PRAYAS

JEE 2025

Lecture - 01

Physics

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Wave Optics

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Topics *to be covered*

1 Hygen's theory, wavefront

2 Interference of light.

3

4

$$I \propto A^2$$

Light $\equiv$?

I_0

$$y_1 = A\sin(wt - kx)$$

$$y_2 = A\sin(wt - kx + 180°)$$

$$y_1 + y_2 = 0$$

I_0

particle | wave

Newton | Hygen's

$\rightarrow$ wave

S_1

s

SLT

ri

particle

Photoelectric

exp

Wave

YDSE

wavefront $\longrightarrow$ Locus of all the particles vibrating in same phase.

Same

$t=0$ Center

Vt

t

$$\phi_P - \phi_s = 0$$

$$\Delta\phi_{PS} = \Delta\phi_{QT} = \Delta\phi_{PR}$$

spherical w/f

P

S

→ w/f

$$I_A = \frac{P}{4\pi r^2}$$

P (watt)
point source

T

U

A

R

W/f cylindrical

P

$$I = \frac{P}{2\pi r l}$$

$$I \propto \frac{1}{r}$$

l

r

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$P, Q, R \longrightarrow$ Same phase

$S, T, U \longrightarrow$ '' ''

P

$(w|f)_2$

A

$(w|f)_1$

point
source

B $\quad$ C

R

Q

$A, B, C \longrightarrow$ same phase

$P, Q, R \longrightarrow \quad "$

Point
Source

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ray
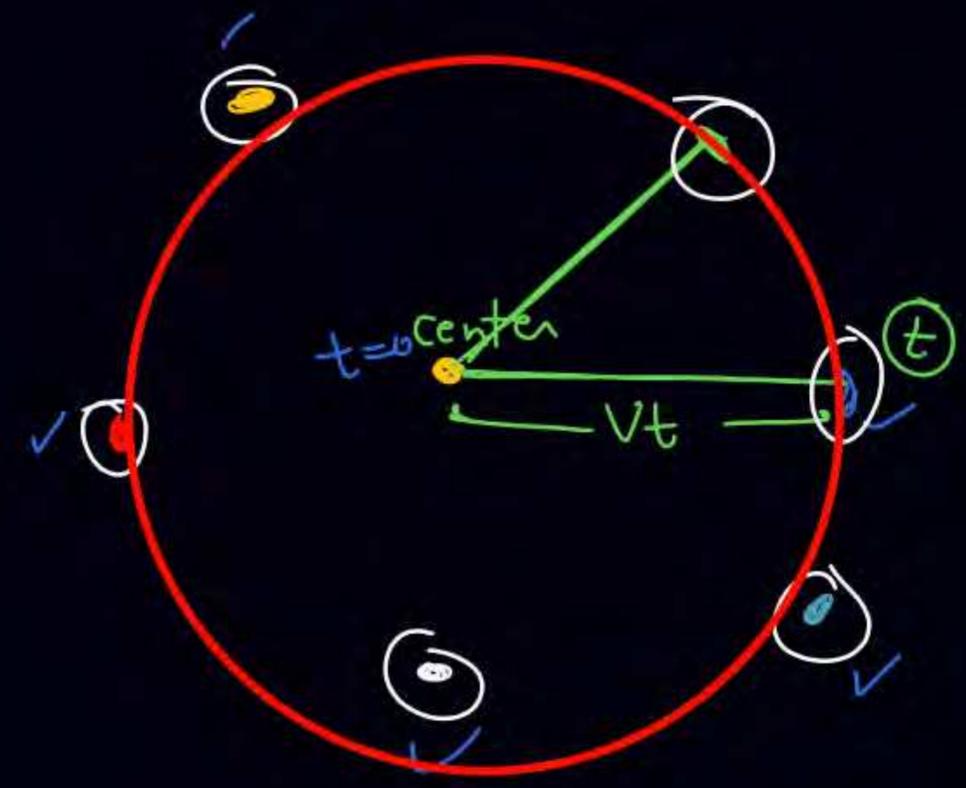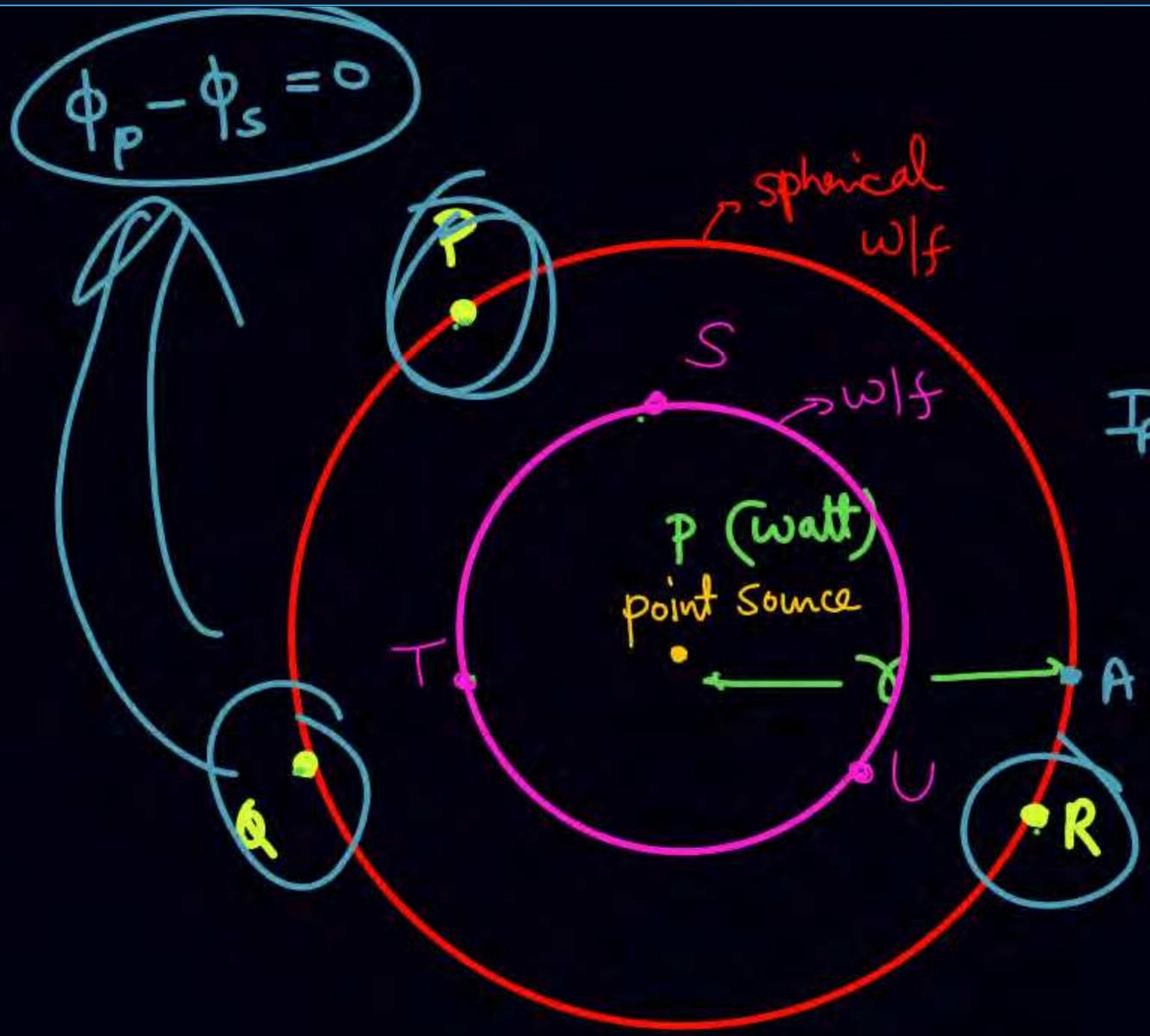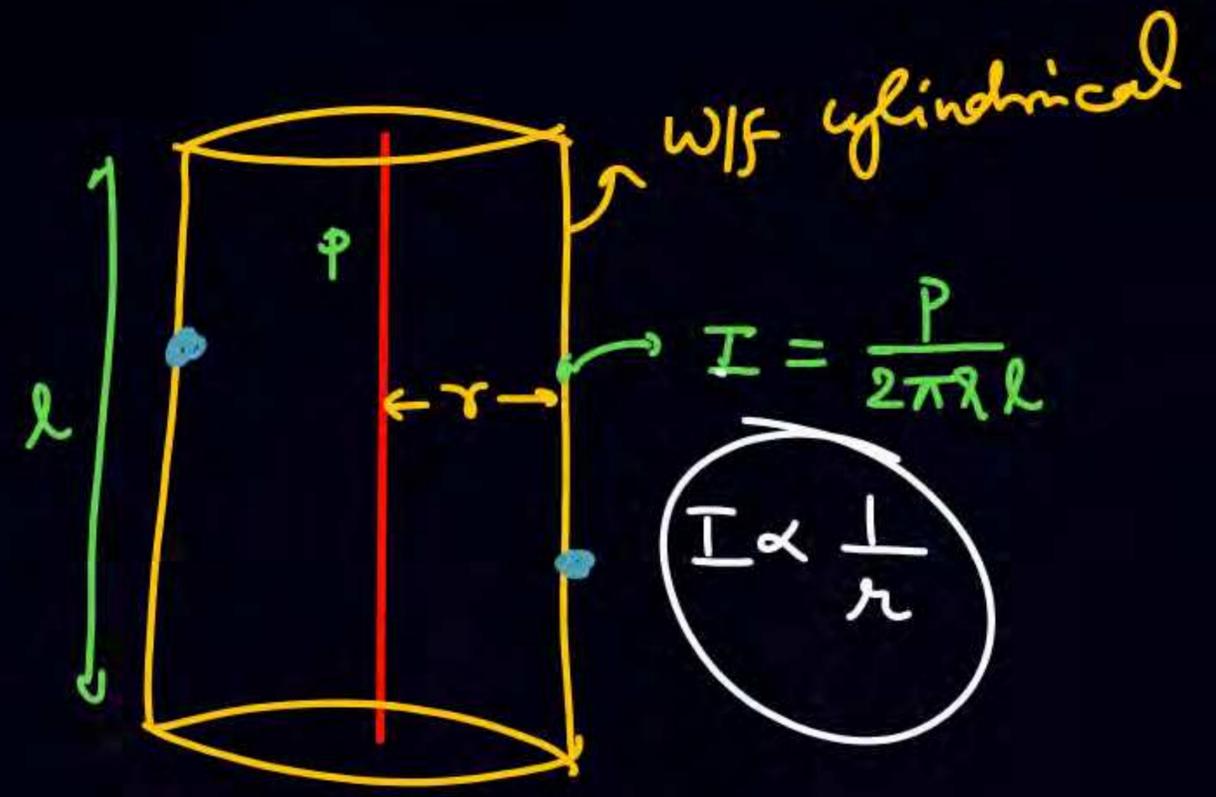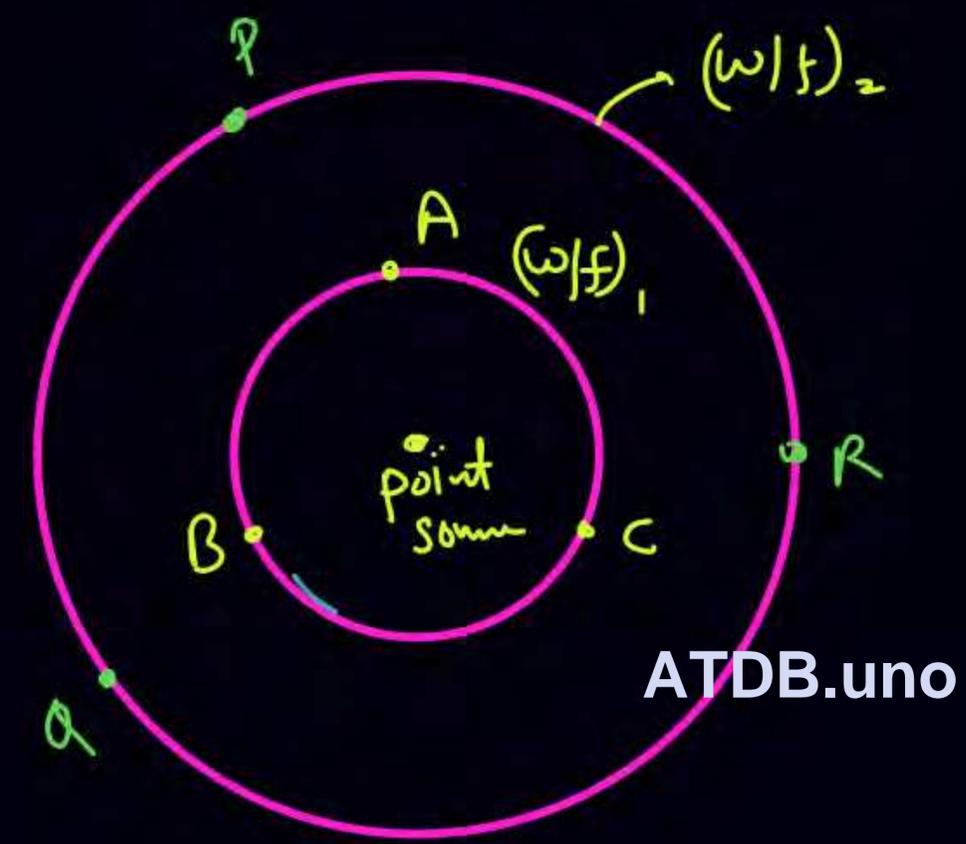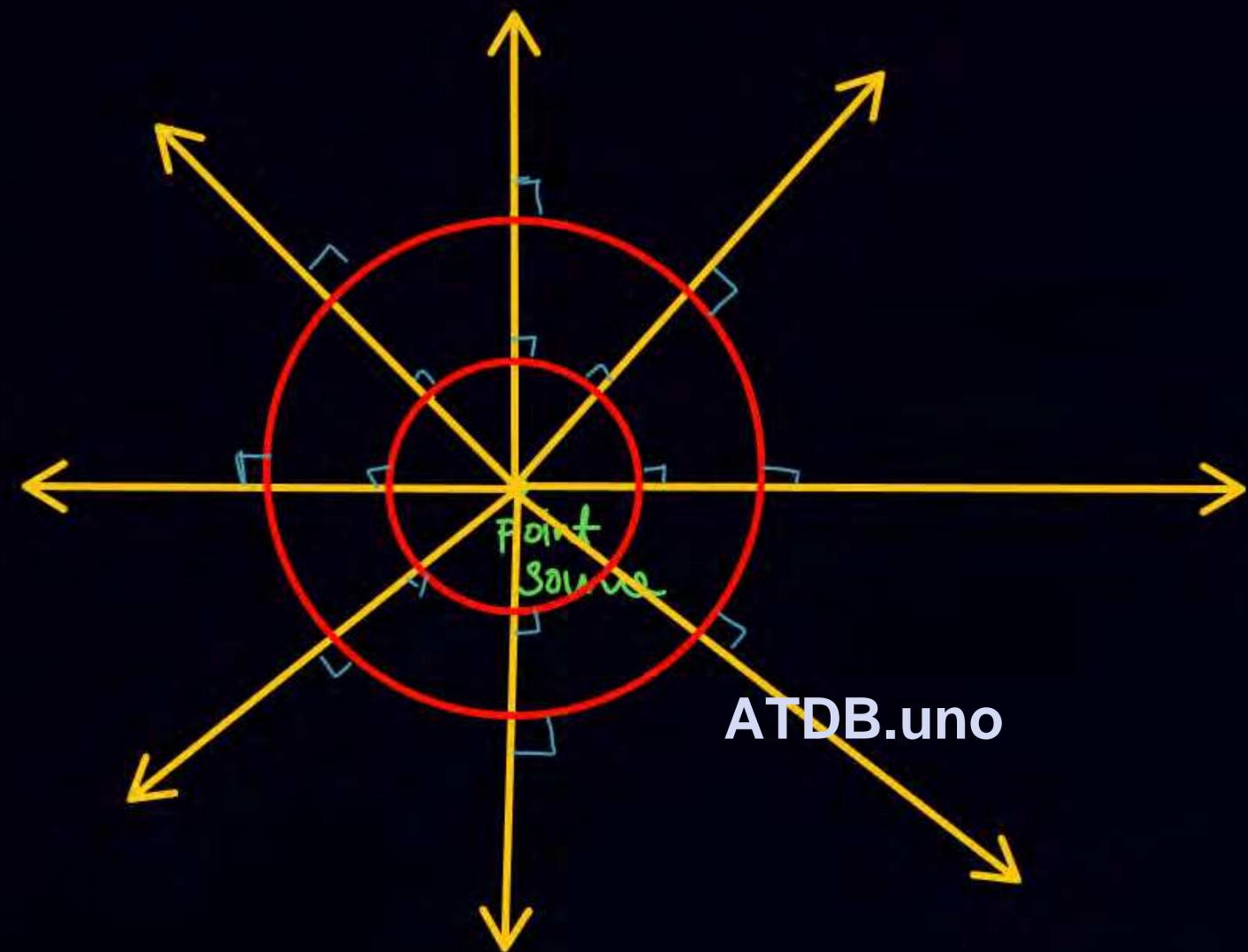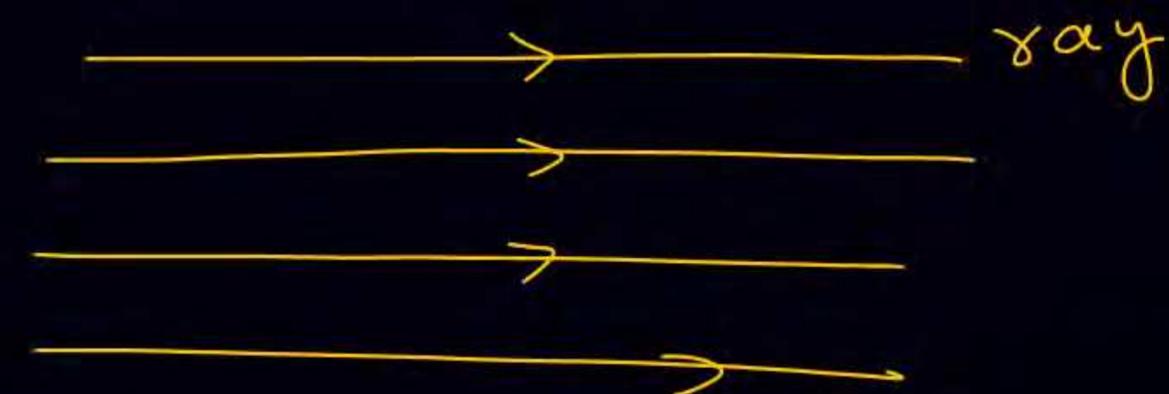
<u>wave front</u> $\longrightarrow$ Locus of all the particles vibrating in the same phase.

* for point source $\longrightarrow$ w/f $\equiv$ spherical

* " Linear " $\longrightarrow$ w/f $\Rightarrow$ cylindrical

* Rays are always perpendicular to wavefront (w/f).

* Wavefront are always perpendicular to rays.

* Time taken by wave from one w/f to another w/f for all rays are same.

* Phase diff b/w all the points on a same wavefront is zero.

Wavefront $\longrightarrow$ Locus of all the particle vibrating in same phase

① **point source**

$$I \propto A^2$$
$$I \propto \frac{1}{\gamma^2}$$
$$A \propto \frac{1}{\gamma}$$

$$I \propto \frac{1}{\gamma^2}$$

Spherical w/f

② **line source**

$$I \propto \frac{1}{\gamma}$$
$$I \propto A^2$$
$$A \propto \frac{1}{\sqrt{\gamma}}$$
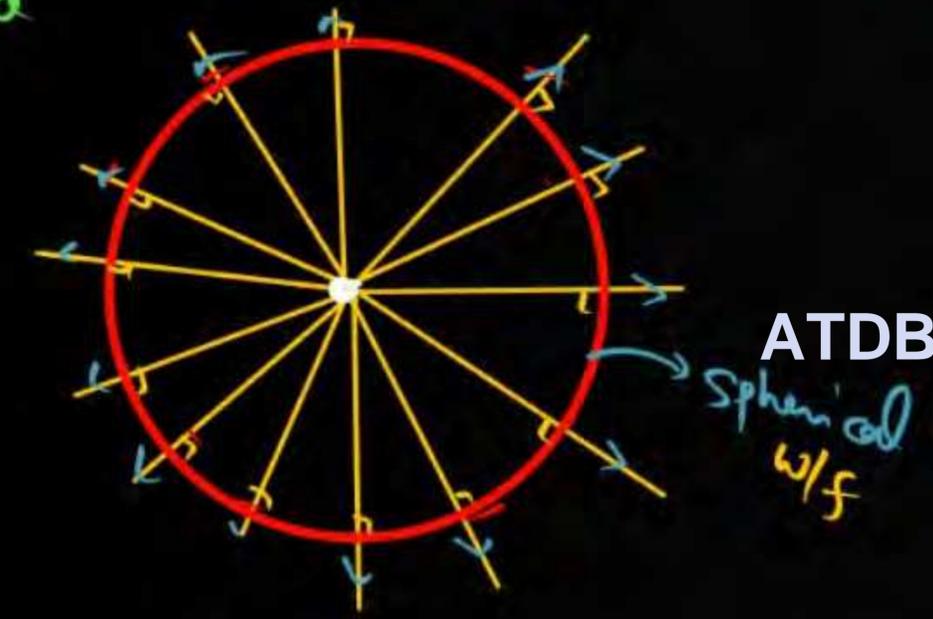
cylindrical w/f

③ **parallel beam**

$$I \propto \gamma^0$$

w/f   w/f
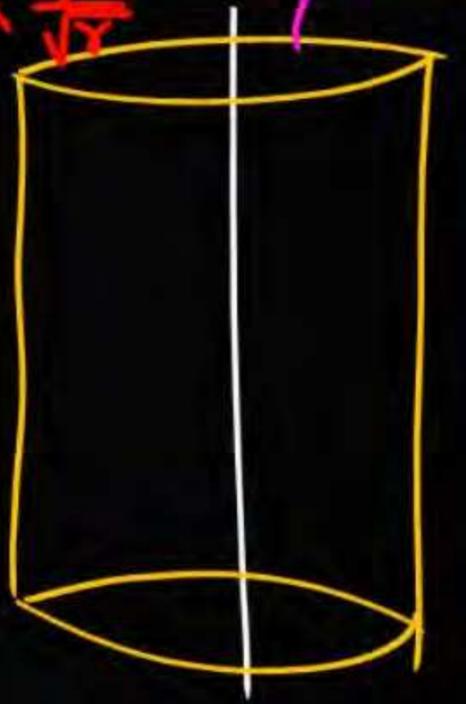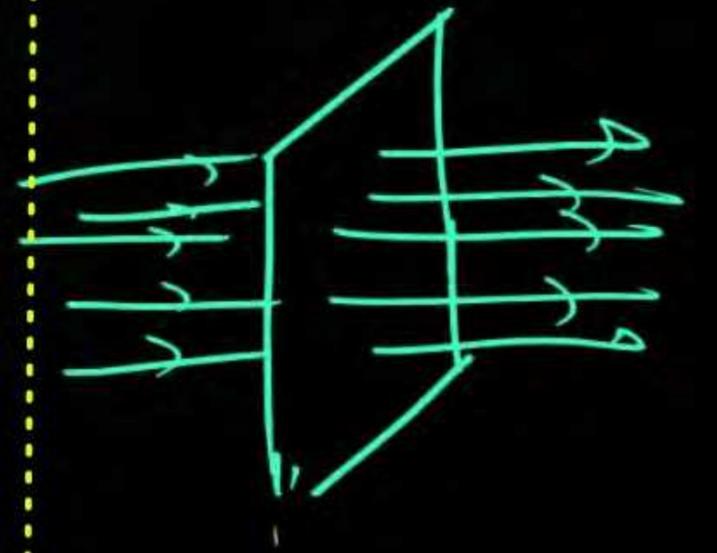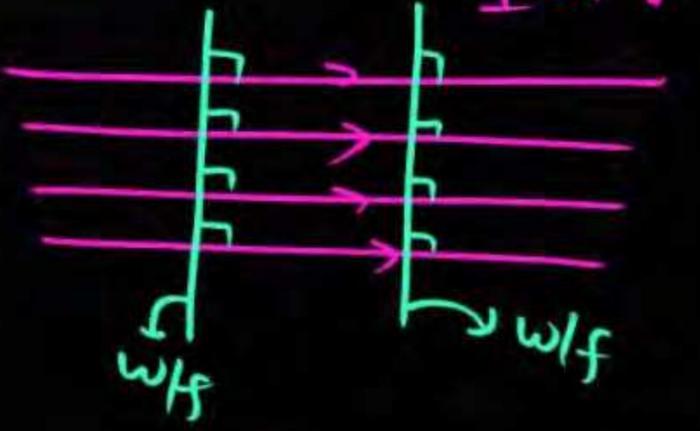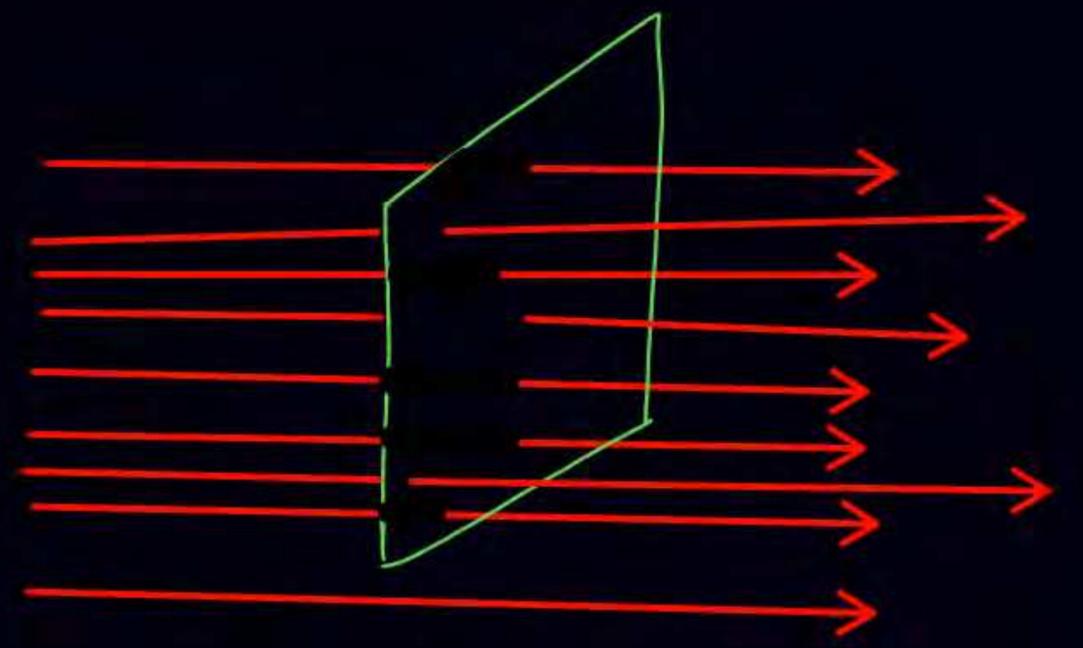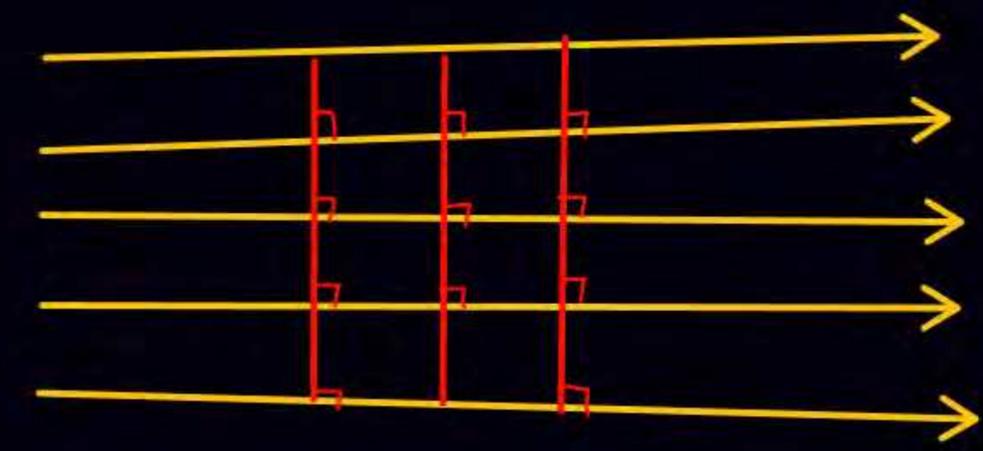
Draw the wavefront

(1)

Q₂
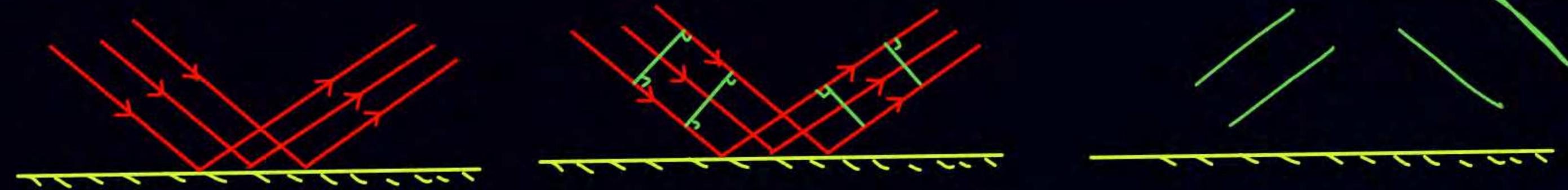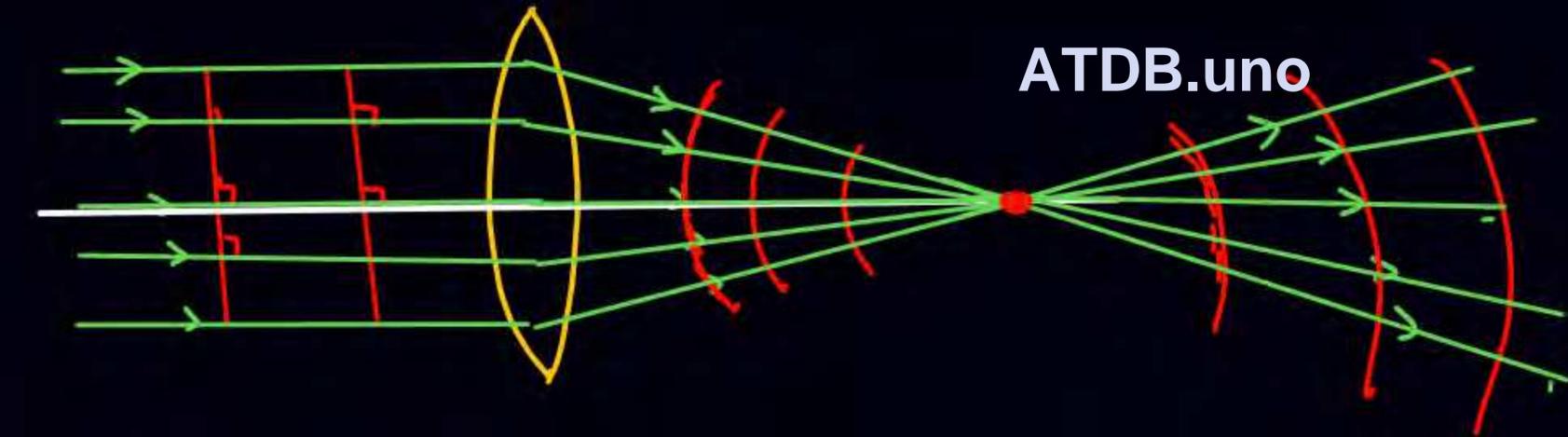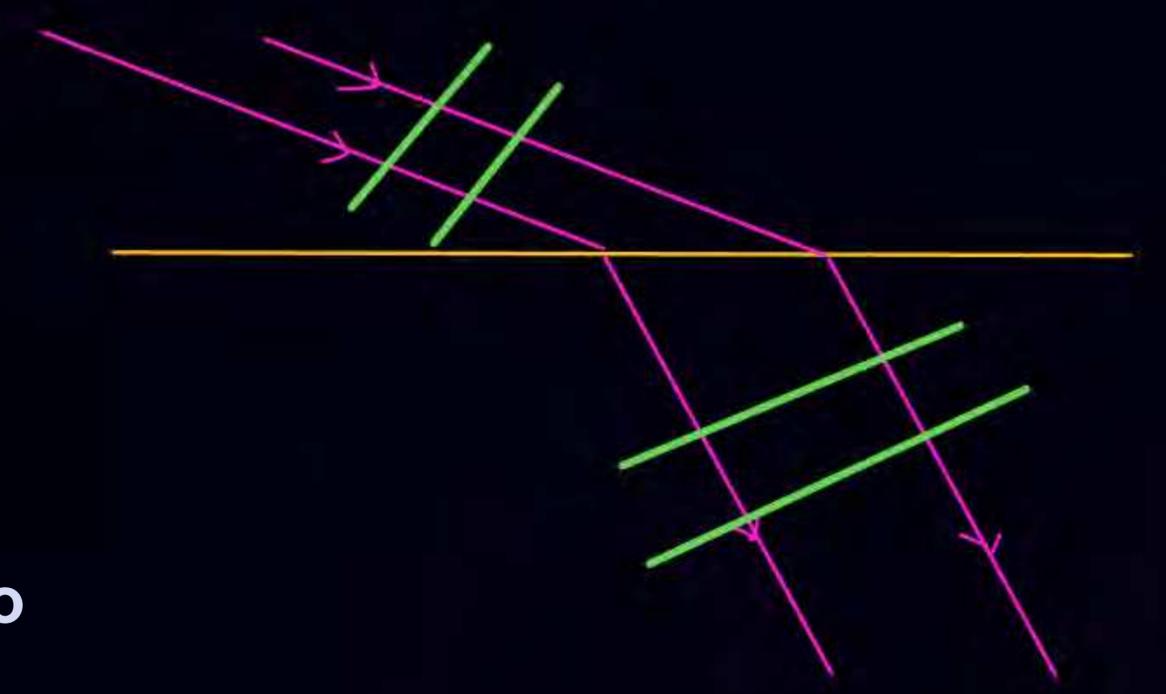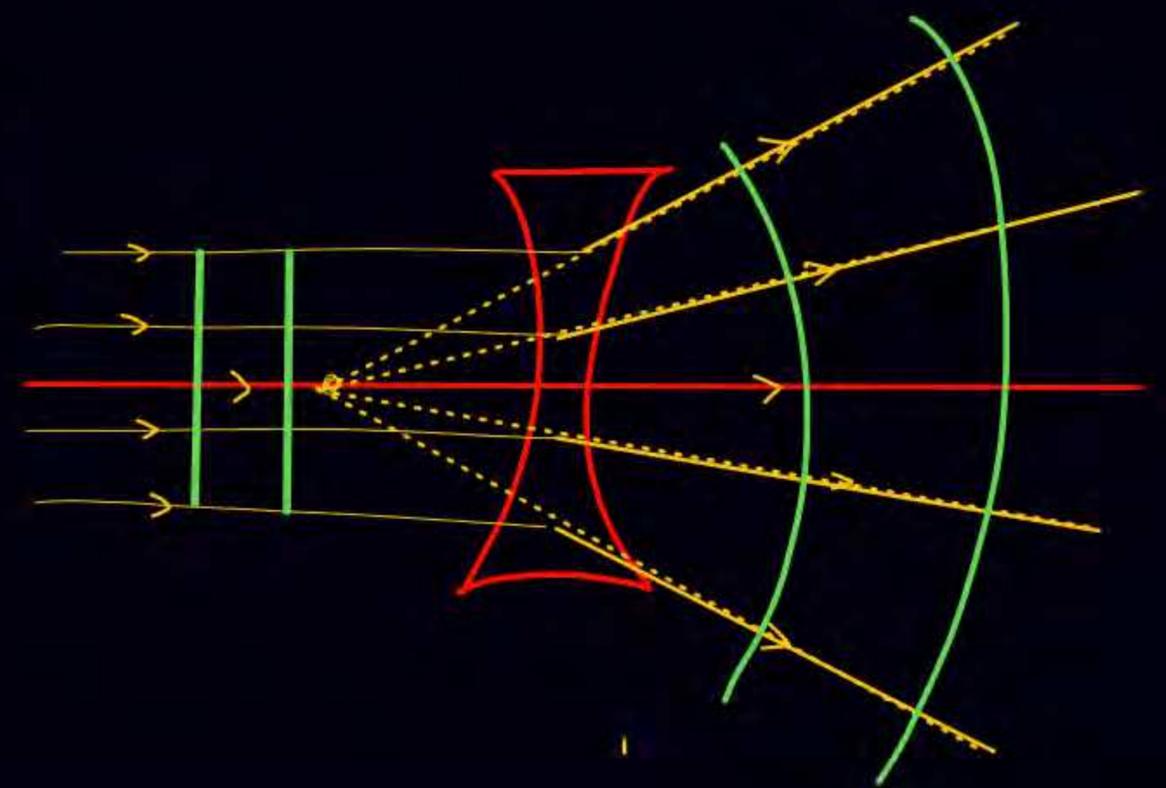
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Q
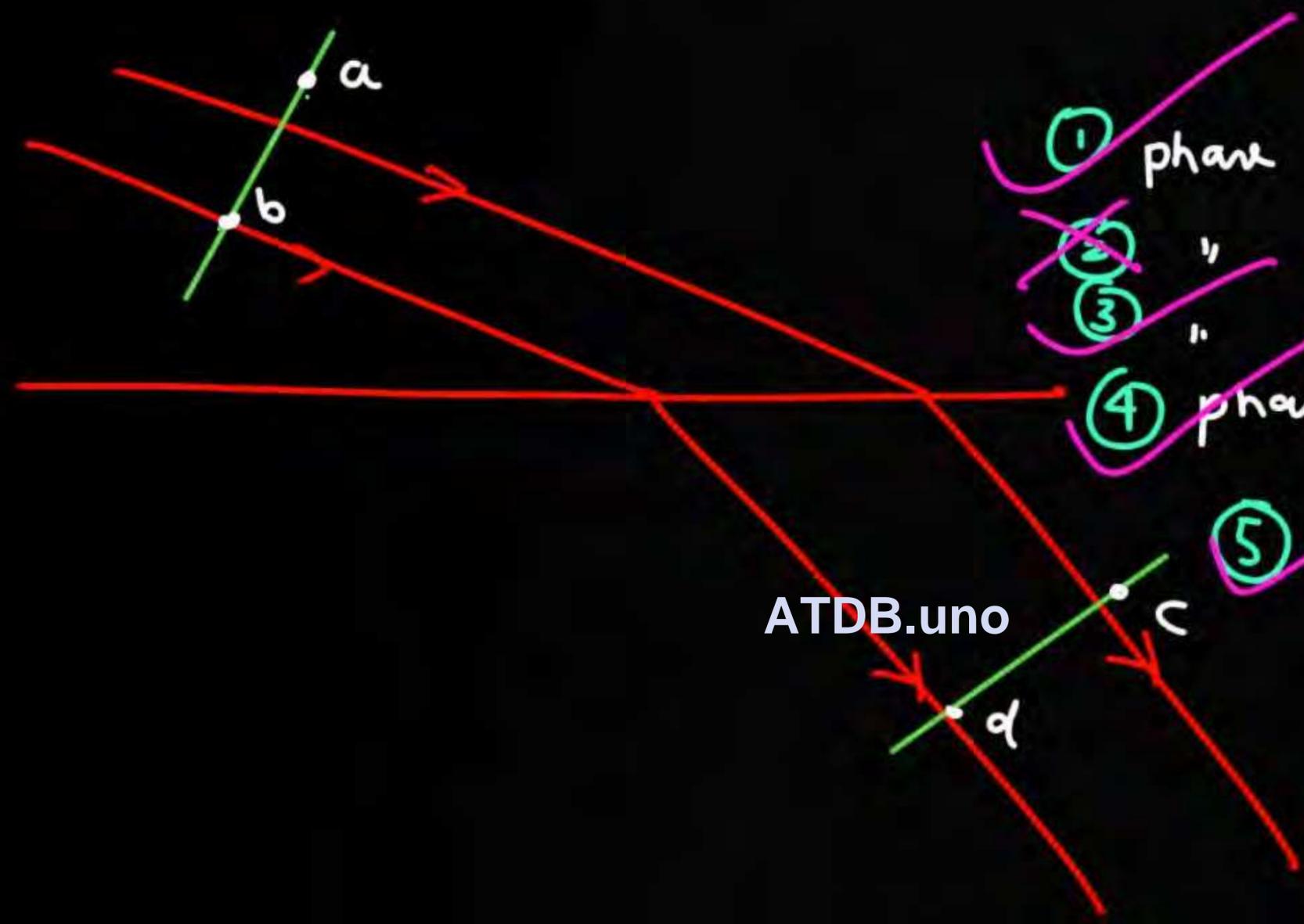
① phase a & b must be same

② " a & c must be same

③ " d & c " "

④ phase diff. b/w a & b is zero

⑤ phase diff b/w a & c is
equal to phase diff b/w
b & d

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CHARACTERISTIC OF WAVEFRONT

(a)   The phase difference between various particles on the wavefront is zero.

(b)   These wavefronts travel with the speed of light in all directions in an isotropic medium.

(c)   A point source of light always gives rise to a spherical wavefront in an isotropic medium.

(d)   In an anisotropic medium it travels with different velocities in different directions.

(e)   Normal to the wavefront represents a ray of light.

(f)   It always travels in the forward direction of the medium.

S.No.	Wavefront	Shape of light source	Diagram of shape of wavefront	Variation of amplitude with distance	Variation of intensity with distance
1.	Spherical	Point source		$A \propto \dfrac{1}{d}$ or $A \propto \dfrac{1}{r}$	$I \propto \dfrac{1}{r^2}$
2.	cylindrical	Linear or slit		$A \propto \dfrac{1}{\sqrt{d}}$ or $A \propto \dfrac{1}{\sqrt{r}}$	$I \propto \dfrac{1}{r}$
3.	Plane	Extended large source situated at very large distance		$A = $ constant	$I = $ constant
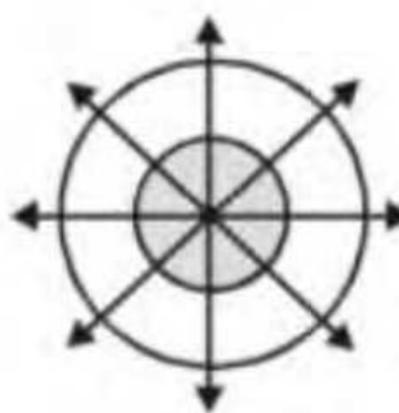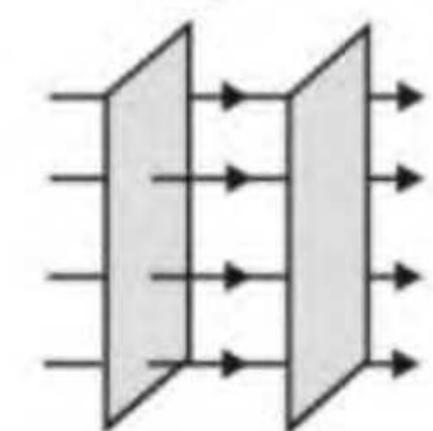
WAVE THEORY OF LIGHT

(i)   The locus of all ether particles vibrating in same phase is known as wavefront.

(ii)   Light travels in the medium in the form of wavefront.

(iii) When light travels in a medium then the particles of medium start vibrating and consequently a disturbance is created in the medium.

(iv)   Every point on the wave front becomes the source of secondary wavelets. It emits secondary wavelets in all directions which travel with the speed of light (v),

The tangent plane to these secondary wavelets represents the new position of wave front.

(a)

(b)
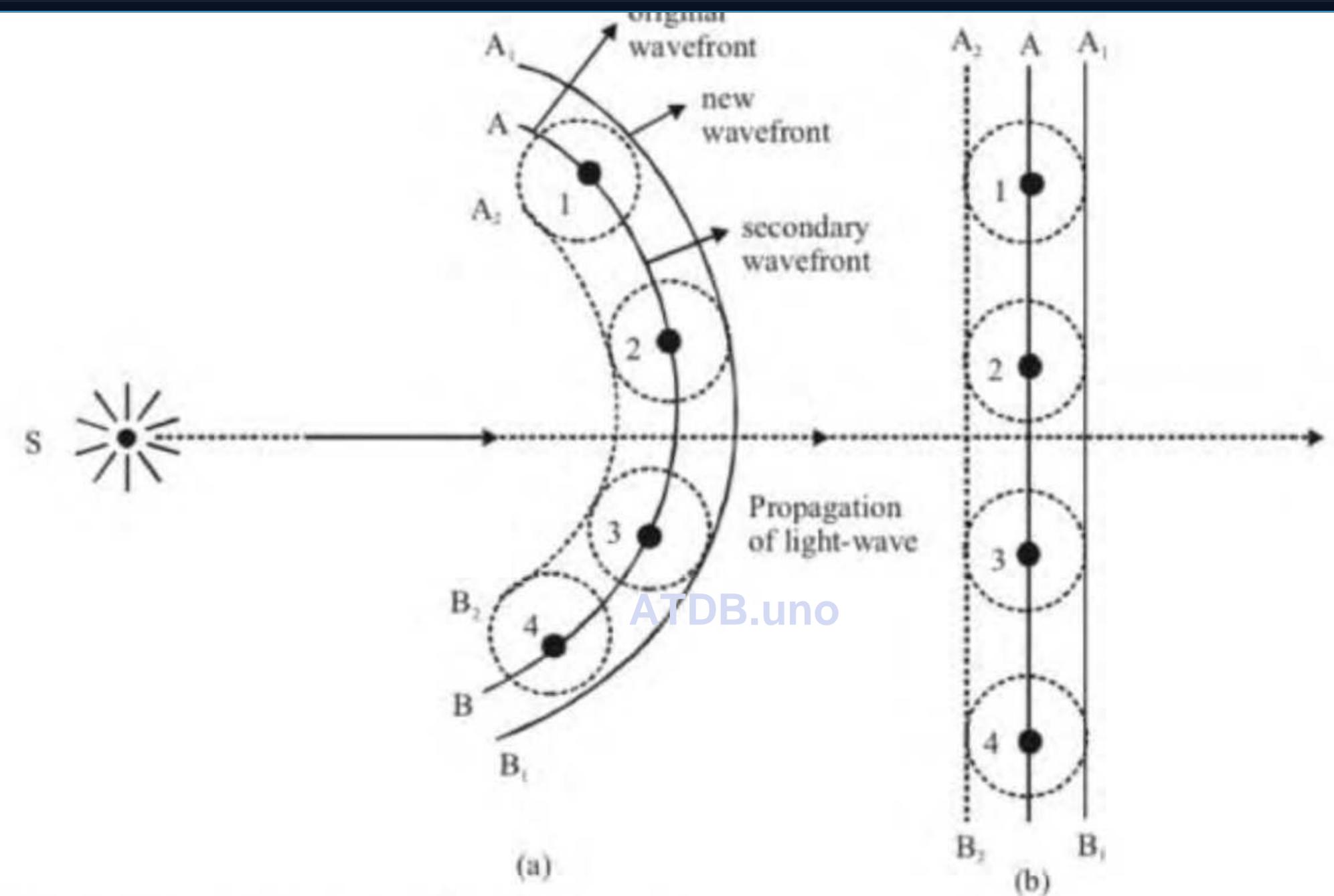
The phenomena explained by this theory

(i)   Reflection, refraction, interference, diffraction, polarisation and double refraction.

(ii)   Rectilinear propagation of light.

(iii)   Velocity of light in rarer medium being grater than that in denser medium.

* All points on a wavefront viberates in same phase with same frequency

* Every point on a wavefront is an independent fresh source which produce secondary waveforms.

* Each points on a wavefront is a source of new disturbance called secondary wavelets. these wavelets are spherical, & wavelets from these points spread out in each direction with speed of wave, if we draw a common tangent to all these sphere we obtain the new position of wavefront at a later time.

w/f

new w/f.

Vt

new w/f

w/f
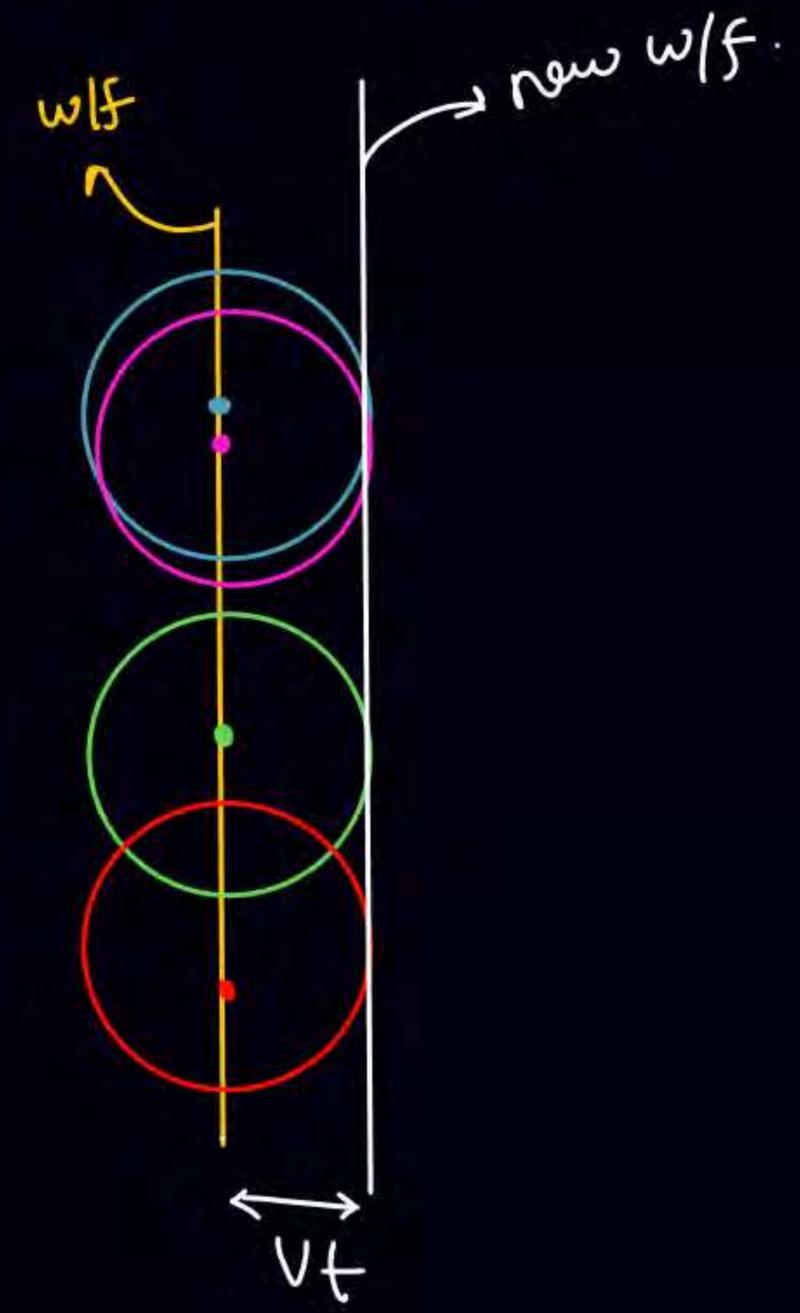
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Coherent source $\longrightarrow$ Two source are said to be coherent if they produce wave of same frequency with const phase difference.

* phase difference is const (independent on time)

* — Two independent source cannot be coherent.

— For sustained interference sources must be coherent.

— To create two coherent source, it is essential that they should derived from same common source.

— Incoherent source $\equiv$ phase diff $= f(t)$

①

②

*

S₁

S₂

①

②

S

S'
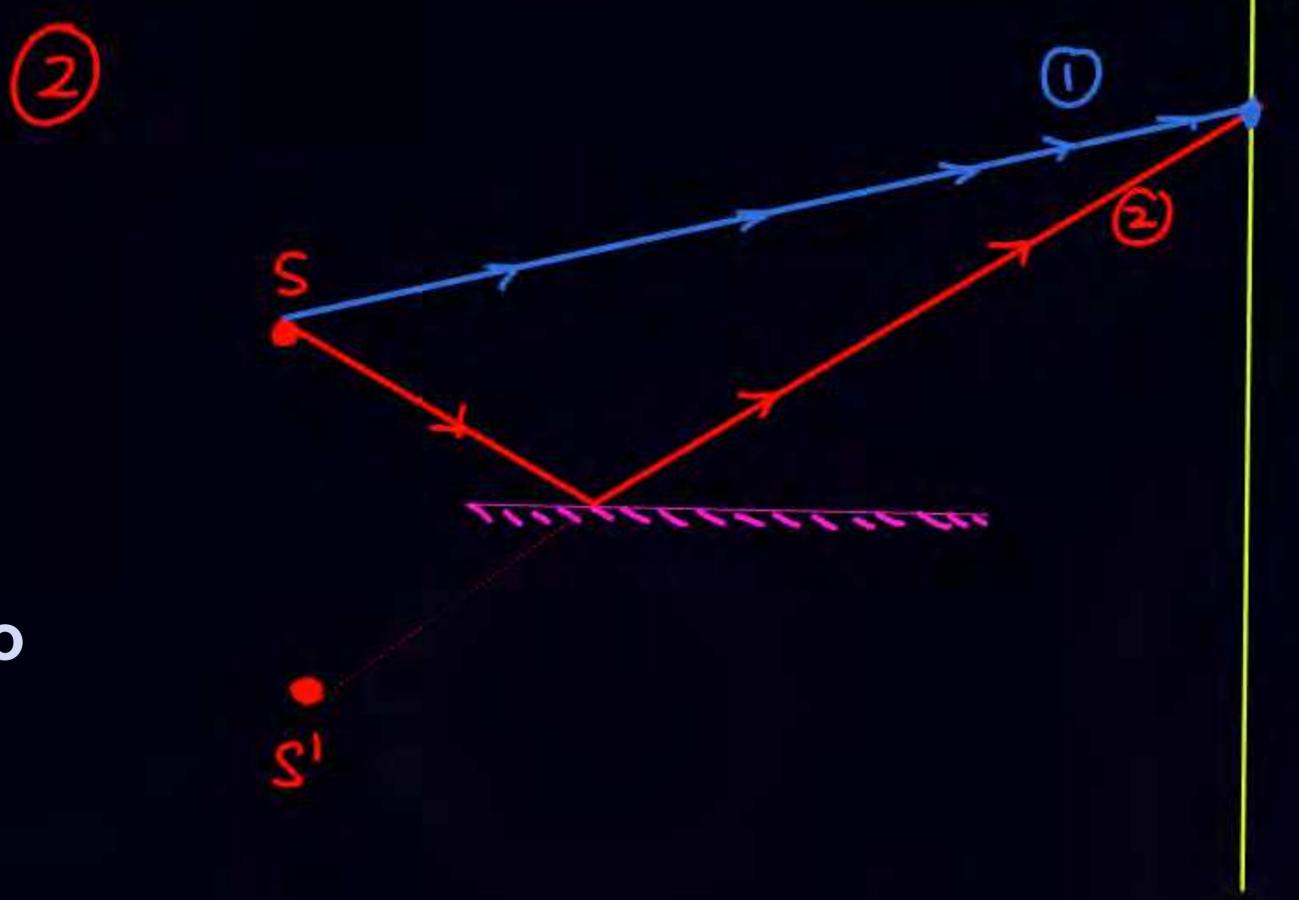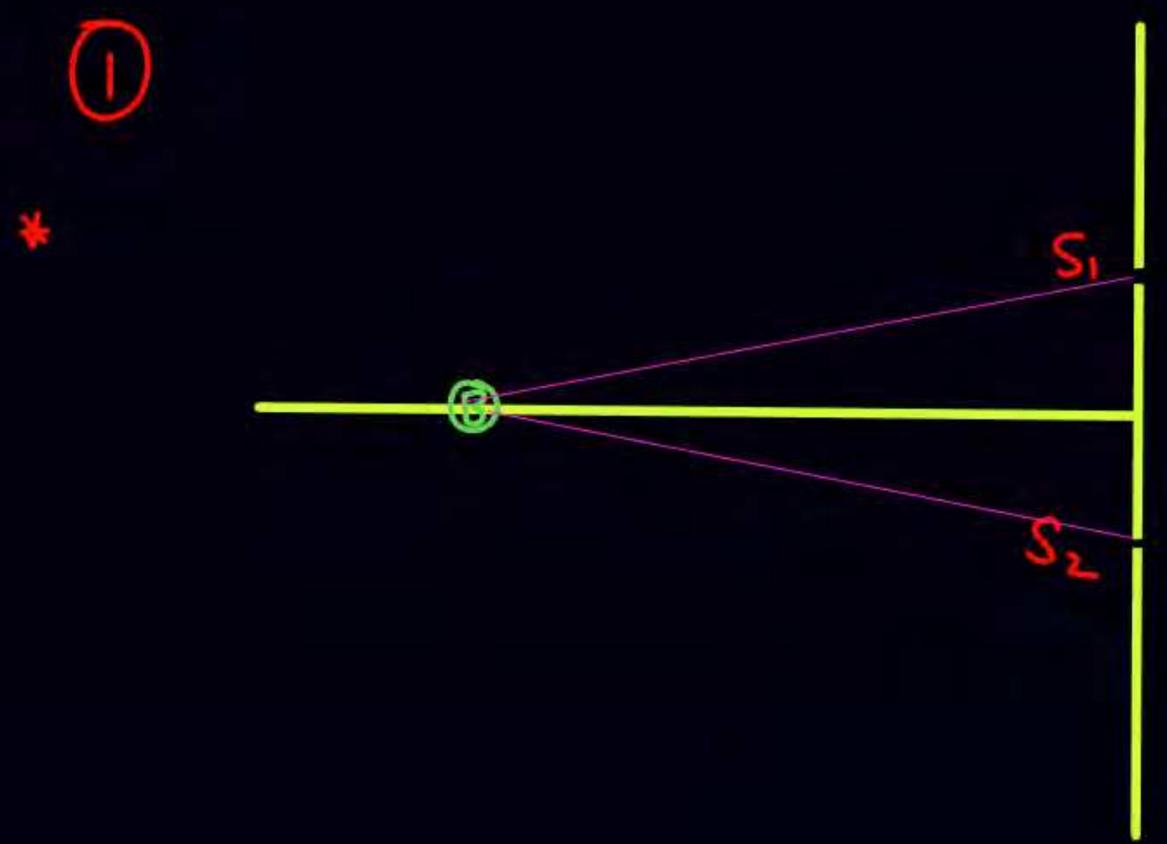
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1. INTERFERENCE OF LIGHT

When two light waves of same frequency with zero initial phase difference or constant phase difference superimpose over each other, then the resultant intensity in the region of superposition is different from the sum of intensity of individual waves.

This modification in intensity in the region of superposition is called interference.

Interference of light

$$\longrightarrow (\text{freq. same})$$

when two waves from coherent source superimpose they gives definite intensity pattern in definite location at space.

$$y_{net} = y_1 + y_2$$

$$y_1 = A_1 \sin(\omega t - kx)$$

$$y_2 = A_2 \sin(\omega t - kx + \phi)$$

$$\longrightarrow \text{phase diff} = \phi \equiv \Delta\phi$$

$$y_{net} = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx + \phi)$$

$$= A_{net} \sin(\omega t - kx + \alpha)$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$$

$$\longrightarrow (\text{phase diff})$$

$$\boxed{\text{Intensity} \propto A^2}$$

if/ put $x = 0$

$$y_1 = A_1 \sin\omega t$$

$$y_2 = A_2 \sin(\omega t + \phi)$$

Superposition of SHM

$$y_{net} = y_1 + y_2$$

$$y_{net} = A_{net} \sin(\omega t + \alpha)$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$$

$$y_1 = A_1 \sin(\omega t - kx)$$
$$y_2 = A_2 \sin(\omega t - kx + \phi)$$

$$\left(\text{Phase difference} \equiv \phi \right)$$

$$\rightarrow \text{const}$$

$$y = a \sin\theta + b\cos\theta$$

$$y = \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2+b^2}} \sin\theta + \frac{b\cos\theta}{\sqrt{a^2+b^2}} \right)$$

$$y = \sqrt{a^2 + b^2} \left(\sin(\theta + \alpha) \right)$$

$$y = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx + \phi)$$

Let $kx - \omega t = B \Rightarrow$

$$y_{net} = A_1 \sin B + A_2 \sin(B + \phi)$$

$$= A_1 \sin B + A_2 \sin B \cos\phi + A_2 \cos B \sin\phi$$

$$= (A_1 + A_2 \cos\phi) \sin B + (A_2 \sin\phi) \cos B$$

$$y_{net} = (A_1 + A_2 \cos\phi) \sin(\omega t - kx) + (A_2 \sin\phi) \cos(\omega t - kx)$$

$$= a \sin(\omega t - kx) + b \cos(\omega t - kx)$$

$$\tan\alpha = \frac{b}{a}$$
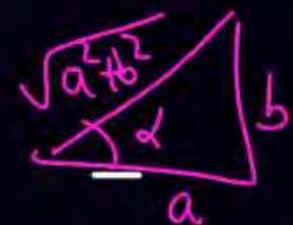
$$y_{net} = a\sin(\omega t - kx) + b\cos(\omega t - kx) = \sqrt{a^2 + b^2}\,\sin(\omega t - kx + \alpha)$$

$$= A_{net}\sin(\omega t - kx + \alpha)$$

$$a = A_1 + A_2\cos\phi$$

$$b = A_2\sin\phi$$

$$A_{net} = \sqrt{a^2 + b^2} = \sqrt{(A_1 + A_2\cos\phi)^2 + (A_2\sin\phi)^2} = \sqrt{A_1^2 + A_2^2\cos^2\phi + 2A_1A_2\cos\phi + A_2^2\sin^2\phi}$$

$$\boxed{A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}}$$

$\longrightarrow$ phase difference

$$\tan\alpha = \frac{b}{a} = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$

Result

$$y_1 = A_1 \sin(\omega t - kx)$$

$$y_2 = A_2 \sin(\omega t - kx + \phi)$$

$$y_{net} = A_{net} \sin(\omega t - kx + \alpha)$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$$

$$y_1 = A_1 \sin(\omega t - kx + \theta_1)$$

$$y_2 = A_2 \sin(\omega t - kx + \theta_2)$$

$$y_{net} = A_{net} \sin(\omega t - kx + \alpha)$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$$

$$\Delta\phi \equiv \phi = (\theta_2 - \theta_1)$$

Phase difference

SHM Superposition

$$y_1 = A_1 \sin(\omega t)$$

$$y_2 = A_2 \sin(\omega t + \phi)$$

$$y_{net} = A_{net} \sin(\omega t + \alpha)$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi} \longrightarrow \text{phase diff.}$$

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$I \propto A^2$$

$$\boxed{\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}}$$

path diff.

$$\Delta\phi \longrightarrow \text{phase diff.}$$

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda}\Delta x}$$

Constructive Interference

$$\boxed{\phi = 2n\pi}$$

$$A_{net} \longrightarrow max$$

$$I_{net} \longrightarrow max$$

$$\cos\phi = +1$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \times 1}$$

$$A_{net} = A_1 + A_2$$

$$I_{net} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$\text{If } A_1 = A_2 = A_0$$
$$I_1 = I_2 = I_0 \Big\} \longrightarrow A_{net} = 2A_0$$
$$I_{net} = 4I_0$$

$$\Delta\phi = 2n\pi$$

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

(C.I.)

$$\frac{2n\pi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\boxed{\Delta x = n\lambda}$$

Destructive Interference

$$\boxed{\phi = (odd)\pi}$$

$$A_{net} \longrightarrow min$$
$$I_{net} \longrightarrow min$$

$$\cos\phi = -1$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 - 2A_1A_2} = |A_1 - A_2|$$

$$I_{net} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$\text{If } A_1 = A_2 = A_0$$
$$I_1 = I_2 = I_0$$

$$A_{net} = 0$$
$$I_{net} = 0$$

$$\cos\phi = -1$$

$$\Delta\phi = (odd)\pi = (2n+1)\pi$$

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\frac{(2n+1)\pi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\Delta x = (2n+1)\frac{\lambda}{2}$$

$$\boxed{\Delta x = (odd)\lambda/2}$$

$* \quad \dfrac{\Delta \phi}{2\pi} = \dfrac{\Delta x}{\lambda}$

$* \quad A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$ — phase diff

$* \quad I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\phi$

$** \quad I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$* \quad I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\dfrac{A_1 + A_2}{A_1 - A_2}\right)^2$

$** \quad$ If $I_1 = I_2 = I_0 \Rightarrow \boxed{I_{net} = 4 I_0 \cos^2(\phi/2)}$

$* \quad$ If $\begin{matrix} A_1 = A_2 = A_0 \\ I_1 = I_2 = I_0 \end{matrix}\Bigg\} \longrightarrow$ (C.I.) $\begin{array}{l} I_{net} = 4 I_0 \\ A_{net} = 2 I_0 \end{array}$

$\longrightarrow$ (D.I) $\Rightarrow \begin{array}{l} A_{net} = 0 \\ I_{net} = 0 \end{array}$

If $I_1 = I_2 = I_0$

$I_{net} = I_0 + I_0 + 2\sqrt{I_0 I_0}\,\cos\phi$

$I_{net} = 2 I_0 \left(1 + \cos\phi\right)$

$\boxed{I_{net} = 4 I_0 \cos^2(\phi/2)}$

for C.I. $\Rightarrow \Delta x = n\lambda$

D.I $\Rightarrow \Delta x = \text{odd}\,\dfrac{\lambda}{2}$

initial phase same (μ)

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

S_1

x_1

S_2

x_2

P
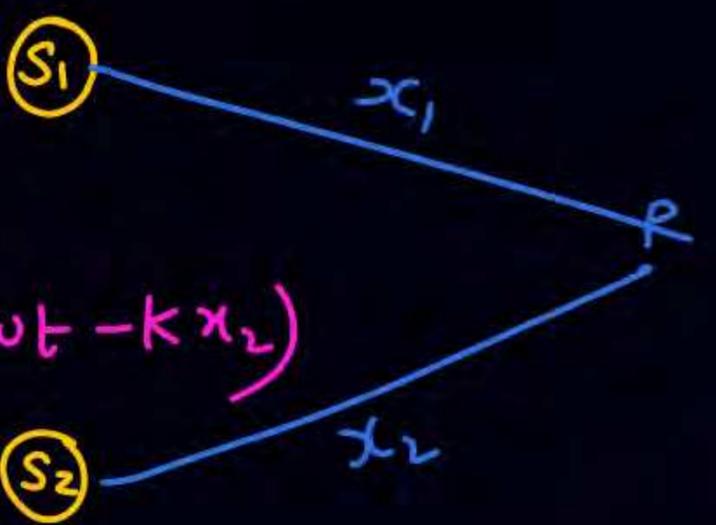
$$y_1 = A_1 \sin(\omega t - kx_1)$$

$$y_2 = A_2 \sin(\omega t - kx_2)$$

phase diff $= (\omega t - kx_1) - (\omega t - kx_2)$

$$\Delta\phi = k(x_2 - x_1)$$

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)}$$

Phase diff.   path diff

Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are :

I तथा 4I तीव्रता वाले दो कलासंबंद्ध एकवर्णीय प्रकाश पुंजों को अध्यारोपित किया जाता है। परिणामी पुंज की संभावित अधिकतम तथा न्यूनतम तीव्रताएं होगी

(A) 5I and I (B) 5I and 3I   (C*) 9I and I (D) 9I and 3I

$$I_1 = I$$
$$I_2 = 4I$$

$$I_{max} = \left(\sqrt{I} + \sqrt{4I}\right)^2 = 9I$$

$$I_{min} = \left(\sqrt{I} - \sqrt{4I}\right)^2 = I$$

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Two waves of sound having intensities I and 4I interfere to produce interference pattern. The phase difference between the waves is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is

दो ध्वनि तरंगें जिनकी तीव्रताऐं क्रमश: I व 4I है, व्यतिकरित होकर व्यतिकरण प्रतिरूप बनाती है। बिन्दु A पर तरंगों के बीच कलान्तर $\pi/2$ तथा बिन्दु B पर π है। तब बिन्दु A तथा B पर परिणामी तीव्रताओं के मध्य अन्तर होगा:-

(A) 2I   (B) 4I   (C) 5I   (D) 7I

$$\cos\pi = -1$$

Ans. (B)

Ⓐ

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$I_A = I + 4I + 2\sqrt{I \cdot 4I} \cdot \cos\pi/2$$

$$I_A = 5I$$

Ⓑ

$$I_{net} = I + 4I + 2\sqrt{I \cdot 4I}\cos\pi$$

$$= 5I - 4I = I$$

The ratio of maximum to minimum intensity due to superposition of two waves is $\frac{49}{9}$. Then the ratio of the intensity of component waves is :-

दो तरंगों के अध्यारोपण के कारण अधिकतम तथा न्यूनतम तीव्रता का अनुपात $\frac{49}{9}$ है। घटक तरंगों की तीव्रता का अनुपात होगा:-

(A) $\frac{25}{4}$

(B) $\frac{16}{25}$

(C) $\frac{4}{49}$

(D) $\frac{9}{49}$

Ans. (A)

$$\frac{I_{max}}{I_{min}} = \frac{49}{9} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$$

$$\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{10}{4} = \frac{5}{2}$$

$$\frac{7}{3} = \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}$$

$$\frac{I_1}{I_2} = \frac{25}{4}$$

Two coherent sources emit light waves which superimpose at a point where these can be expressed as

$$E_1 = E_0 \sin(\omega t + \pi/4) \equiv I$$

$$E_2 = 2E_0 \sin(\omega t - \pi/4) \equiv 4I$$

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$= I + 4I + 2\sqrt{I \cdot 4I}\cos 90$$
$$= 5I$$

Here, E_1 and E_2 are the electric field strengths of the two waves at the given point.

If I is the intensity of wave expressed by field strength E_1, find the resultant intensity.

Three coherent waves of equal frequencies having amplitude 10 μm, 4μm and 7 μm respectively, arrive at a given point with successive phase difference of π/2. The amplitude of the resulting wave in μm is given by

समान आवृत्ति परन्तु 10 μm, 4μm तथा 7 μm आयाम वाली तीन कलासंगत तरंगें π/2 के क्रमागत कलान्तर से एक बिन्दु पर मिलती है। परिणामी तरंग आयाम μm में होगा:-
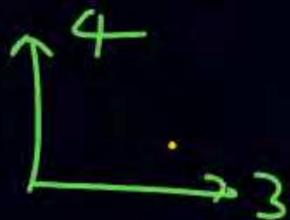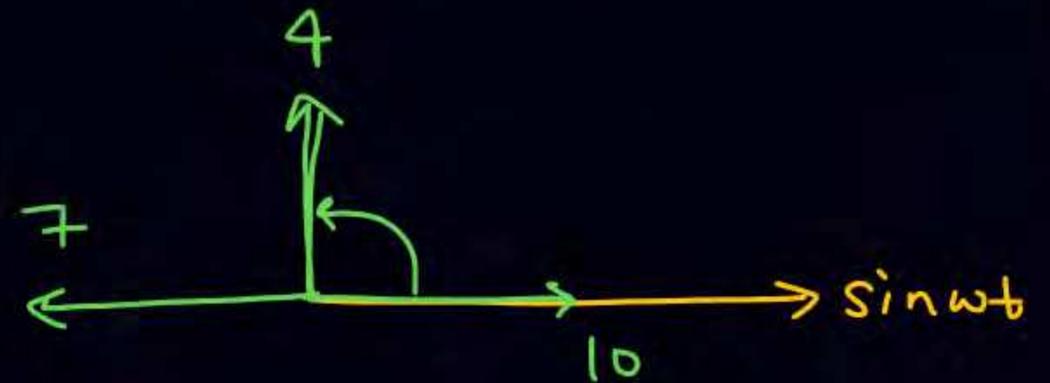
(A) 5   (B) 6   (C) 3   (D) 4

$$y_1 = 10 \sin(\omega t - kx)$$
$$y_2 = 4 \sin(\omega t - kx + 90°)$$
$$y_3 = 7 \sin(\omega t - kx + 180°)$$

$$y_{nt} = y_1 + y_2 + y_3 =$$

$$x = 0$$

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π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is.

बराबर आवृत्तियों तथा तीव्रता I_0 की चार आवर्त तरंगों की कला के कोण 0, π/3, 2π/3 तथा π है। जब इन तरंगों को अध्यारोपित (superpose) किया जाता है तो परिणामी तरंग की तीव्रता nI_0 है। तब n का मान है।

[JEE-Advance-2015]

Ans. 3

$x = 0$

$y_1 = A \sin(\omega t + 0)$

$y_2 = A \sin(\omega t + 60)$

$y_3 = A \sin(\omega t + 120)$

$y_3 = A \sin(\omega t + 180)$

$A\sqrt{3}$

$I \propto (A\sqrt{3})^2$
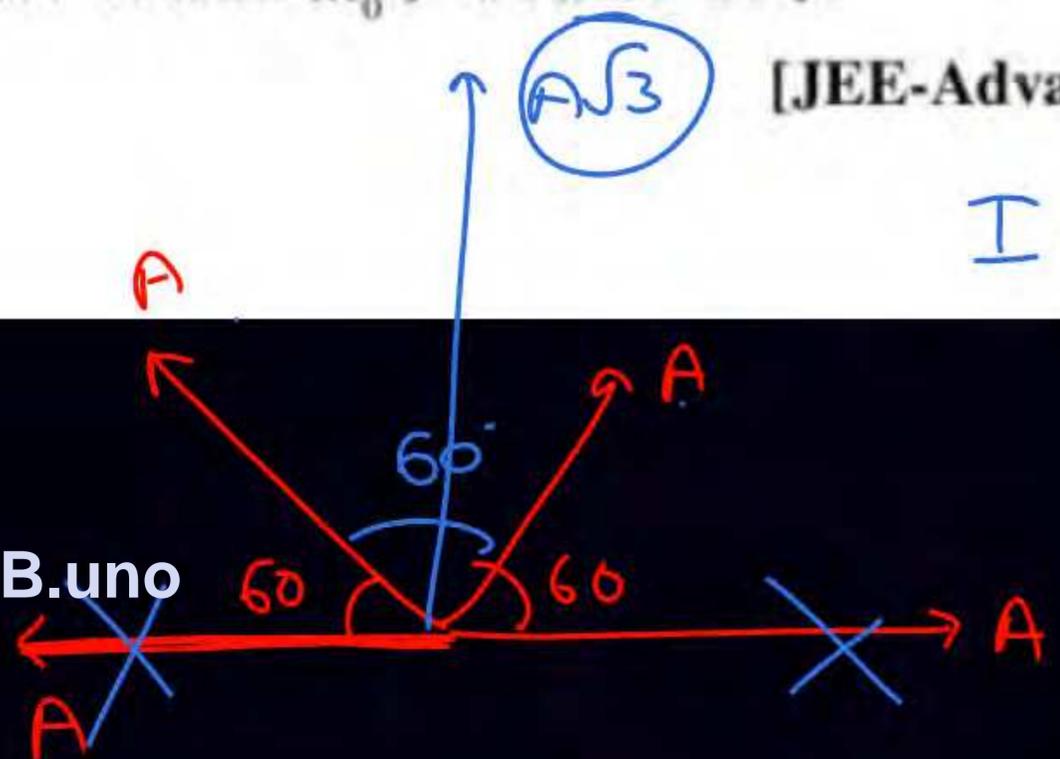
1. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \dfrac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

[IIT-JEE 2010]

जब दो प्रगामी तरंगे $y_1 = 4\sin(2x - 6t)$ तथा $y_2 = 3\sin\left(2x - 6t - \dfrac{\pi}{2}\right)$ अध्यारोपित होती है तो परिणामी तरंग का आयाम कितना होगा ?

$$\sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \times \cos \pi/2} \qquad = 5$$

Ans. 5

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All sides of the polygon are equal and of length 1m. The intensity of sound at M due to both the individual sources is I_0. What will be the intensity of sound at point M when both the sources are on.

दो ध्वनि स्त्रोत 1 m तरंगदैर्ध्य वाली ध्वनि उत्सर्जित कर रहे है तथा चित्रानुसार बिन्दु P व Q पर स्थित है। बहुभुज की सभी भुजायें 1m लम्बी है। दोनों अलग–अलग स्त्रोतों के कारण M पर ध्वनि की तीव्रता I_0 है। जब दोनों स्त्रोत चालू कर दिये जाते है तो बिन्दु M पर ध्वनि की तीव्रता होगी:–

(A) $4I_0$ (B) $2I_0$ (C) I_0 (D) $(1/2)I_0$

Ans. (A)

wavelength of sound emitted by them. Number of maxima located on the elliptical boundary around it will be :

S_1, S_2 दो कला सम्बद्ध ध्वनि स्त्रोत हैं, जो x–अक्ष के अनुदिश 4λ दूरी पर स्थित हैं। जहाँ λ उत्सर्जित ध्वनि की तरंगदैर्ध्य है। इनके चारों ओर एक दीर्घवृताकार परिसीमा पर स्थित उच्चिष्टों की संख्या होगी :–
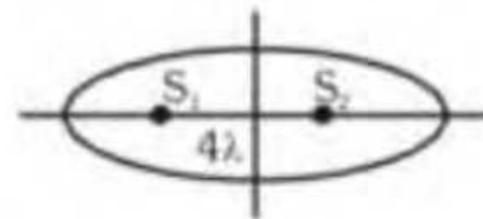
(A) 16   (B) 12   (C) 8   (D) 4

Home Work

— Complete your notes & backlogs