
PRAYAS
JEE 2025
Lecture - 01
Physics
Gravitation
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Topics to be covered

1. Gravitation
2.
3.
4.

Gravitation

$$q \longrightarrow m$$

$$F = \frac{K q_1 q_2}{r^2} \longrightarrow F = \frac{G m_1 m_2}{r^2}$$

$$k \longrightarrow G$$

$$\frac{1}{4\pi\epsilon_0} \longrightarrow G$$

$$\epsilon_0 \longrightarrow \frac{1}{4\pi G}$$

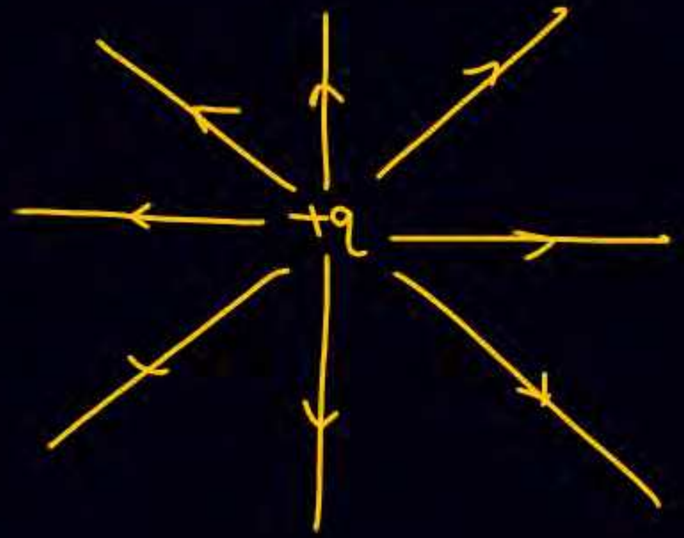
Electrost.

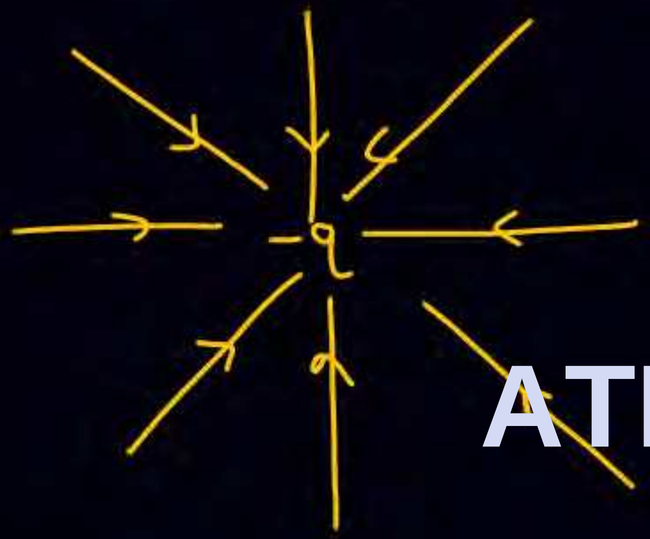
Gravitation

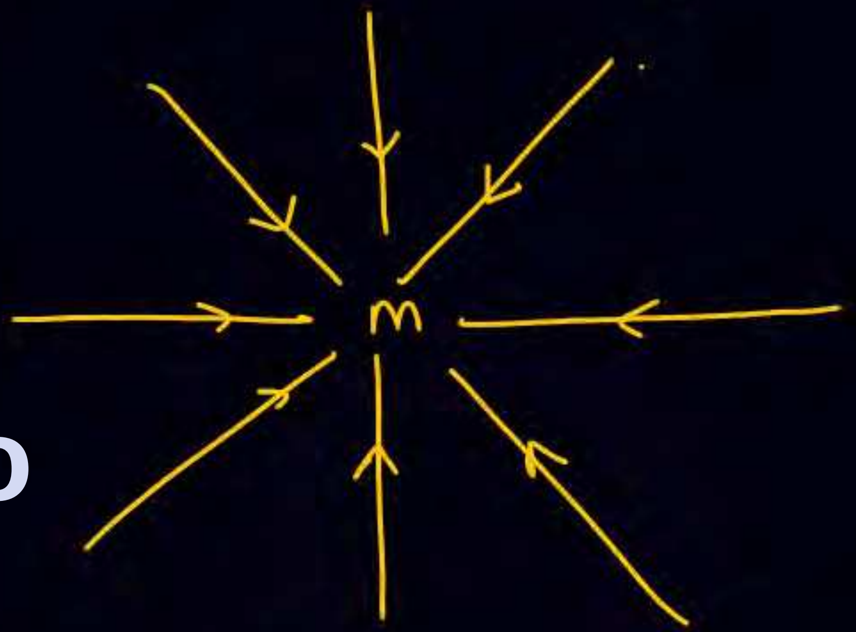
* Attraction & Repulsion

\- Only attraction

* Electric field $E = \frac{F}{q_0}$ → Gravitational field $E_g = \frac{F}{m_0}$







* $\vec{F} = q_0 \vec{E}$

* $\vec{F} = m \vec{E_g}$

Electrost.

* $q_1 \xleftrightarrow{\quad r \quad} q_2$

$$U = \frac{K q_1 q_2}{r}$$ with sign

* $U = qV$

$$V = \frac{U}{q}$$

$$\Delta V = \frac{\Delta U}{q}$$

Electric potential

Gravitation

Gravitational potential =

only Attraction

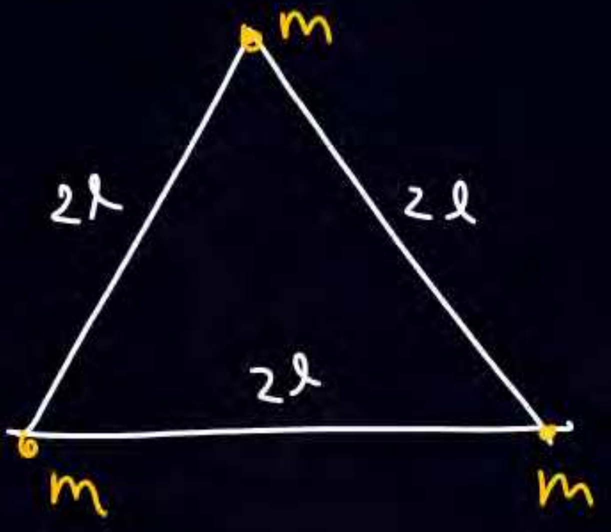
$m_1 \xleftrightarrow{\quad r \quad} m_2$

$$U = -\frac{G m_1 m_2}{r}$$

नहीं भूलना है

Q

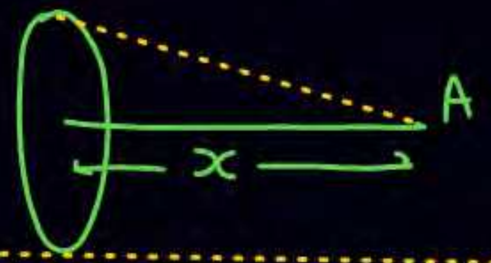
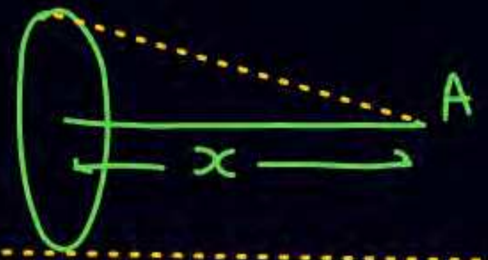
$\xrightarrow{(WD)_{ext}}$



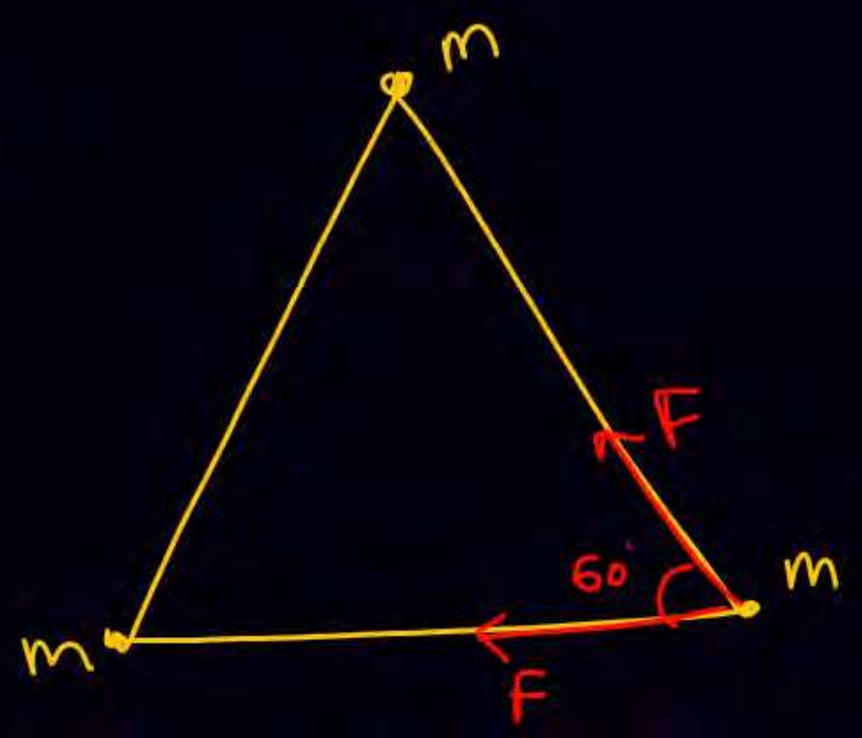
$$U_i = -\frac{Gmm}{l} \times 3$$

$$U_f = -\frac{Gmm}{2l} \times 3$$

$$(WD)_{ext} = U_f - U_i$$

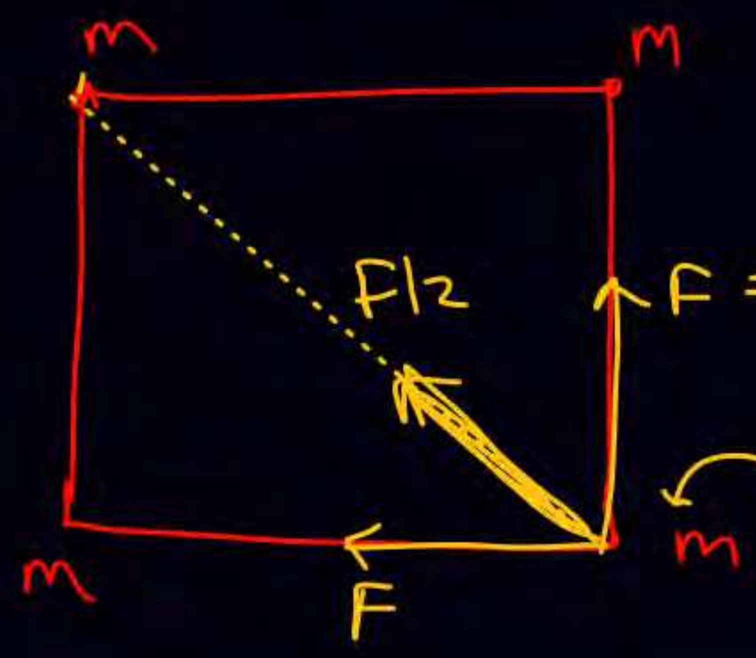
	E_g	E.F (charge)	Grav. potential	Electric pot
① point mass	$E_g = \frac{Gm}{r^2}$	$\frac{KQ}{r^2}$	$V = -\frac{Gm}{r}$	$V = \frac{Kq}{r}$
② Ring (Axis)	$\frac{Gmx}{(R^2+x^2)^{3/2}}$ 	$\frac{KQx}{(R^2+x^2)^{3/2}}$	$V = -\frac{Gm}{(r^2+x^2)^{\frac{1}{2}}}$ 	$\frac{Kq}{(r^2+x^2)^{1/2}}$
③ Disc	$\frac{\sigma}{2} 4\pi G (1-\cos\theta)$	$\frac{\sigma}{2\epsilon_0}(1-\cos\theta)$	$-\frac{\sigma}{2} 4\pi G (1-\cos\theta)\sqrt{R^2+x^2}$	$\frac{\sigma}{2\epsilon_0}(1-\cos\theta)\sqrt{R^2+x^2}$
④ ∞ sheet	$\frac{\sigma}{2} 4\pi G$	$\frac{\sigma}{2\epsilon_0}$	$\lvert\Delta V\rvert = -\frac{\sigma}{2} 4\pi G (r_2 - r_1)$	$\frac{\sigma}{2\epsilon_0}(r_{\text{बड़ा}} - r_{\text{छोटा}})$
⑤ ∞ wire	$\frac{2G\lambda}{r}$	$\frac{2K\lambda}{r}$	$\Delta V = -2G\lambda \ln\left(\frac{r_2}{r_1}\right)$	$\Delta V =$ $\Delta V = 2K\lambda \ln\left(\frac{r_2}{r_1}\right)$

Q



$F_{net} = F\sqrt{3}$

Q

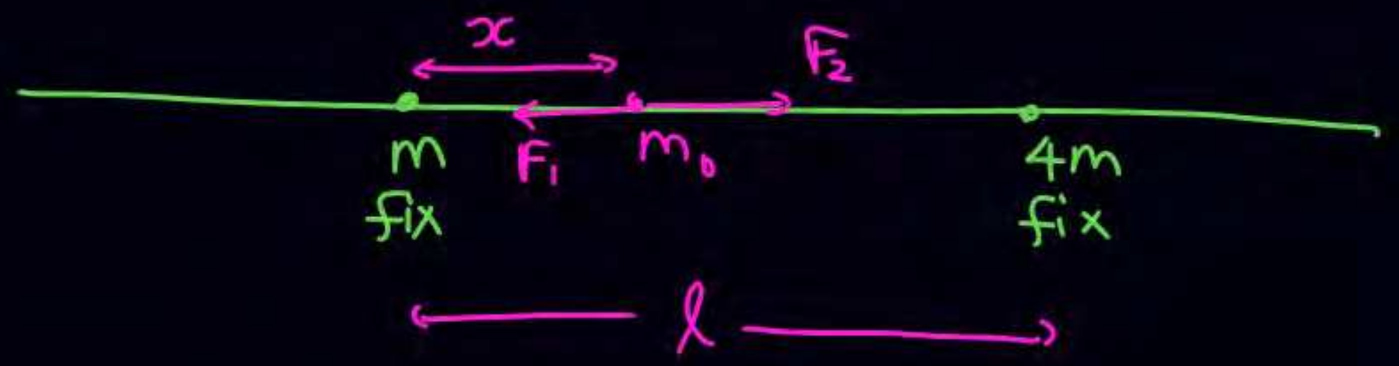


$F = Gmm/r^2$

$F_{net} = ?$

$F_{net} = F\sqrt{2} + F/2$

Q



$$F_1 = F_2$$

$$\frac{G m m_0}{x^2} = \frac{G 4m m_0}{(l-x)^2}$$

Grav.

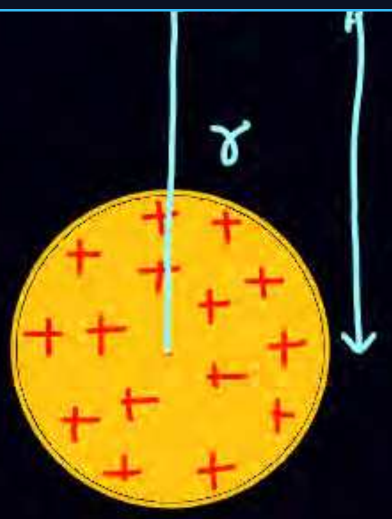
hollow sphere

Inside $E = 0$ $V = KQ/R$ $\longrightarrow$ Inside $E_g = 0$, $V = -\frac{Gm}{R}$

outside $E = \frac{KQ}{r^2}$, $V = \frac{KQ}{r}$ $\longrightarrow$ Outside $E_g = \frac{Gm}{r^2}$

$V = -\frac{Gm}{r}$

Solid charged sphere



Outside

$$E_A = \frac{KQ}{r^2}$$

$$V_A = \frac{KQ}{r}$$

Inside

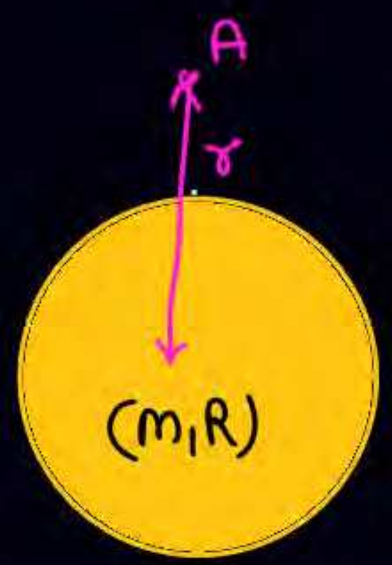
$$E = \frac{\rho r}{3\epsilon_0}, \quad \frac{KQr}{R^3}$$

$$V = \frac{KQ}{2R^3}(3R^2 - r^2)$$

At center $E = 0, \quad V = \frac{3}{2}\frac{KQ}{R}$

Cavity $\equiv E = \frac{\rho \overrightarrow{O_1O_2}}{3\epsilon_0}$

Solid sphere (Earth)

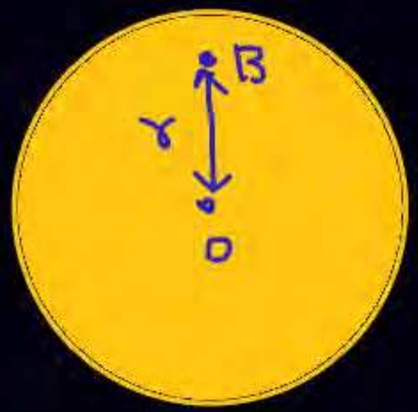


① outside $(E_A)_g = \frac{Gm}{r^2}$

$$V = -\frac{Gm}{r}$$

$$V_{surface} = -\frac{Gm}{R}$$

② Inside



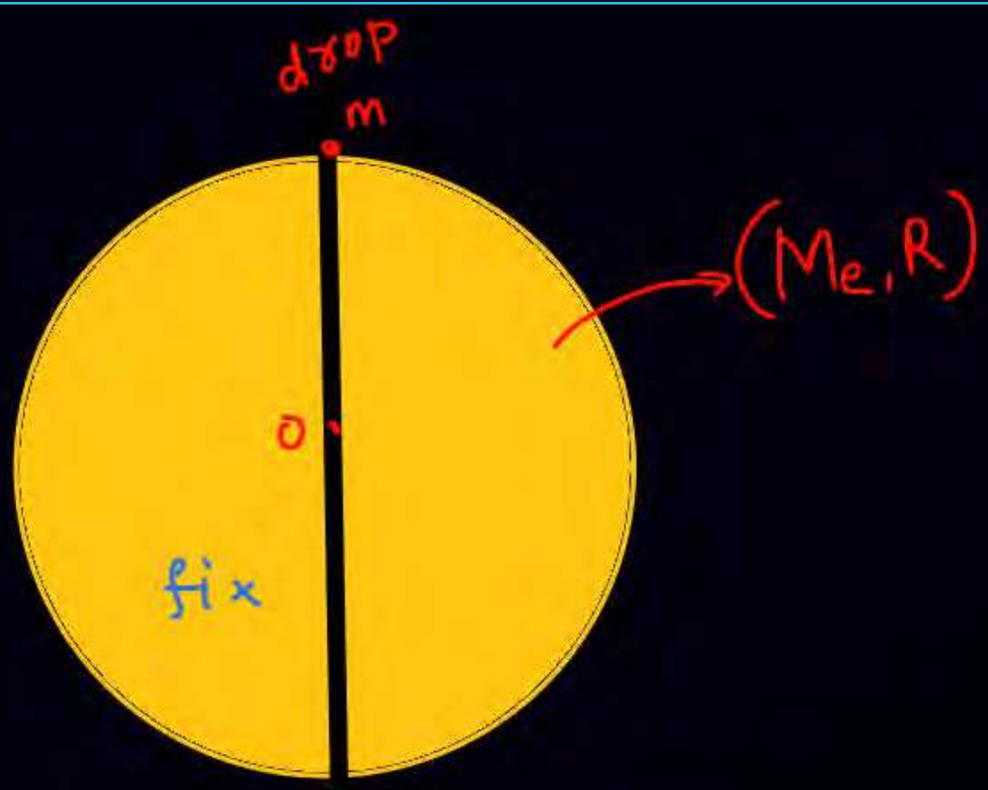
$$(E_B) = \frac{Gmr}{R^3}$$

$$V = -\frac{GM}{2R^3}(3R^2 - r^2)$$

At center $E_g = 0, \quad V = -\frac{3}{2}\frac{Gm}{R}$

Cavity $\equiv E = \frac{\rho(O_1O_2)}{3} 4\pi G$

Q



Find speed of mass m when it reaches at center.

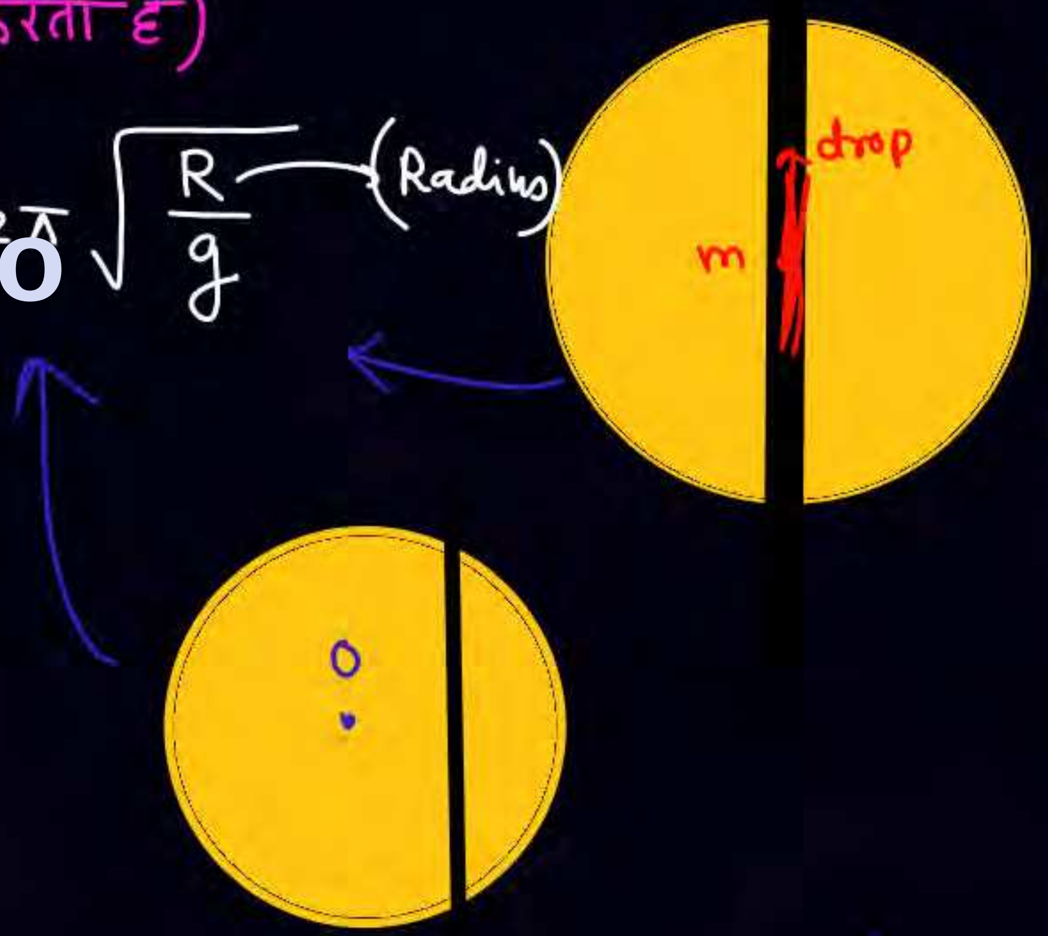
Sol

$$K_i + U_i = K_f + U_f$$

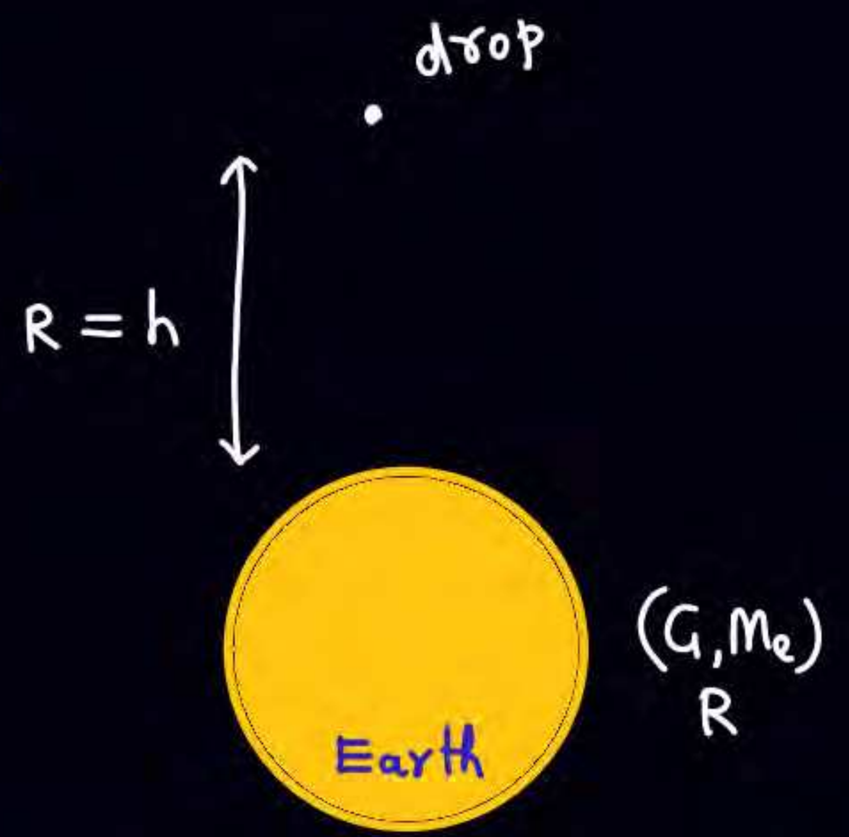
$$0 + m\left(-\frac{GM_e}{R}\right) = \frac{1}{2}mv_0^2 + m\left[-\frac{3}{2}\frac{GM_e}{R}\right]$$

SHM (करता है)

$$T = 2\pi\sqrt{\frac{R}{g}}$$ (Radius)



Q



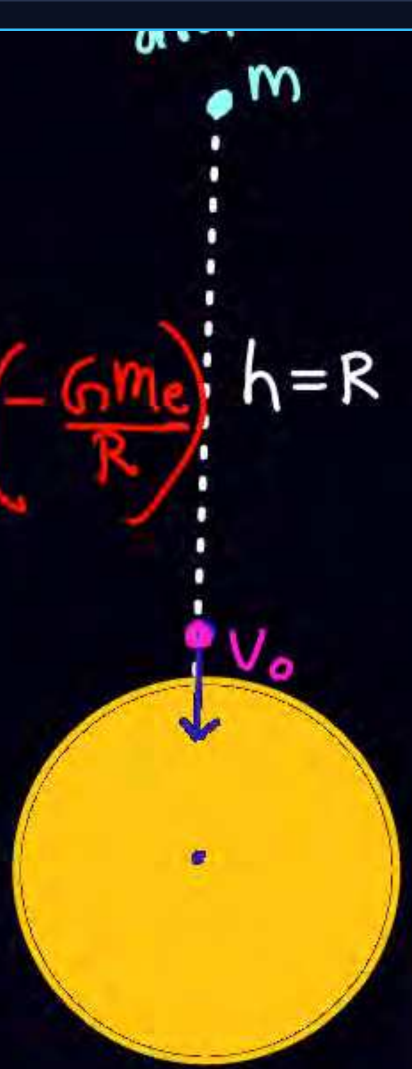
find speed of particle when it hit the surface of earth.

Sol^n

$$K_i + U_i = K_f + U_f$$

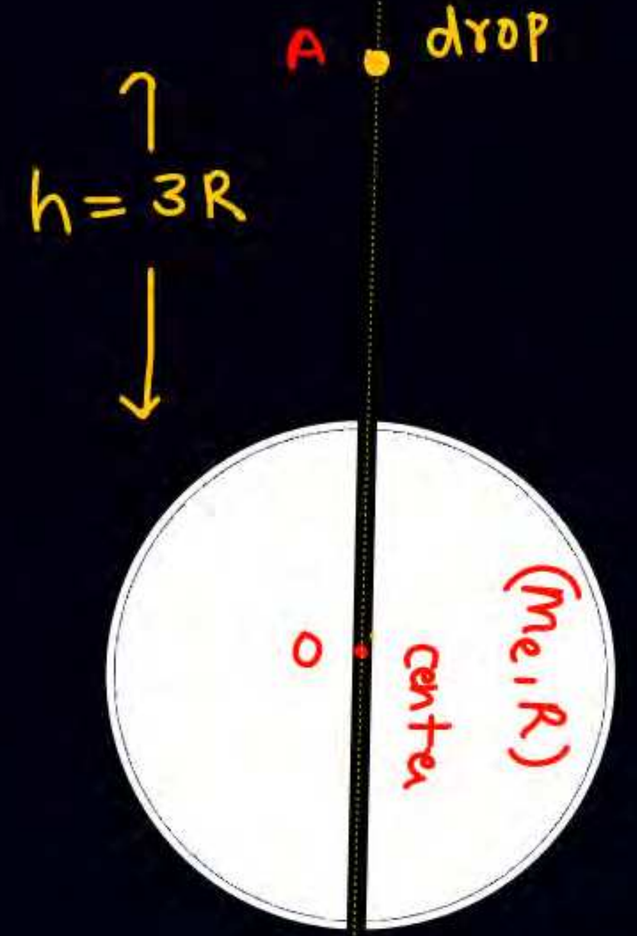
$$0 + m\left(-\frac{GM_e}{h+R}\right) = \frac{1}{2}mV_f^2 + m\left(-\frac{GM_e}{R}\right)$$

$$0 - m\frac{GM_e}{2R} = \frac{1}{2}mV_f^2 - m\frac{GM_e}{R}$$



Q

find speed of mass 'm' when it reaches at center of earth



Sol^n

$$K_A + U_A = K_O + U_O$$

$$0 + m\,(V_A) = \frac{1}{2} m v_0^2 + m\,V_O$$

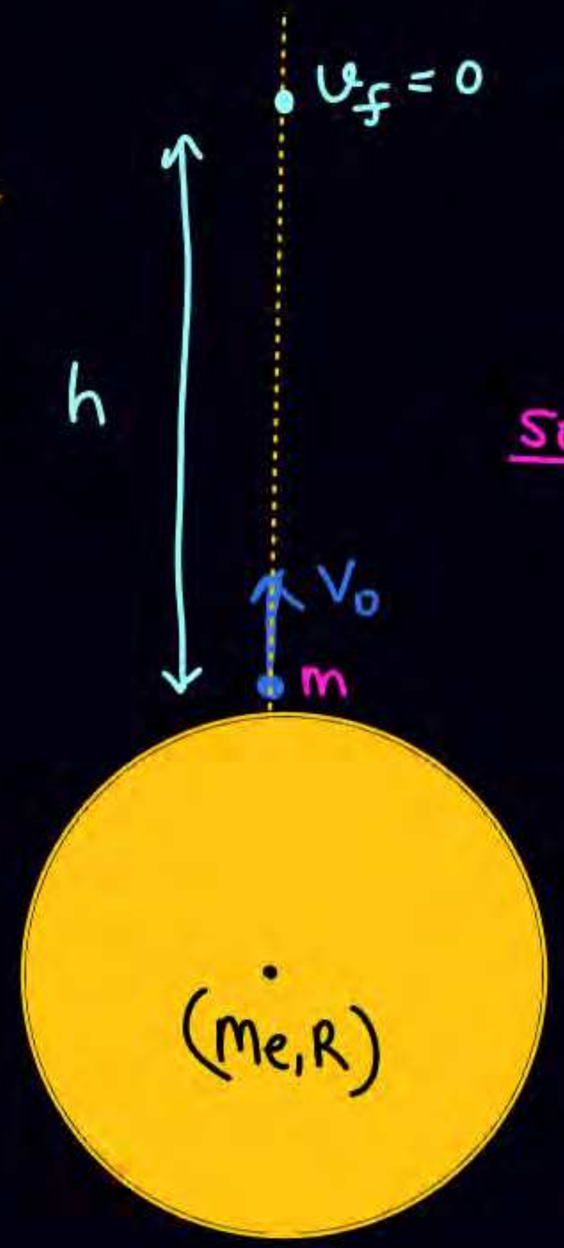
$$m \times \left(-\frac{GM_e}{4R}\right) = \frac{1}{2} m v_0^2 + m\left(-\frac{3}{2}\frac{GM_e}{R}\right)$$

$$\frac{1}{2} m v_0^2 = \frac{3}{2}\frac{GM_e m}{R} - \frac{1}{4}\frac{GM_e m}{R}$$

$$\frac{1}{2}\cancel{m} v_0^2 = \frac{5}{4}\frac{GM_e \cancel{m}}{R}$$

$$\boxed{V = \sqrt{\frac{5GM_e}{2R}}}$$

Q find h_{max} from surface of earth.



sol^n

$$\frac{1}{2}mV_0^2 + m\left(-\frac{Gm_e}{R}\right)$$

$$= 0 + m\left(-\frac{Gm_e}{R+h}\right)$$

Q Find min value of V_0 required so that particle reaches at ∞

sol^n

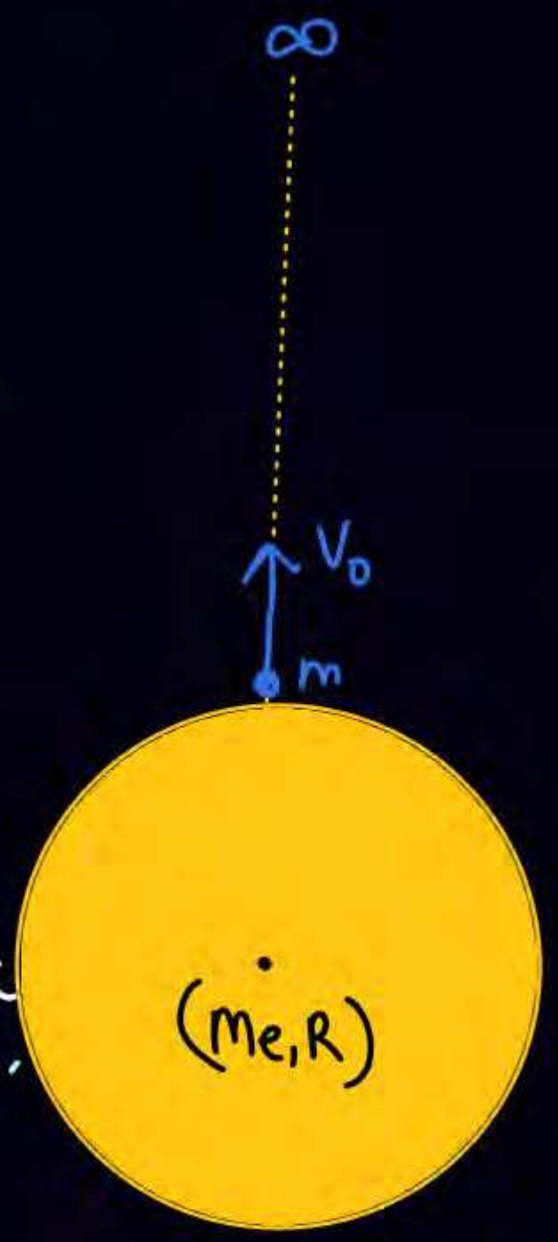
$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + m\left(-\frac{Gm_e}{R}\right) = 0 + 0$$

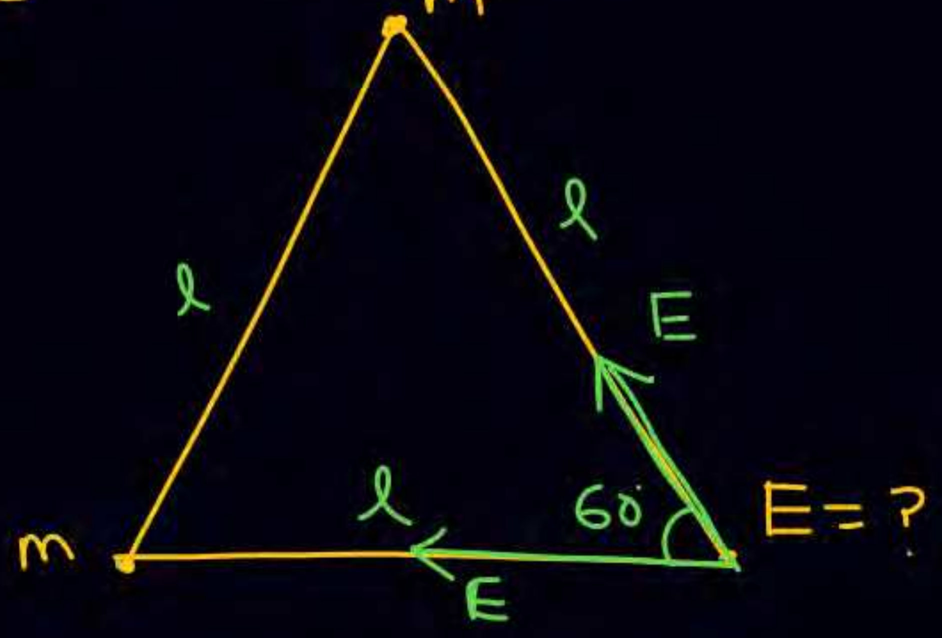
$$V_0 = \sqrt{\frac{2Gm_e}{R}} = V_{escape}$$

$$= 11.2\ km/sec$$

$$V_0 = \sqrt{\frac{2Gm_e \cdot R}{R^2}} = \sqrt{2g_0R}$$



Q



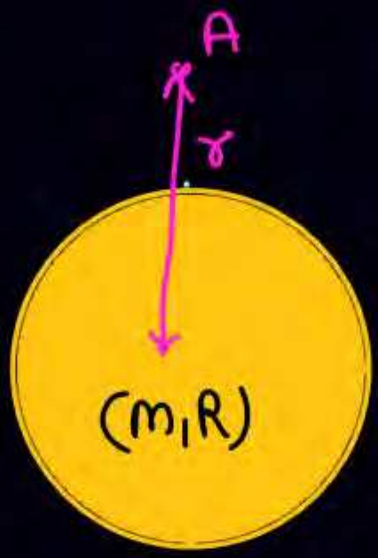
$$E_{net} = E\sqrt{3}$$

Q

$$(E_g)_A = 4E\sin\theta$$

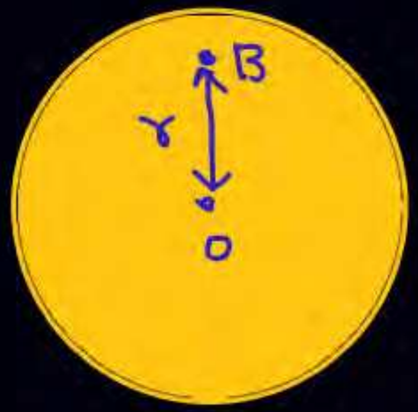
solid sphere (Earth)

① outside. $(E_A)_g = \frac{Gm}{r^2}$



② Inside

$(E_B)_g = \frac{Gmr}{R^3}$

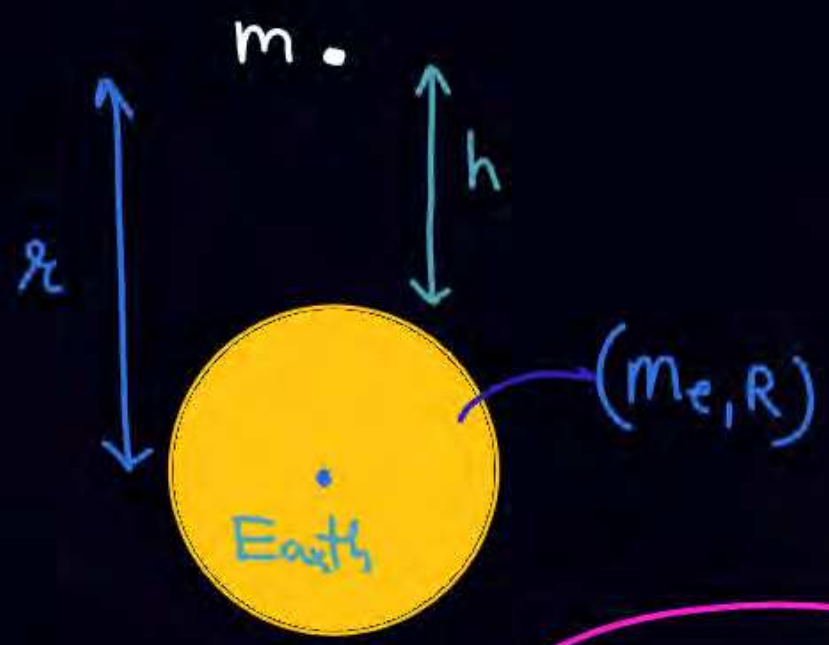


At center $\quad E_g = 0$

$\vec{F} = q\vec{E} \longrightarrow \vec{F} = m\vec{E}_g$

$F = m\frac{GM_e}{r^2}$

$F = \frac{GM_e m}{(R+h)^2}$



If h is very very small

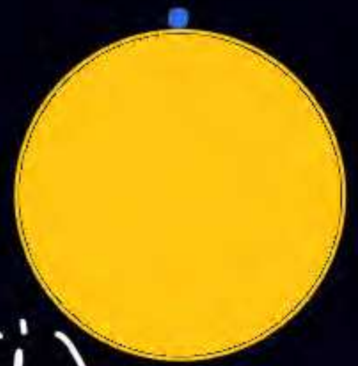
$F = \frac{GM_e}{R^2} m = gm = mg$

$g = \frac{GM_e}{R^2} = 9.8\, m/s^2$

Variation of E_g (Grav-field) or g' (acc due to gravity) due to earth

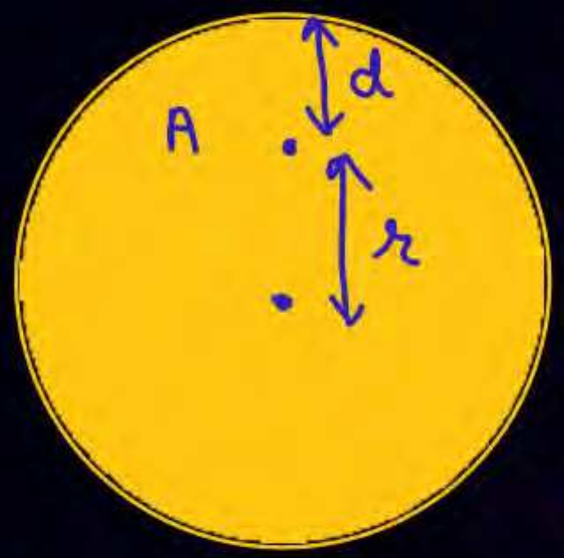
① At surface

$$E_g = \frac{GM_e}{R^2} = g_0$$



$$g_0 = \frac{GM_e}{R_e^2} = 9.8\ m/s^2$$

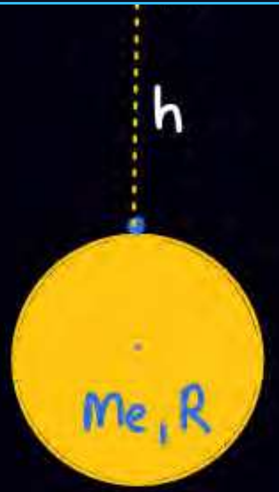
② Inside (at depth 'd' from surface)



$$E_g = \frac{GMr}{R^3} = \frac{Gm}{R^3}(R-d) = \frac{Gm}{R^2}\left(1-\frac{d}{R}\right)$$

$$E_g = g_{inside} = g_0\left(1-\frac{d}{R}\right)$$

③ At height



$$E_g = \frac{GM}{r^2} = \frac{Gm}{(R+h)^2}$$

If $h << R$ (Approximation)

$$E_g = g' = Gm(R+h)^{-2}$$

$$= \frac{Gm}{R^2}\left(1+\frac{h}{R}\right)^{-2}$$

$$= \frac{Gm}{R^2}\left(1-\frac{2h}{R}\right)$$

$$\boxed{g' = g_0\left(1-\frac{2h}{R}\right)}$$

काम का डब्बा

① * $g_{surface} = g_0 = \frac{GM}{R^2}$

② * $g_{inside} = g_0\left(1 - \frac{d}{R}\right)$ depth

③ * At height $\Rightarrow g = \frac{GM}{(R+h)^2}$

$g = g_0\left(1 - \frac{2h}{R}\right)$, when $(h << R)$

If rotation of earth is Consider

④ $g = g_0 - R\omega^2\cos^2\theta$ $\left(\begin{array}{ll}\theta = 90^\circ & \text{pole} \\ \theta = 0 & \text{Equator}\end{array}\right)$

⑤ Escape Velocity $\frac{1}{2}mv_0^2 - m\frac{GM}{R^2} = 0 + 0$

$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2g_0R}$

Q

planet A — planet B

(M, R) — (8M, 2R)

① $$\left(\frac{g_A}{g_B}\right)_{\text{at surface}} = \frac{\left(\frac{GM}{R^2}\right)}{\frac{G\,8M}{(2R)^2}} = \frac{1}{2}$$

② $$\frac{(V_{escape})_A}{(V_{escape})_B} = \frac{\sqrt{\frac{2GM}{R}}}{\sqrt{\frac{2G\,8M}{2R}}} = \frac{1}{2}$$

When rotation of earth is considered

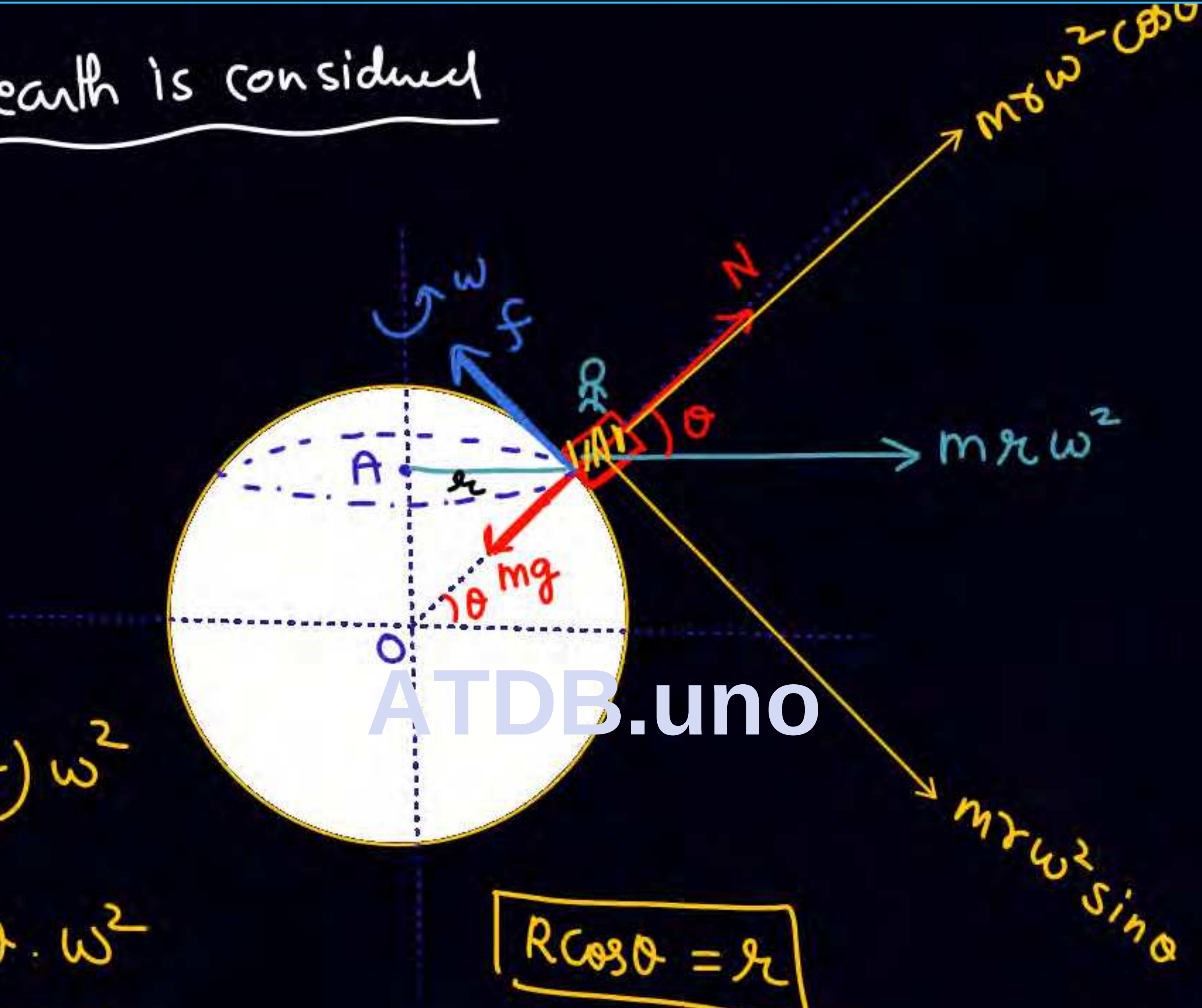
$$mg = N + mr\omega^2\cos\theta$$

$$N = mg - mr\omega^2\cos\theta$$

$$mg_{eff} = mg - m(R\cos\theta)\omega^2$$

$$g_{eff} = g - R\cos^2\theta\cdot\omega^2$$

$$\boxed{g_{eff} = g_0 - R\omega^2\cos^2\theta}$$



$$\boxed{R\cos\theta = r}$$

ques

At pole $\theta = 90^\circ$

$$g_{eff} = g_0$$

At equator $\theta = 0$

$$g_{eff} = g' = g_0 - R\omega^2$$

As we move from equator to pole value of 'g' increases.

dipole $\longrightarrow$ X

$(-q, +q)$ $\longrightarrow$ $+m$,

negative mass $\longrightarrow$ Does not exist

Conductor $\longrightarrow$ X

Attraction & repulsion


THANK
YOU