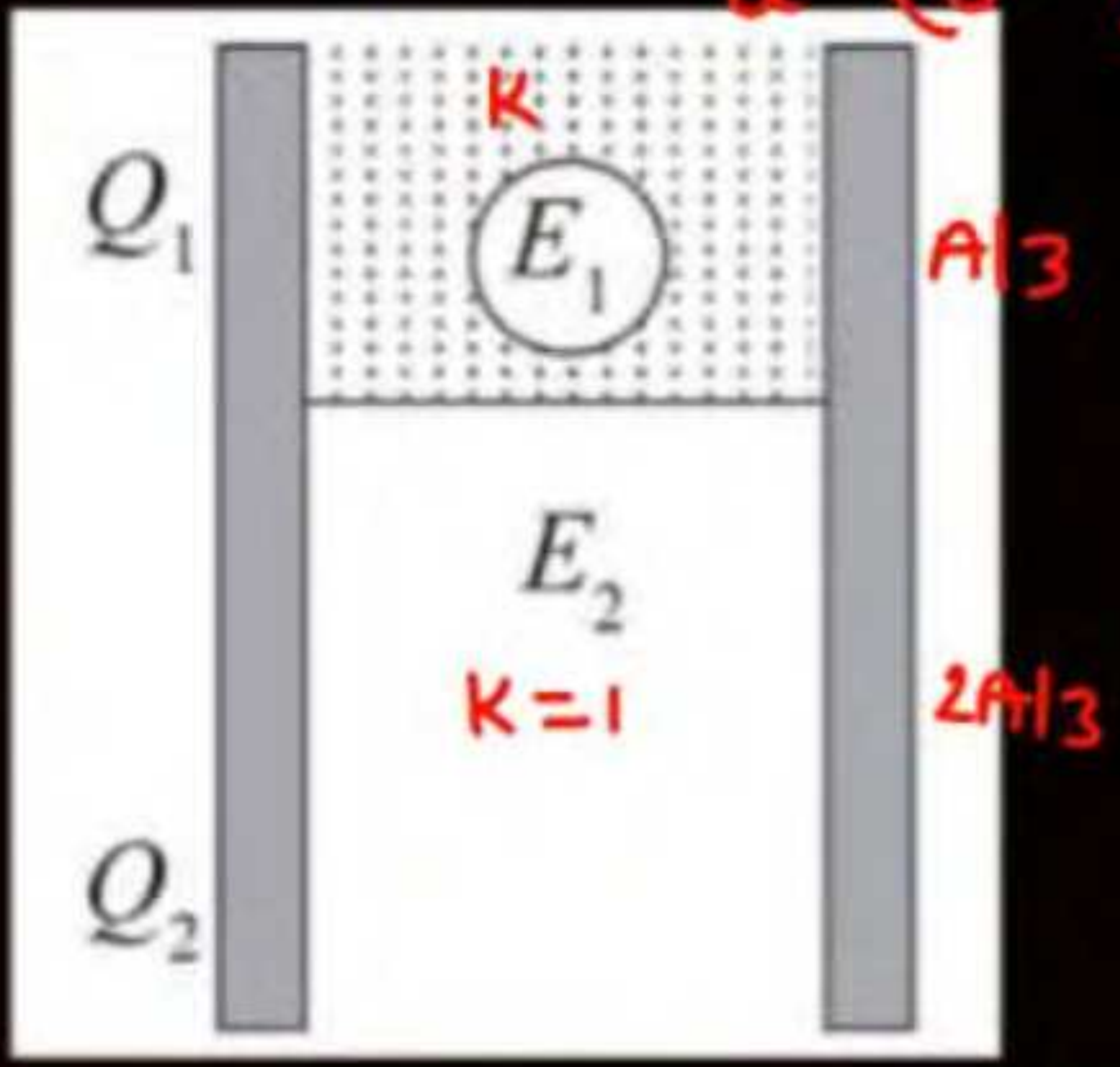
$C_{\text{नीचे}} = \frac{2A}{3}\frac{\epsilon_0}{d}$

$C_{total} = C_{\text{ऊपर}} + C_{\text{नीचे}}$

$C = \frac{A\epsilon_0}{d}\left(\frac{2}{3} + \frac{K}{3}\right)$

15. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects

(JEE Adv. 2014)



(1) $\frac{E_1}{E_2} = 1$

(2) $\frac{E_1}{E_2} = \frac{1}{K}$

(3) $\frac{Q_1}{Q_2} = \frac{3}{K}$

(4) $\frac{C}{C_1} = \frac{2+K}{K}$

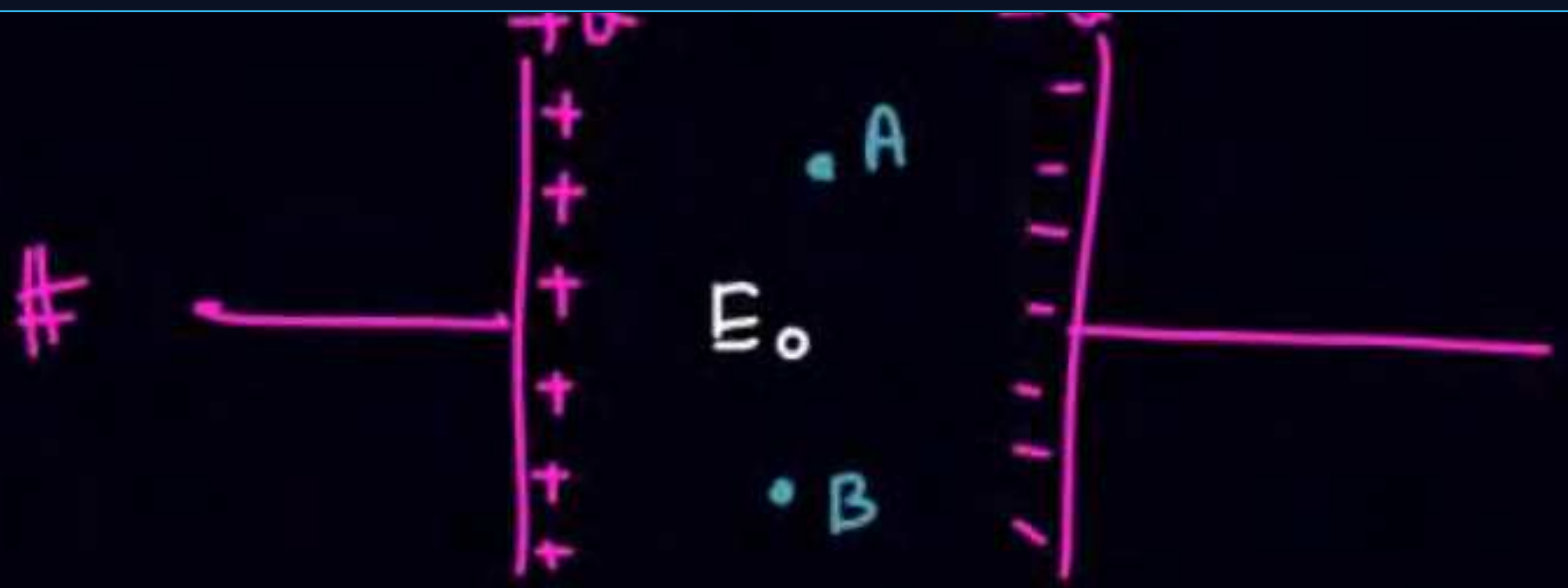
$\frac{\sigma_1}{\sigma_2} = \frac{K}{1}$

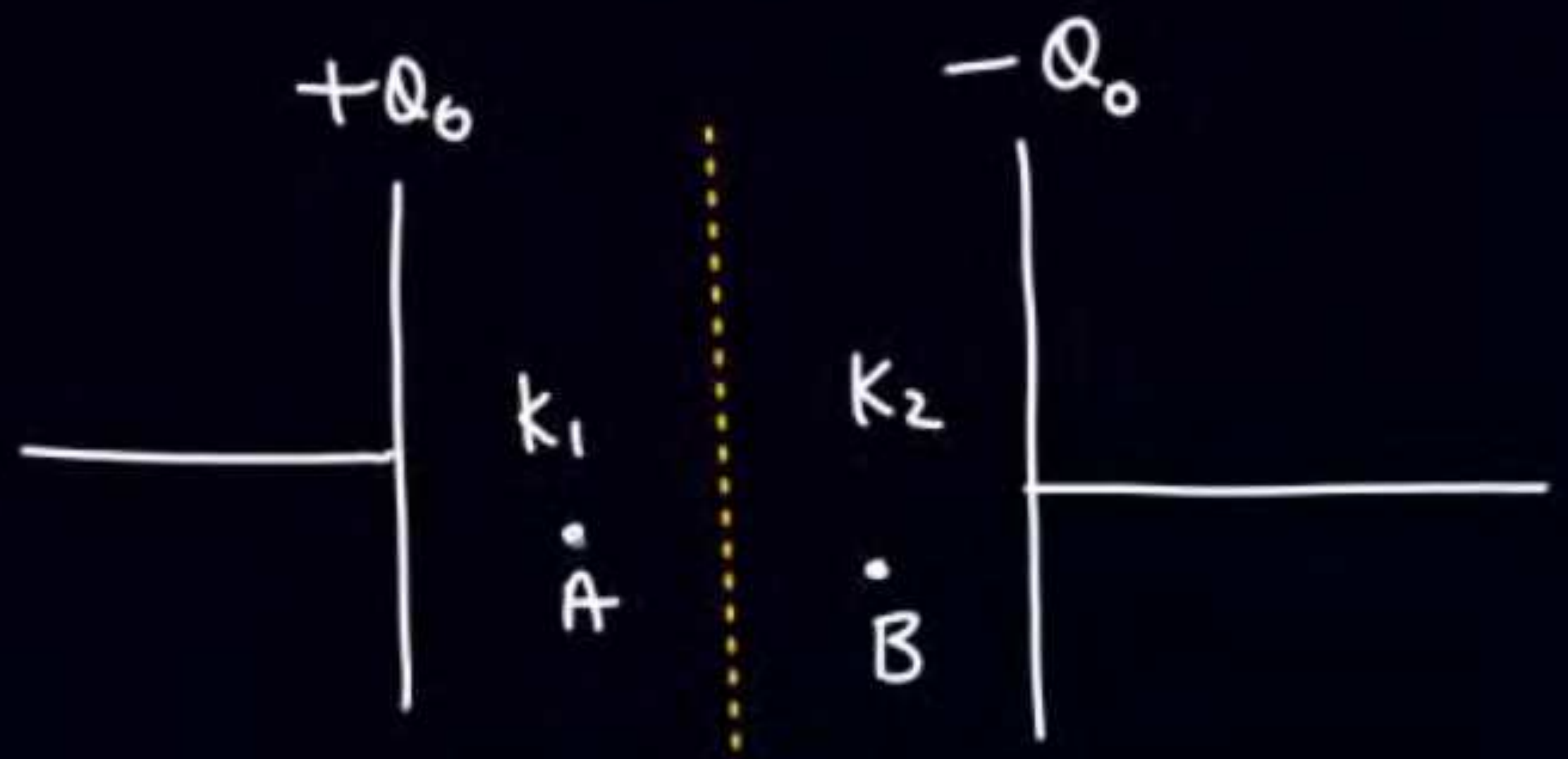
$\frac{Q_1 \; 2A/3}{A/3 \cdot Q_2} \rightarrow \frac{2Q_1}{Q_2} = K$

$\frac{C}{C_1} = \frac{(K+2)}{\cancel{3}\,K/\cancel{3}}$

$= \frac{K+2}{K}$

Ans. (1, 4)



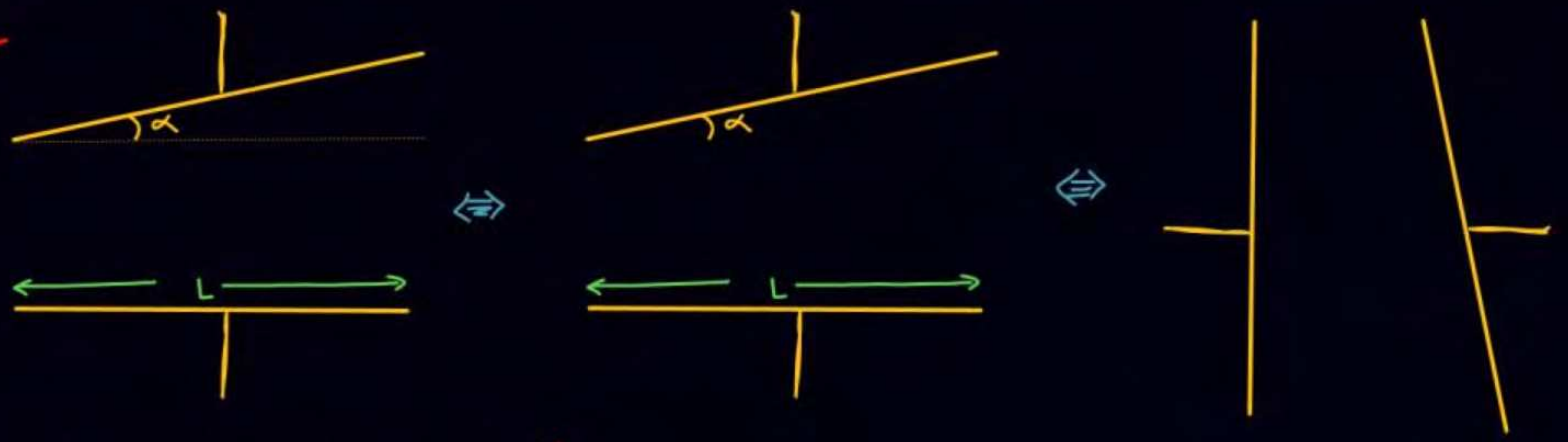


$$E_A = \frac{E_0}{K_1}$$

$$E_B = \frac{E_0}{K_2}$$

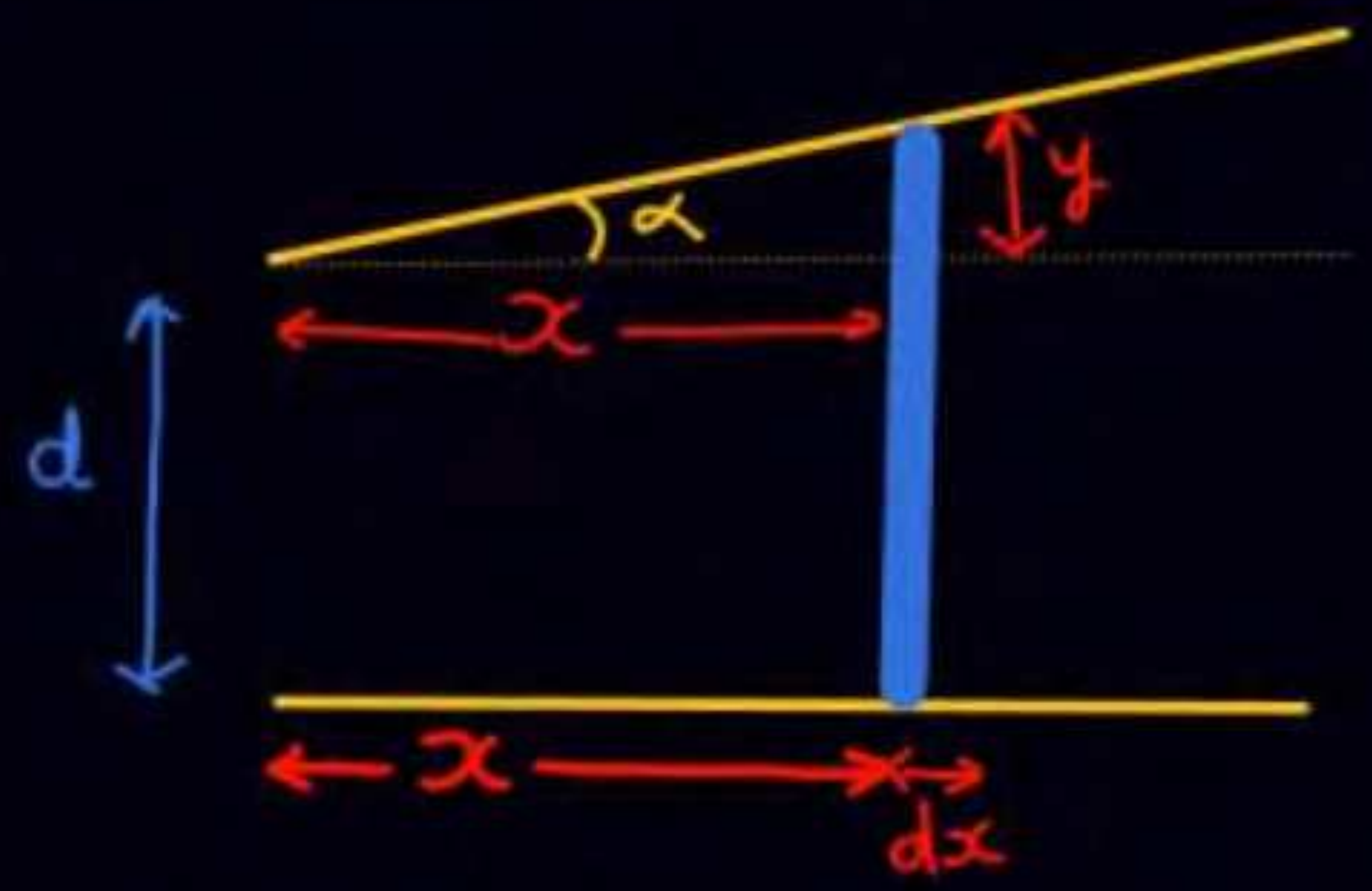
H/W

Q



find C_{eq}. (if α is very small)

Q



$$\tan\alpha = \frac{y}{x} = \alpha$$

$$y = x\alpha$$

$$dc = \frac{L\,dx \cdot \epsilon_0}{d+y} = \frac{L\,dx\,\epsilon_0}{d+x\alpha}$$

$$C_{eq} = \int dc = \int_0^L \frac{L\epsilon_0\,dx}{d+x\alpha} = \frac{L\epsilon_0}{\alpha}\ln\left(\frac{d+L\alpha}{d}\right)$$

$$C_{eq} = \frac{L\epsilon_0}{\alpha}\ln\left(1+\frac{L\alpha}{d}\right)$$ Ans

Log Expansion

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$C_{eq} = \frac{L\epsilon_0}{\alpha} \ln\left(1 + \frac{L\alpha}{d}\right)$$

Log Expansion

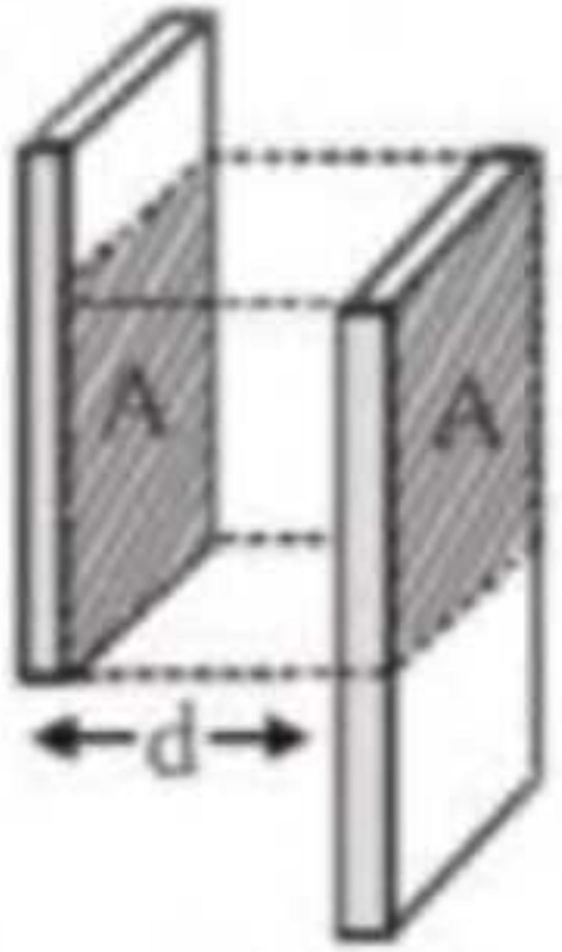
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$C_{eq} = \frac{L\epsilon_0}{\alpha}\left[\frac{L\alpha}{d} - \left(\frac{L\alpha}{d}\right)^2 \frac{1}{2} + \dots\right]$$

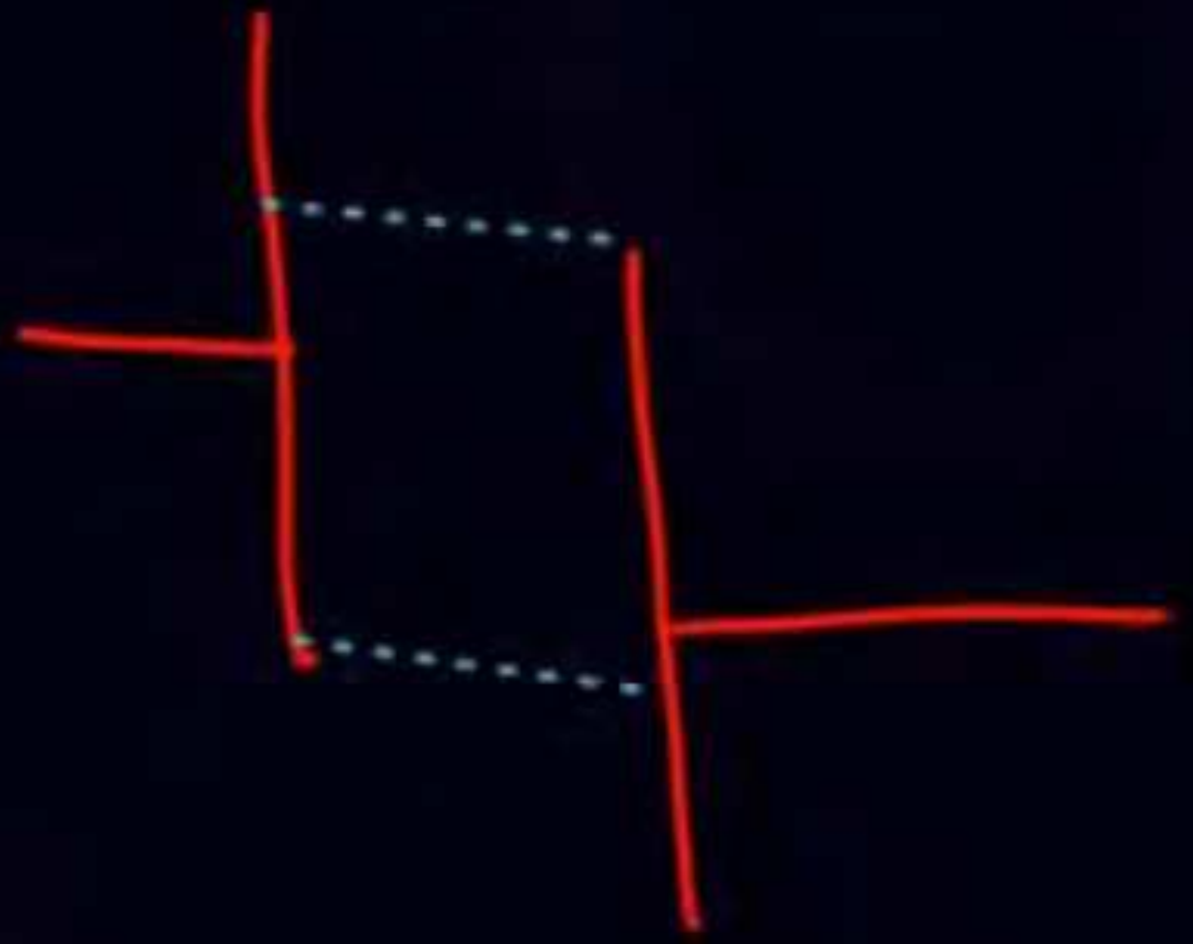
$$C_{eq} = \frac{L\epsilon_0}{\alpha}\,\frac{L\alpha}{d}\left[1 - \frac{L\alpha}{2d} + \dots\right]$$ (α is very small)

$$C_{eq} = \frac{\epsilon_0 L^2}{d}\left[1 - \frac{L\alpha}{2d}\right]$$

- If one of the plates of parallel plate capacitor slides relatively than C decrease (As overlapping area decreases).



$$C = \frac{A_{common}\epsilon_0}{d}$$



$$C = \frac{A\epsilon_0}{d}$$

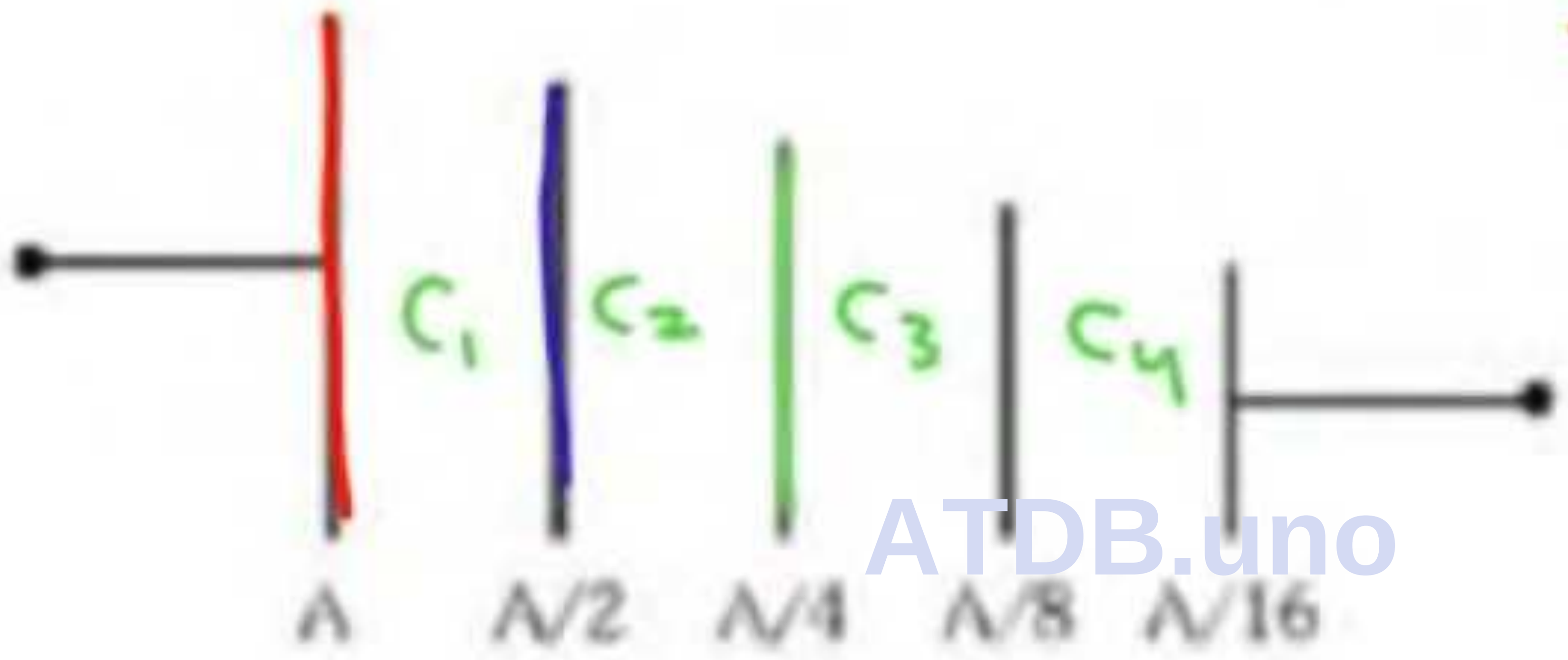
Conducting plates each are placed face to face & equi-spaced at distance d. Area of each plate is half the previous plate. If area of first plate is A. Then the equivalent capacitance of the system shown is :-

पाँच चालक प्लेटों को समान दूरी d पर एक दूसरे के सम्मुख रखा गया है। प्रत्येक प्लेट का क्षेत्रफल पहले वाली प्लेट से आधा है। यदि पहली प्लेट का क्षेत्रफल A हो तो प्रदर्शित निकाय की तुल्यांकी धारिता होगी :-

$C_1 = \frac{(A/2)\epsilon_0}{d}$

$C_2 = \frac{A/4\ \epsilon_0}{d}$

$C_3 = \frac{(A/8)\epsilon_0}{d}$

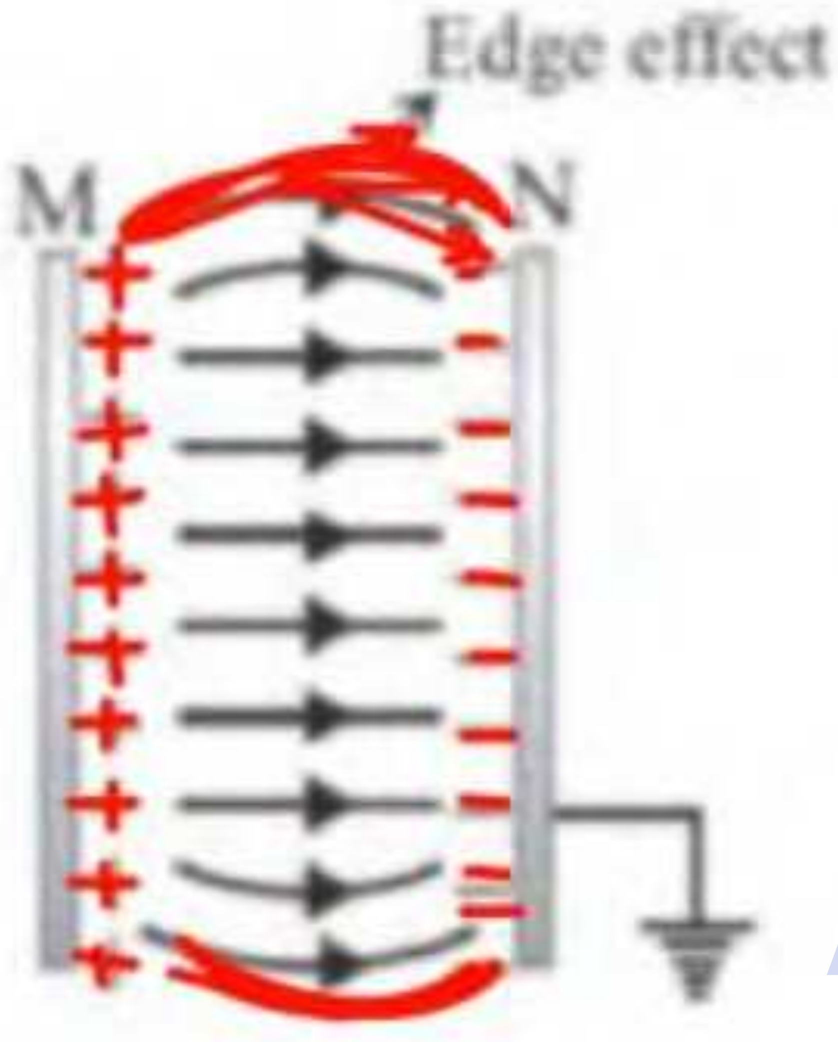


$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$

(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{\varepsilon_0 A}{10d}$ (C) $\frac{\varepsilon_0 A}{20d}$ (D) $\frac{\varepsilon_0 A}{30d}$

$C_4 = \frac{(A/16)\epsilon_0}{d}$

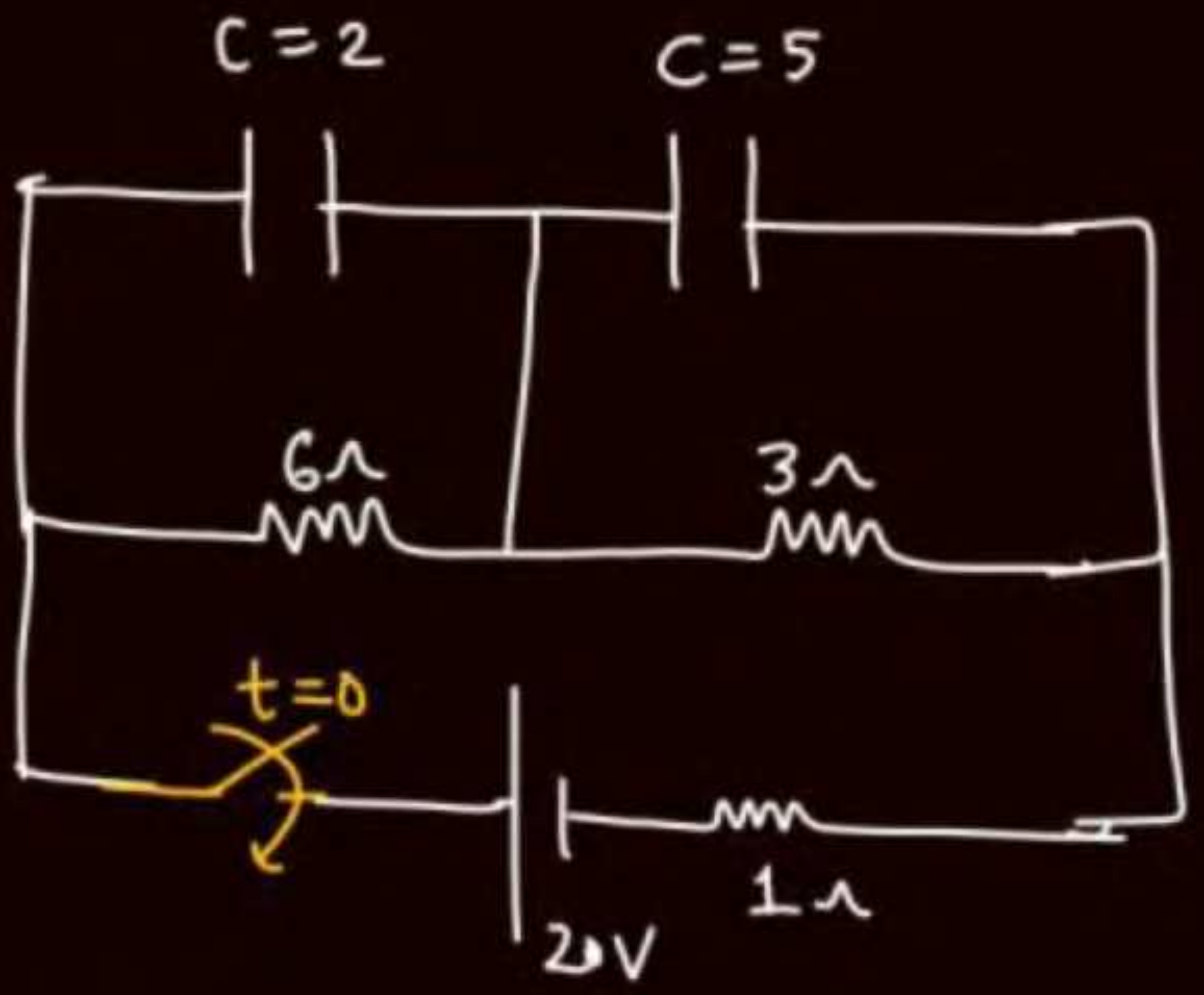
- Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



$\vec{E}$ = uniform in the centre

$\vec{E}$ = non-uniform at the edges

Q



① Find current through battery at $t=0^+$

② Charge on capacitor at steady state and current passing through battery.

Ans ① 20A

② 2A, $q_1 = 24$

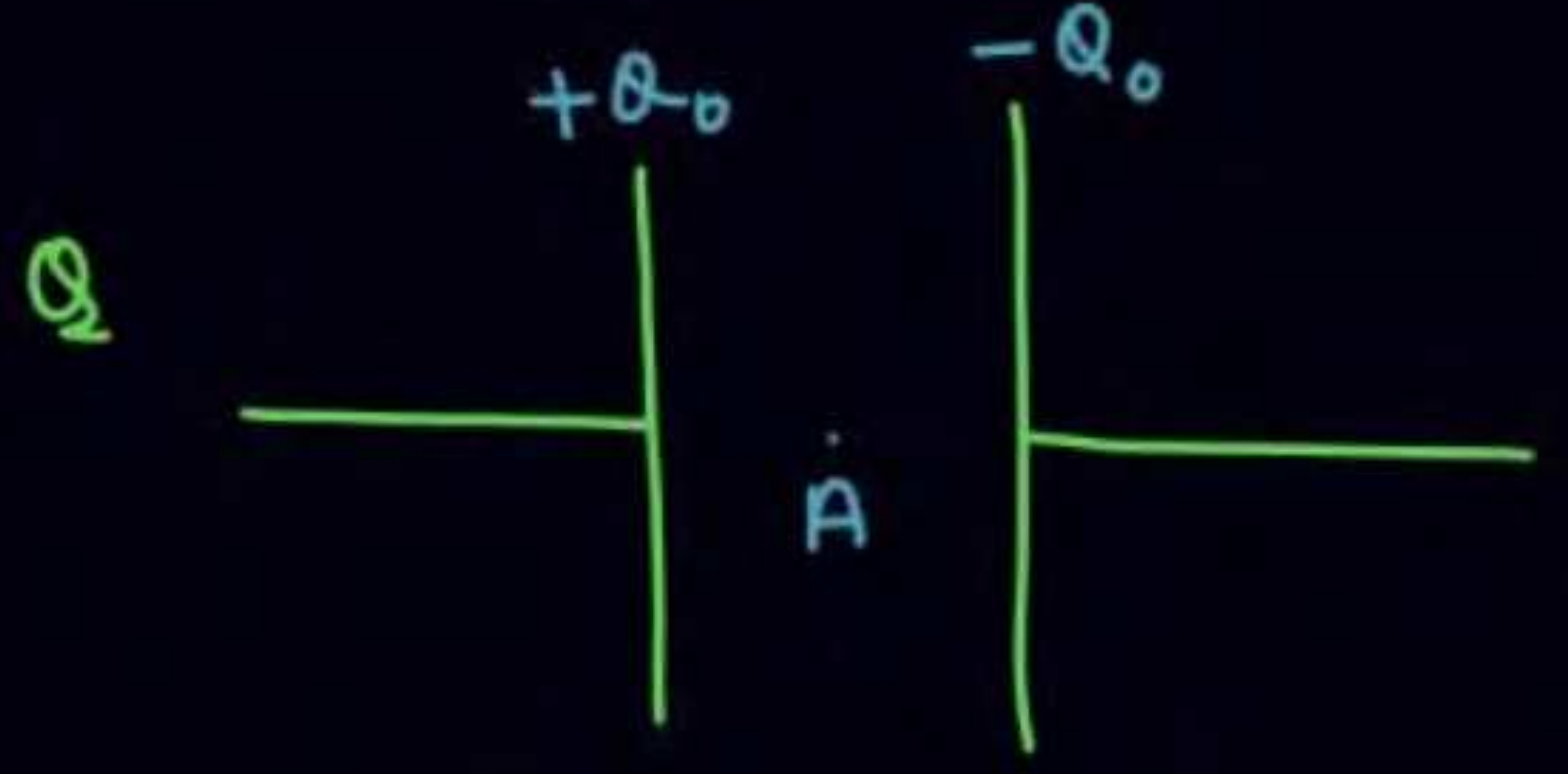
$q_2 = 6 \times 5 = 30$

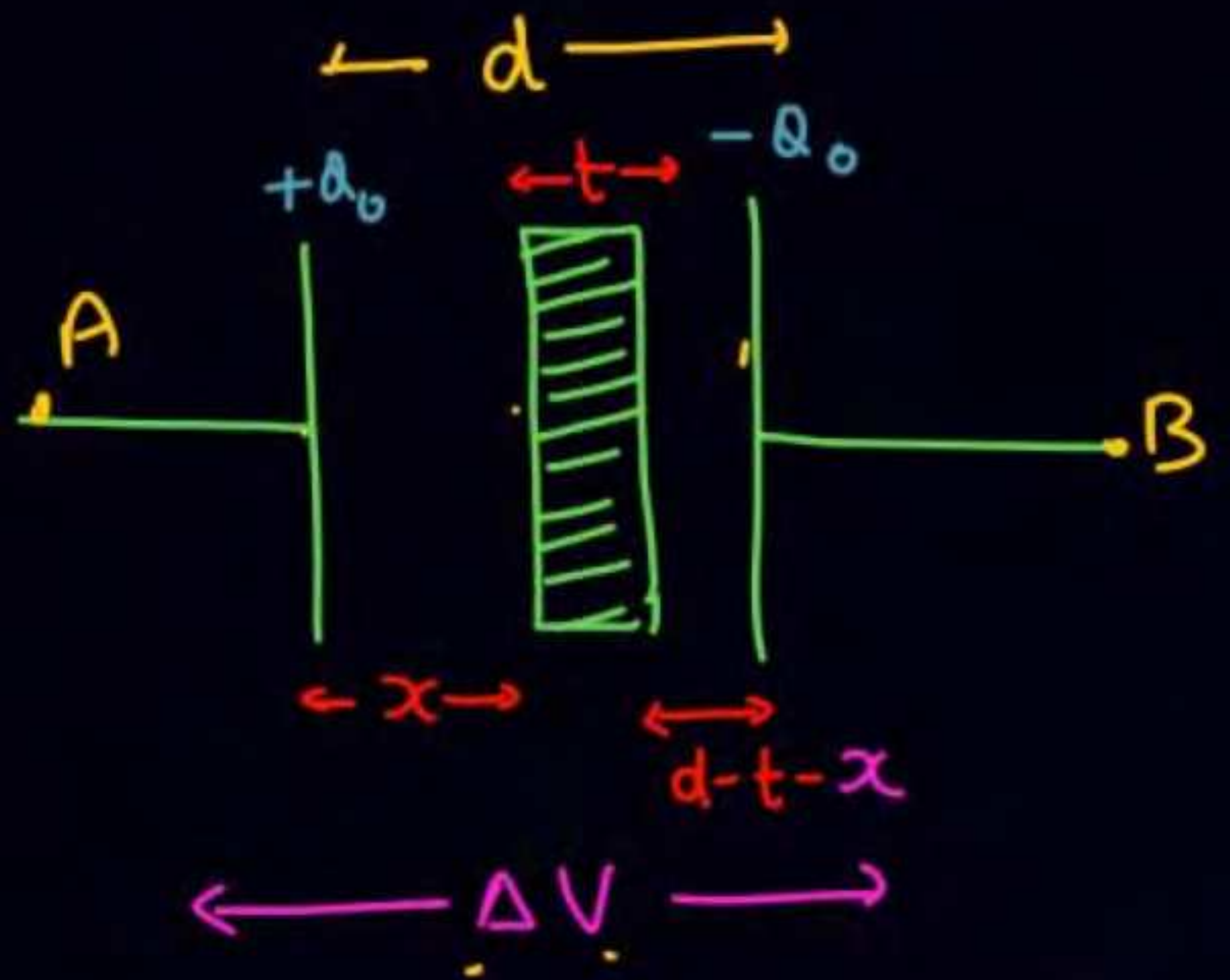
$$E_0 = \frac{Q_0}{A\epsilon_0}$$

$$E_A = \frac{E_0}{K}$$

$$C_{\text{नया}} = \frac{A\epsilon_0}{d - t\left(1 - \frac{1}{K}\right)}$$

$$C = \frac{A\epsilon_0}{d}$$





$$E_A = E_{\text{हवा}} = \frac{Q_0}{A\epsilon_0} = E_0$$

$$V_A - V_B = \Delta V = E_0 x + \frac{E_0}{K}(t) + E_0(d-t-x)$$

$$\Delta V = E_0\left[\frac{t}{K} + (d-t)\right] = \frac{Q_0}{A\epsilon_0}\left[d - t\left(1-\frac{1}{K}\right)\right]$$

$$\Delta V = \frac{Q_0}{A\epsilon_0 / d - t\left(1-\frac{1}{K}\right)} = \frac{Q}{C}$$

$$\boxed{C = \frac{A\epsilon_0}{d - t\left(1-\frac{1}{K}\right)}}$$


THANK
YOU
ATDB.uno